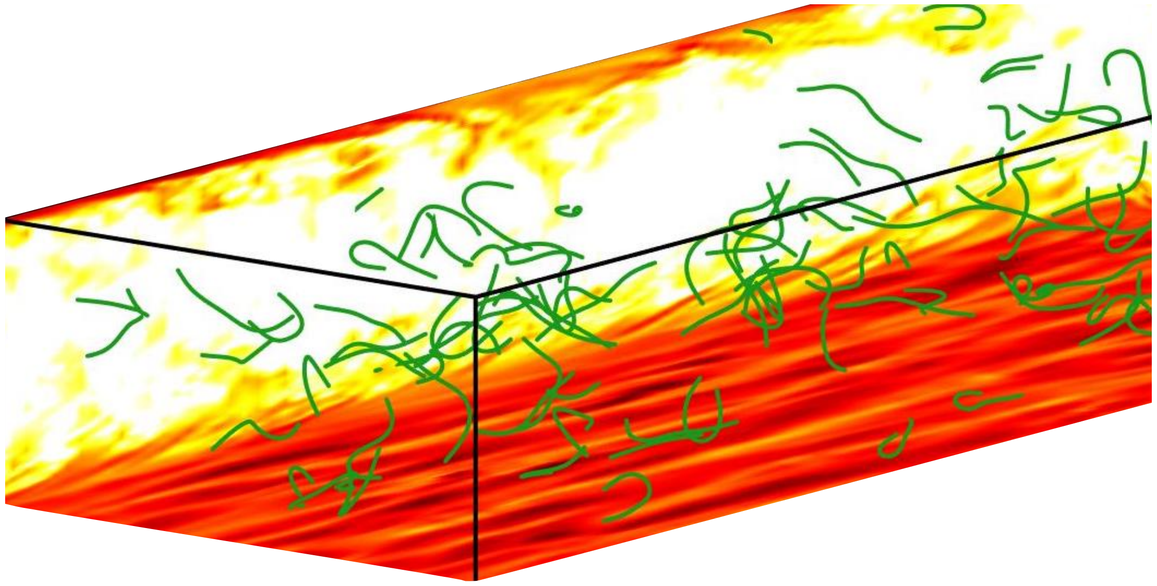
Inertia and flexibility in the turbulent transport of elongated particles



Jérémie Bec

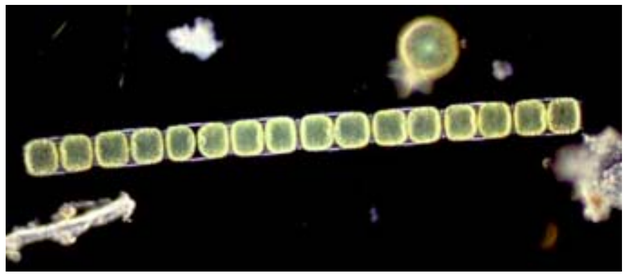
CNRS, Institut de Physique de Nice (InPhyNi), France

& Inria Université Côte d'Azur, Sophia Antipolis, France

with Sofía Allende, Christophe Brouzet, Christophe Henry, De-Fa Sun

Dynamics of elongated particles

Water intakes
(power plants)



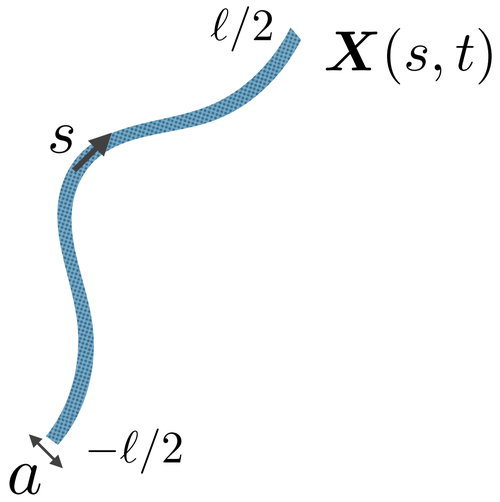
Diatoms colonies
(CO_2 oceanic pump)

Cellulose fibres
(paper industry)

Suspended in turbulent flow
Need for large-scale, long-term predictions

⇒ Need for macroscopic models for **deformable**, **non-ellipsoidal** particles

Cosserat equation & slender body theory



$$\sigma \partial_t^2 \boldsymbol{X} = -\frac{4\pi\,\mu}{\log\lambda}\mathbb{K}\left[\partial_t \boldsymbol{X} - \boldsymbol{u}(\boldsymbol{X},t)\right] + \boldsymbol{F}_{\text{int}} + \sigma \boldsymbol{g}$$

Viscous drag *Internal forces*

(Cox 1970, Keller & Rubinow 1976)

- $\mathbb{K} = \mathbb{I} - (1/2)\,\partial_s \boldsymbol{X}\,\partial_s \boldsymbol{X}^{\mathsf{T}}$: anisotropic drag tensor
- $\lambda = \ell/a \gg 1$: aspect ratio

$$\boldsymbol{F}_{\text{int}} = \partial_s(T\,\partial_s \boldsymbol{X}) - E\,\partial_s^4 \boldsymbol{X}$$

Euler-Bernoulli beam

Tension *Bending elasticity*

- $T(s,t)$ such that $|\partial_s \boldsymbol{X}|^2 = 1$ inextensibility $T|_{\pm\ell/2} = 0$

+ Free-end boundary conditions $\partial_s^2 \boldsymbol{X}\big|_{\pm\ell/2} = 0 \qquad \partial_s^3 \boldsymbol{X}\big|_{\pm\ell/2} = 0$

Parameters and asymptotics

Parameters:

Length	Stokes number	Flexibility	Froude number
$\Lambda = \dfrac{\ell}{\ell_\mathrm{f}}$	$St = \dfrac{\sigma \log\lambda}{4\pi\,\mu\,\tau_\mathrm{f}}$	$\mathcal{F} = \dfrac{4\pi\,\mu\,\ell^4}{\tau_\mathrm{f} E \sigma \log\lambda} = \left(\dfrac{\ell}{\ell_\mathrm{e}}\right)^4$	$Fr = \dfrac{\ell}{\tau_\mathrm{f}^2 g}$

ℓ_e elastic length *(Brouzet et al. 2014)*

Small-size: $\Lambda \to 0$ $$\partial_t^2 \boldsymbol{X} = -\frac{1}{St}\mathbb{K}\left[\partial_t \boldsymbol{X} - \mathbb{A}\,\boldsymbol{X}\right] + \partial_s(T\,\partial_s \boldsymbol{X}) - \frac{1}{St\,\mathcal{F}}\,\partial_s^4 \boldsymbol{X} + \frac{1}{Fr}\hat{\boldsymbol{g}},\ \mathbb{A} = \nabla \boldsymbol{u}$$

Rigid: $\mathcal{F} \to 0$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\bar{\boldsymbol{X}} = -\frac{1}{St}\left(\mathbb{I} - \tfrac{1}{2}\,\boldsymbol{p}\,\boldsymbol{p}^\top\right)\left[\frac{\mathrm{d}}{\mathrm{d}t}\bar{\boldsymbol{X}} - \frac{1}{\ell}\int_{-\ell/2}^{\ell/2} \boldsymbol{u}(\bar{\boldsymbol{X}} + s\,\boldsymbol{p}, t)\,\mathrm{d}s\right] + \frac{1}{Fr}\hat{\boldsymbol{g}}$$

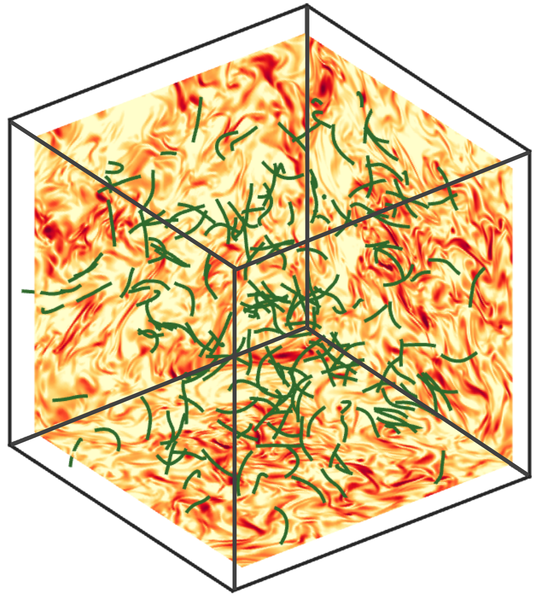
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{p} = \boldsymbol{\Omega} \times \boldsymbol{p}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\Omega} = -\frac{1}{St}\left[\boldsymbol{\Omega} - \frac{12}{\ell^3}\int_{-\ell/2}^{\ell/2} s\,\boldsymbol{p} \times \boldsymbol{u}(\bar{\boldsymbol{X}} + s\,\boldsymbol{p}, t)\,\mathrm{d}s\right]$$

Over-damped: $St \to 0$ $$\partial_t \boldsymbol{X} = \boldsymbol{u}(\boldsymbol{X}, t) + \mathbb{K}^{-1}\left[\partial_s(T\,\partial_s \boldsymbol{X}) - \mathcal{F}^{-1}\,\partial_s^4 \boldsymbol{X} + (St/Fr)\,\hat{\boldsymbol{g}}\right]$$

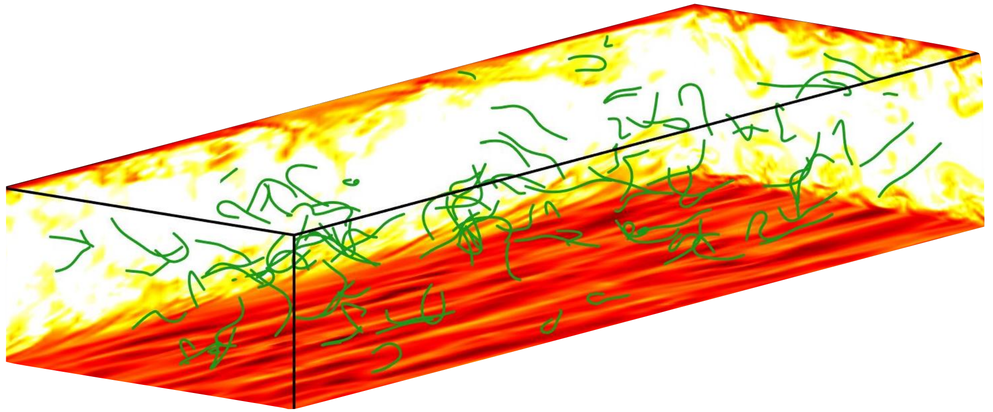
Coupling between dynamics and transport

How do alignment, tumbling, deformation depend on **length**, **flexibility**, and **inertia** and affect migration and transport properties?

in homogeneous, isotropic, developed turbulence?



in inhomogeneous, wall-bounded flow?

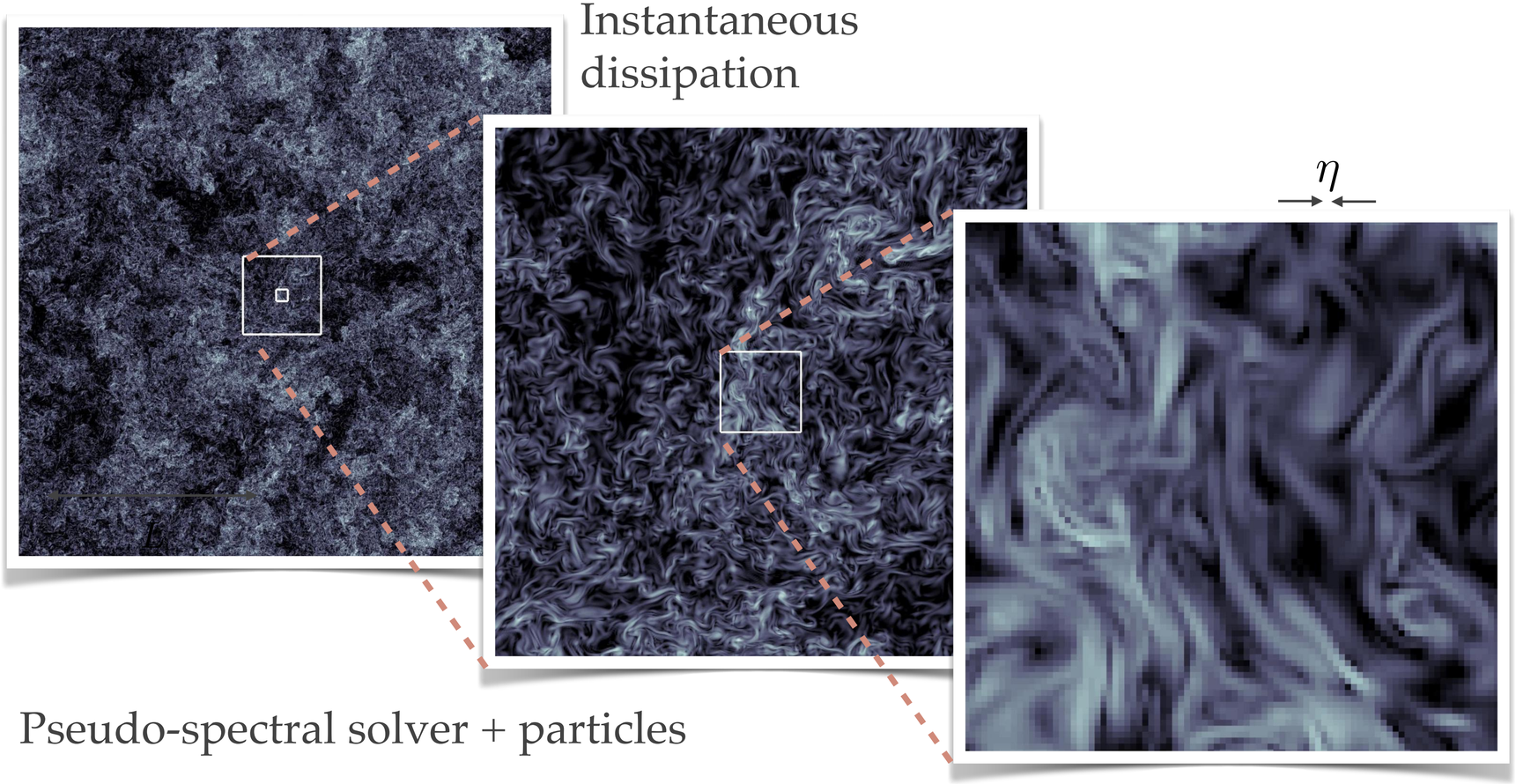


Case I: Homogeneous isotropic turbulence

Direct numerical simulations

Forced, periodic Navier–Stokes

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho_f}\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f},\ \nabla\cdot\boldsymbol{u} = 0$$



RMS velocity:

$u_{\mathrm{rms}} = \langle |\boldsymbol{u}|^2 \rangle / 3$

Dissipation rate:

$$\varepsilon = \frac{\nu}{2}\left\langle \|\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^\top\|^2 \right\rangle$$

Kolmogorov scales:

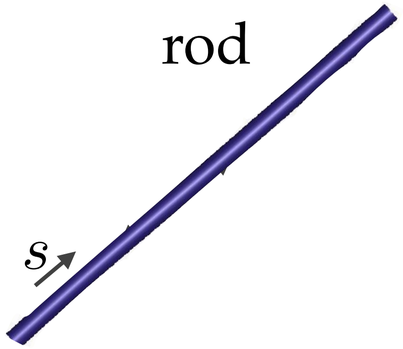
$\eta = \nu^{3/4}/\varepsilon^{1/4},\ \tau_\eta = \nu^{1/2}/\varepsilon^{1/2}$

Reynolds number:

$$Re_\lambda = u_{\mathrm{rms}}^2\sqrt{\frac{15}{\nu\varepsilon}} \approx 120-730$$

HIT: Short fibers are almost always straight

Both flow stretching and elastic bending act to straighten the fibre



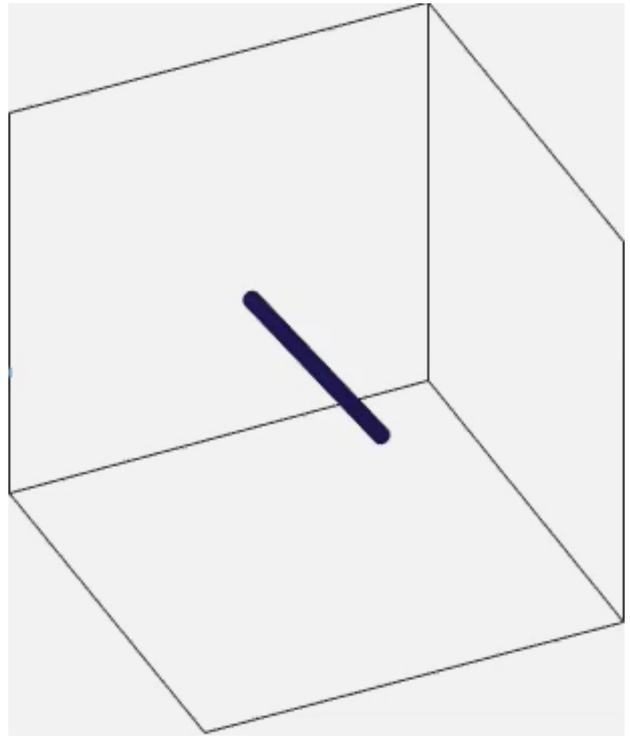
$$\partial_s \boldsymbol{X}(s,t) = \text{const} = \boldsymbol{p}(t)$$

for $\Lambda = \ell/\eta \ll 1$ and $St \ll 1$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{p} = \mathbb{A}\,\boldsymbol{p} - (\boldsymbol{p}^\top \mathbb{A}\,\boldsymbol{p})\,\boldsymbol{p}$$

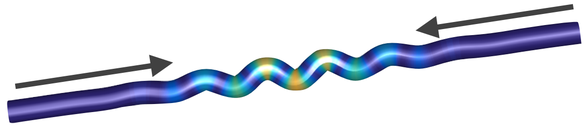
$\Rightarrow$ aligns with the eigendirection associated with the largest Lyapunov

(Parsa et al. 2011, Ni et al. 2014, Cui et al. 2020)



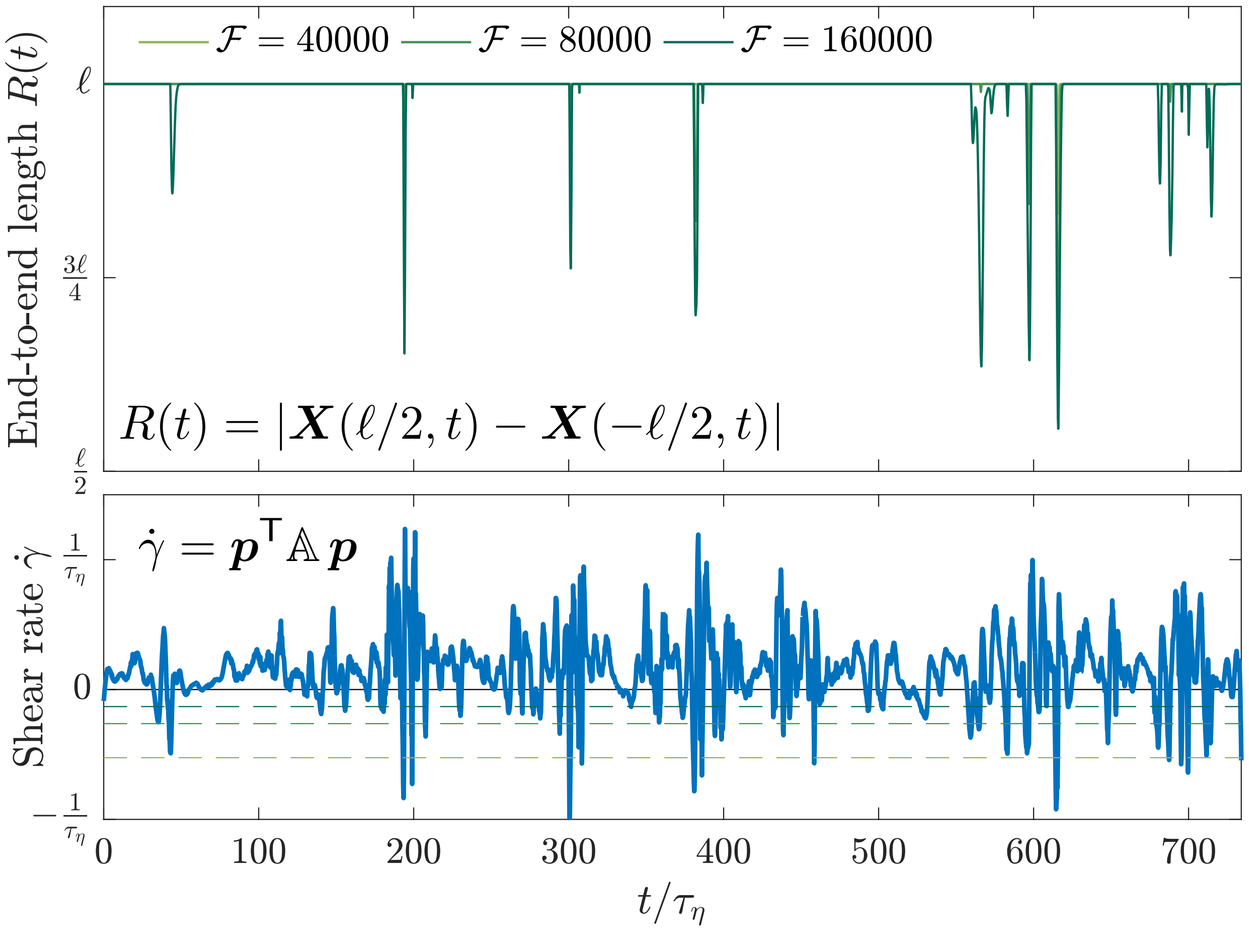
Such a solution can become **unstable**

Euler buckling if the flow is enough compressive along the fibre's direction



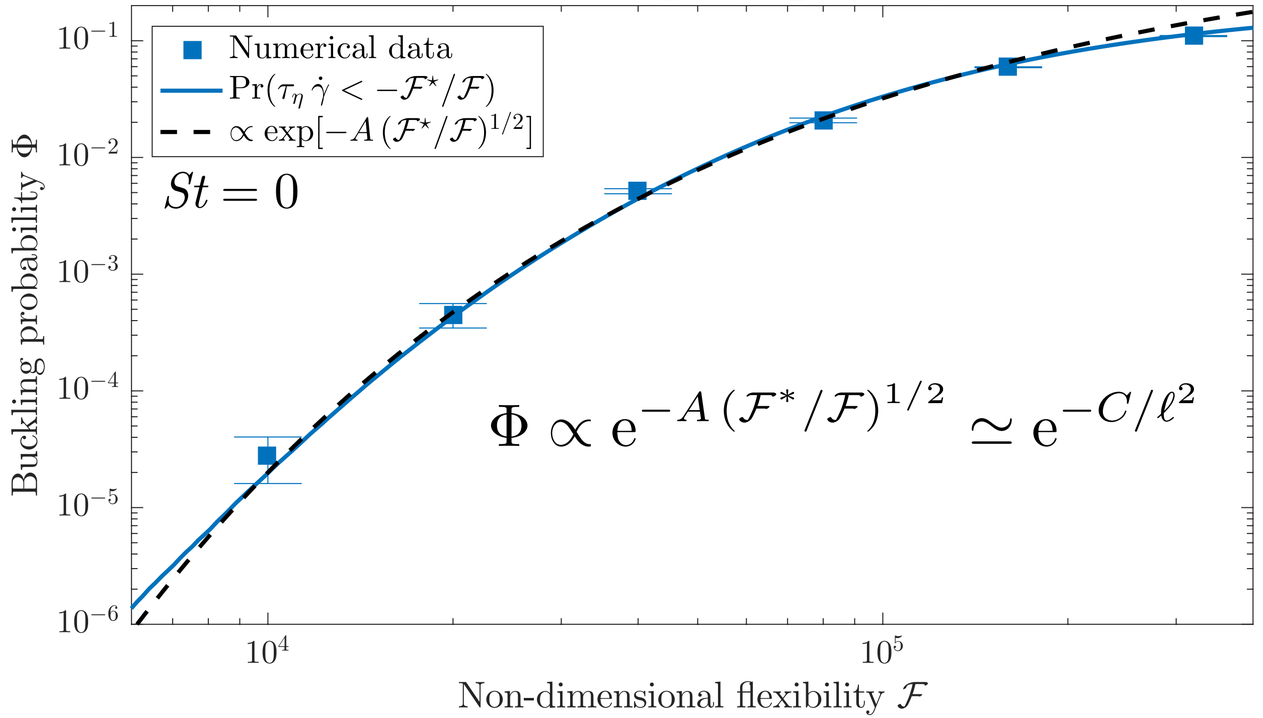
$$\dot{\gamma} = \boldsymbol{p}^\top \mathbb{A}\,\boldsymbol{p} < -\dot{\gamma}^\star$$ ***(Becker & Shelley 2001)***

An activated process



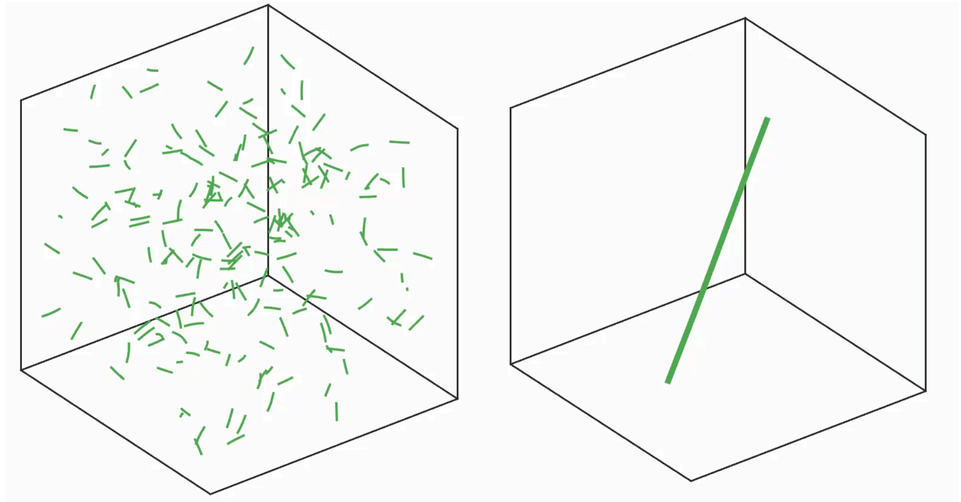
Buckling when $\mathcal{F}^{\mathrm{loc}} = \tau_\eta \mathcal{F}\,|\dot\gamma| > \mathcal{F}^\star$

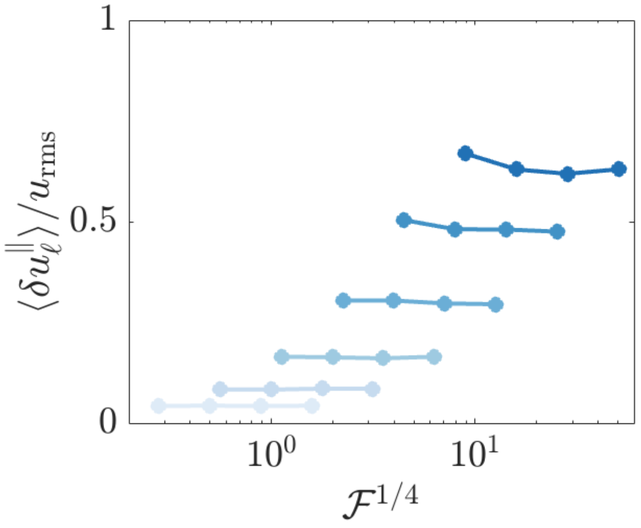
Buckling frequency

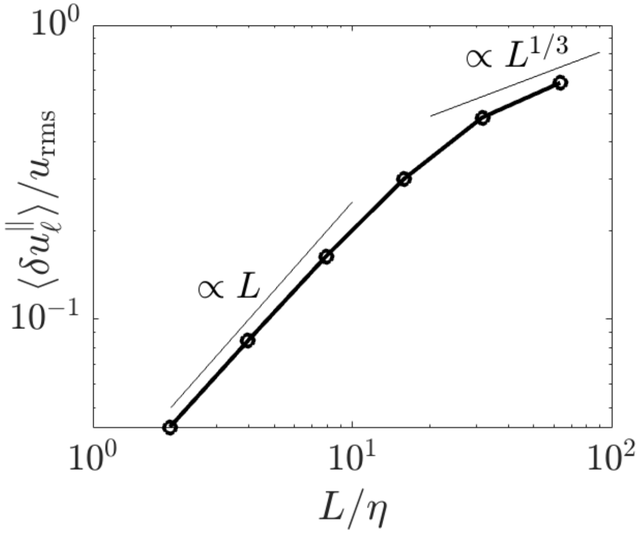


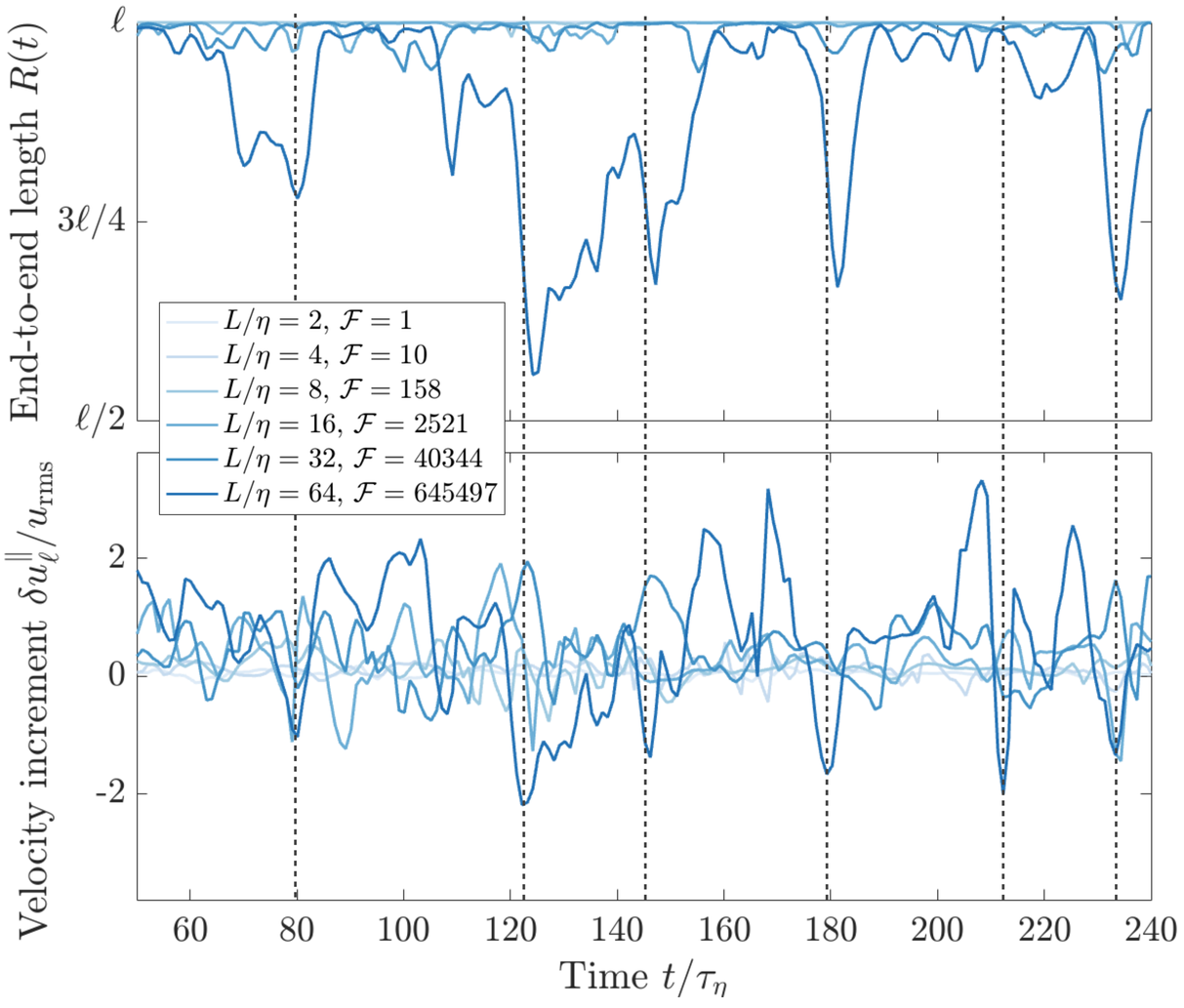
(Allende, Henry & Bec 2018)

Long fibers are also “stretched” by the flow





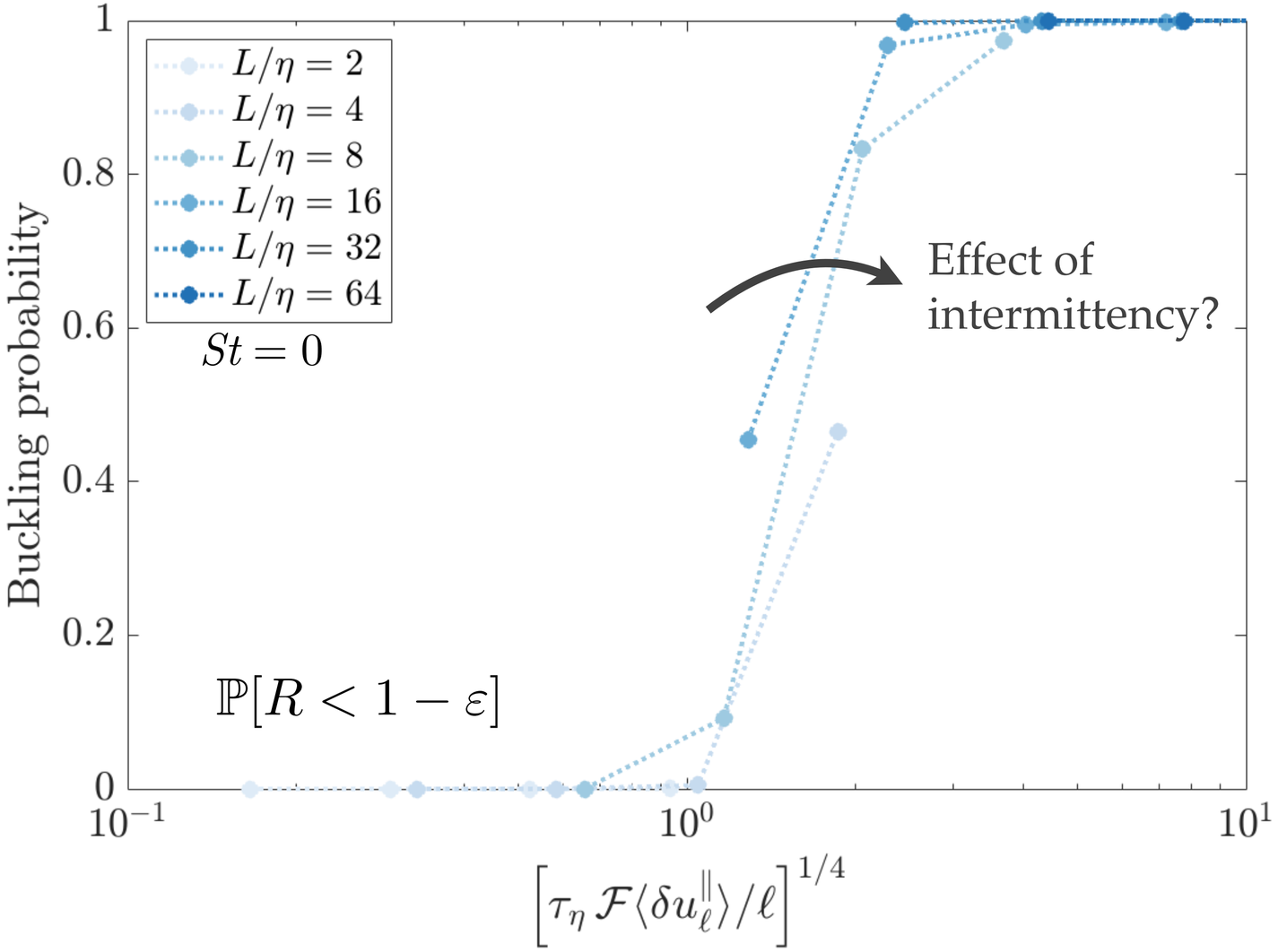


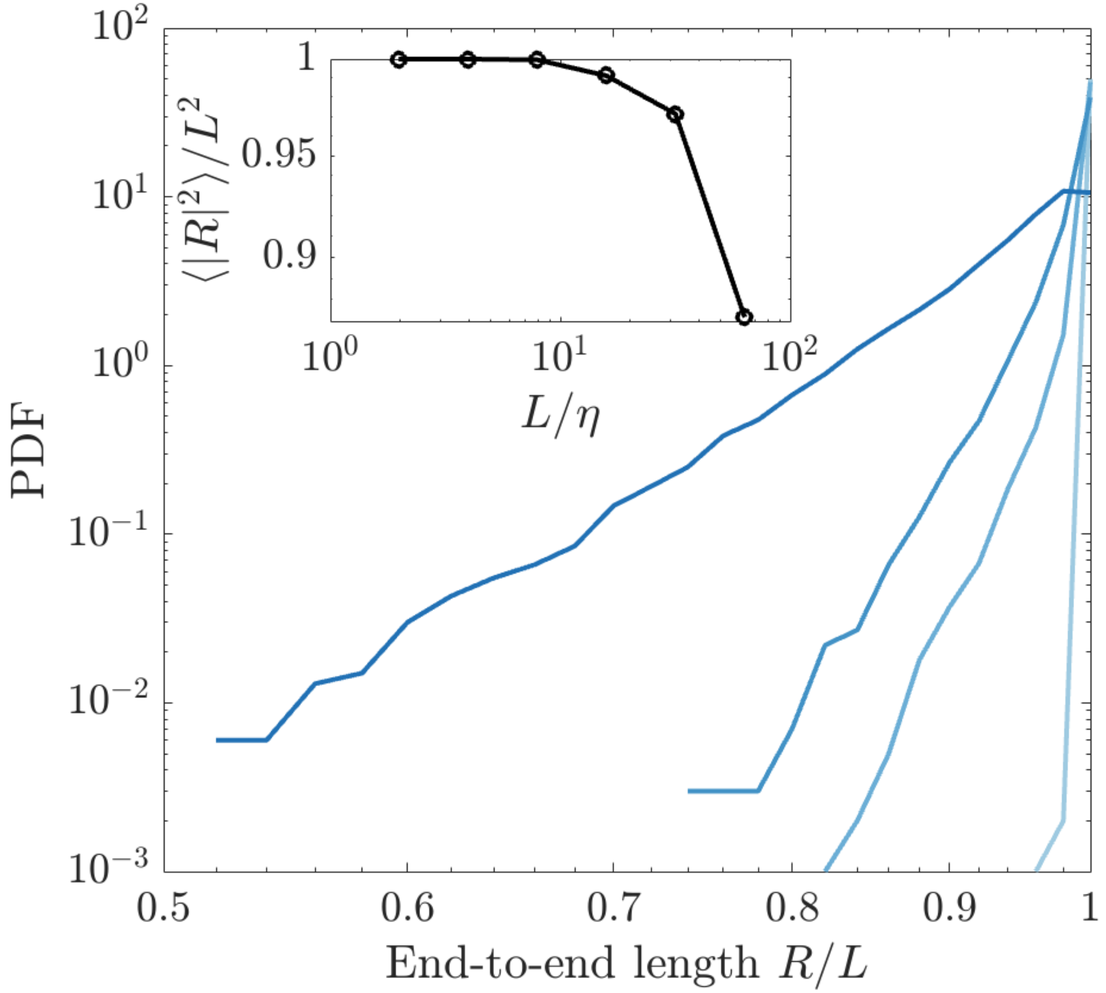


⇒ alignment with a “maximum stretching” direction
(Pujara et al. 2019, Oehmke et al. 2021)

Bending when $\mathcal{F}^{\rm loc}(\ell) = \tau_\eta \mathcal{F} \, |\delta u_\ell^\parallel| / \ell > \mathcal{F}^\star$

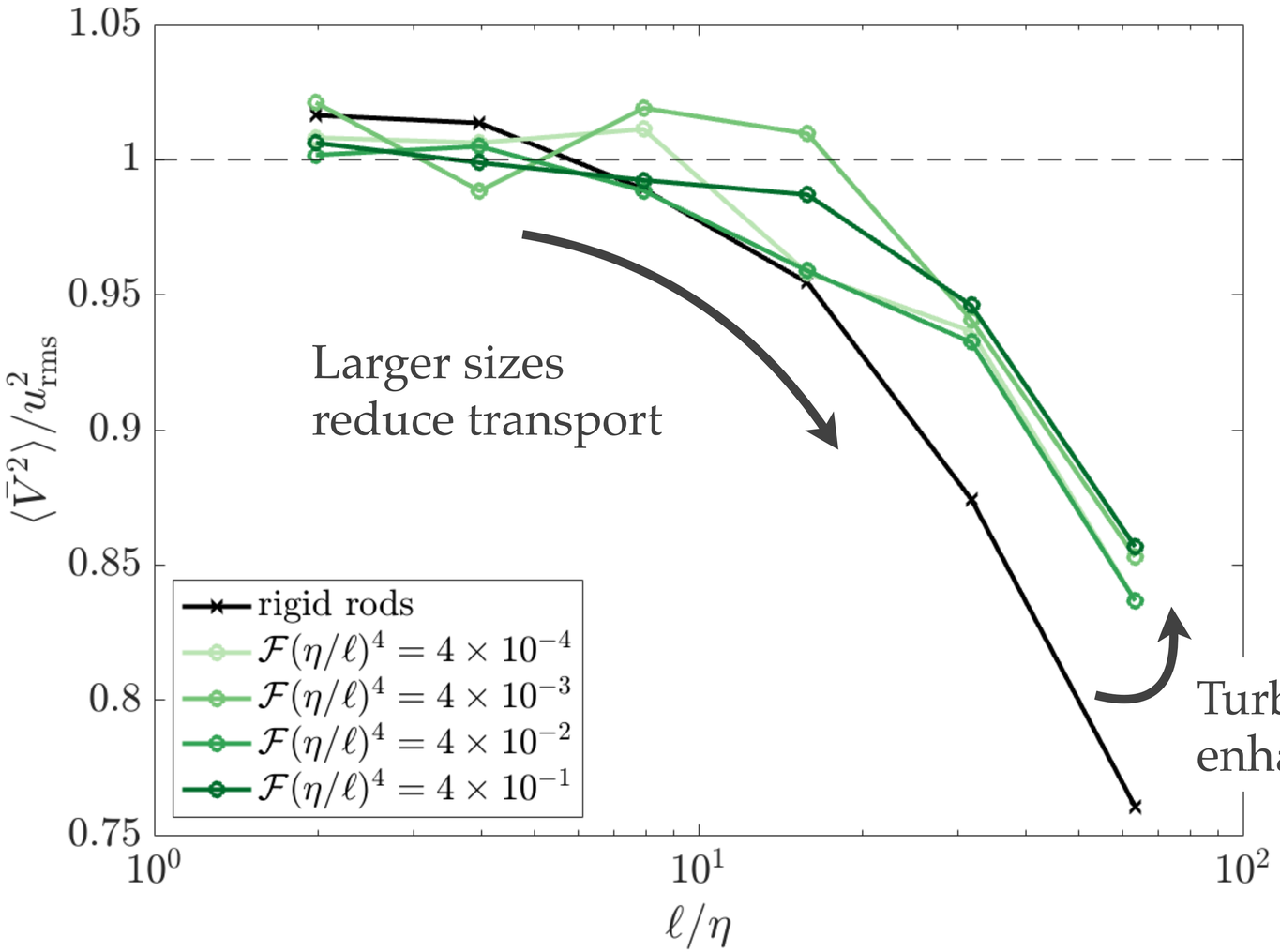
Straight-to-bent transition





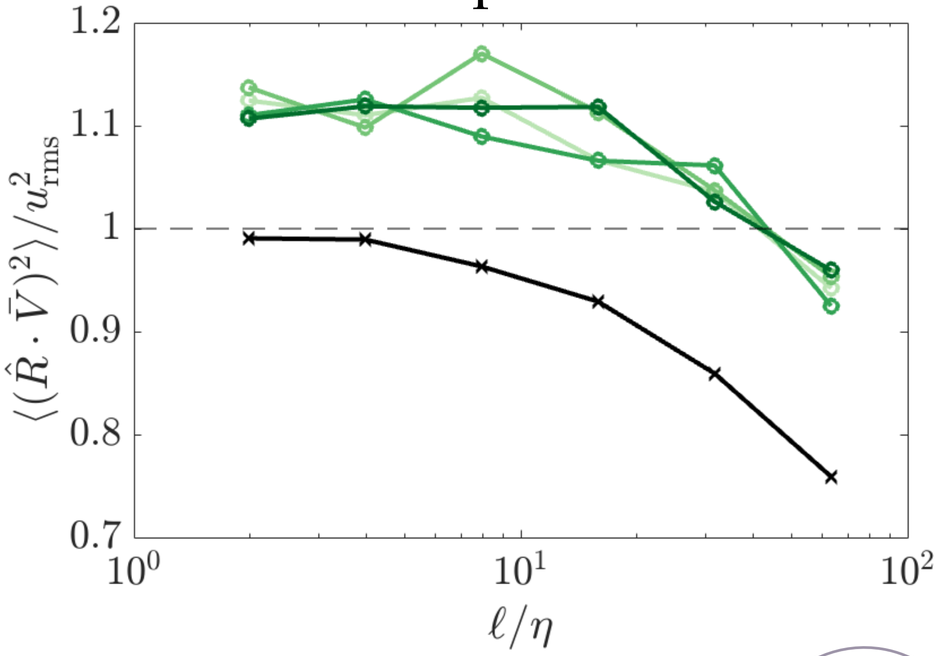
⇒ certainly activated process rather than transition?

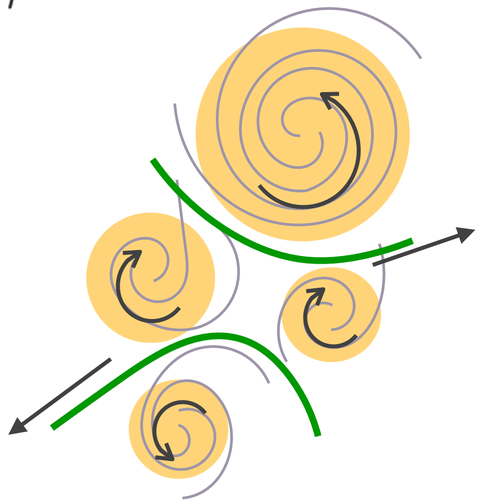
Consequences on transport



Turbulent bending
enhances transport

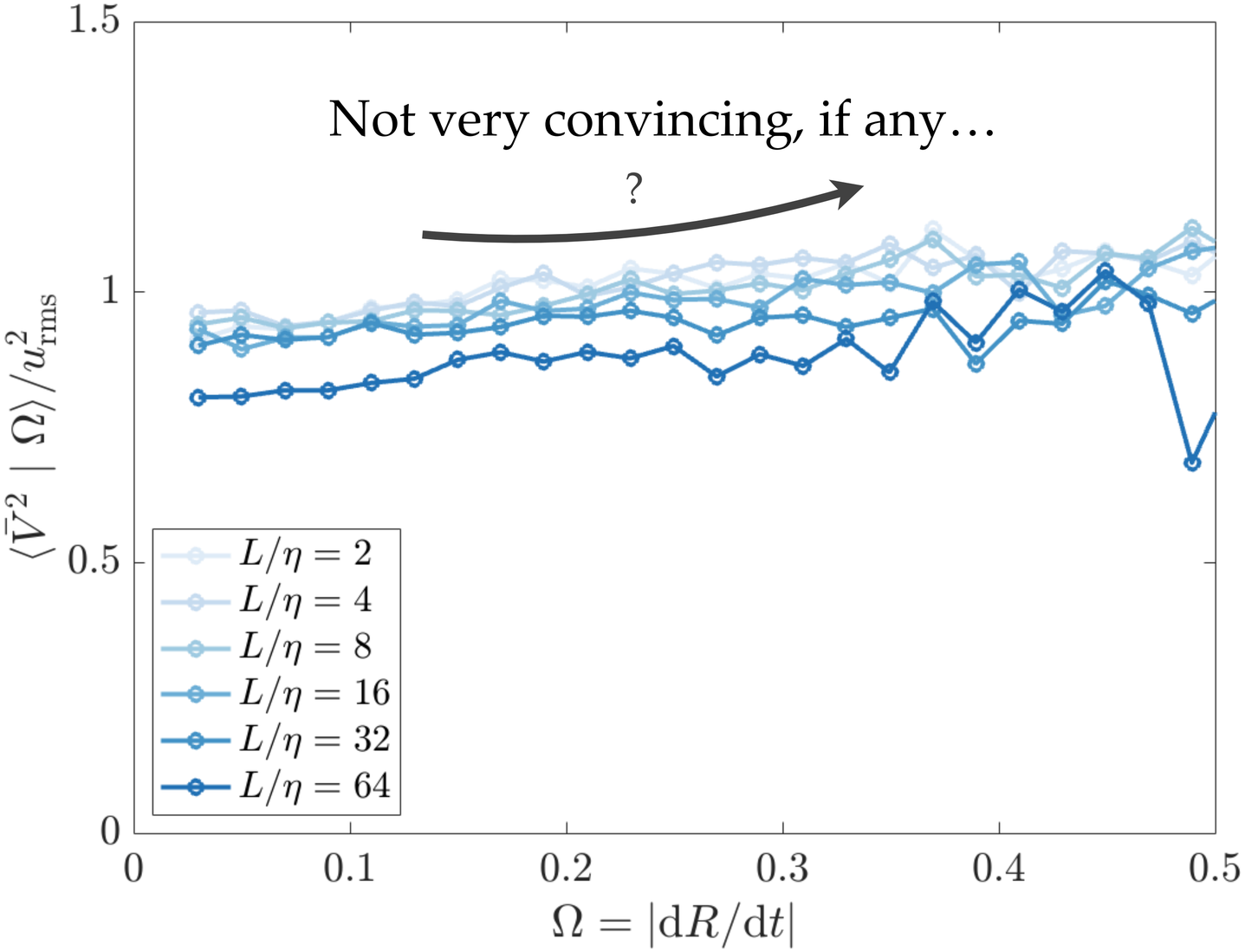
Parallel component increase





“Surfing” effect (due to flow structures) or simple enhanced diffusivity (due to rotational diffusion)?

Translation/rotation coupling?



TODOs:

- more statistics
- larger Reynolds numbers
- longer fibers
- effect of inertia *(Bounoua et al. 2021)*
- gravitational settling…

Flexible fibres in HIT: short summary

- Fibres are most of the time in a straight configuration, aligned with the stretching direction of the flow. They can be described by an effective Jeffery-like equation

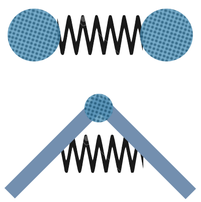
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{R} = \delta\boldsymbol{u}(\boldsymbol{R}) - \left(\boldsymbol{R}^{\mathsf{T}}\delta\boldsymbol{u}(\boldsymbol{R})\right)\boldsymbol{R}$$

- They transition to a bent state when a strong-enough compression occurs

$$\dot{\gamma} = \boldsymbol{R}^{\mathsf{T}}\delta\boldsymbol{u}(\boldsymbol{R})/|\boldsymbol{R}|^2 < -\gamma^*$$

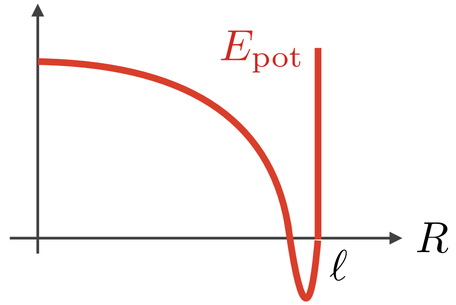
- Bending is very intermittent: rare with localised-in-time, violent episodes

- Possible macroscopic modes



spring with
specific potential

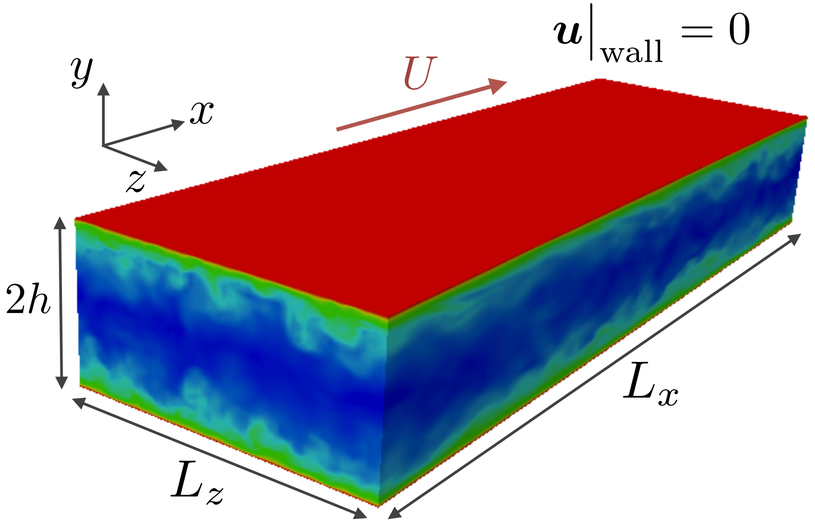
large-size trumbbell



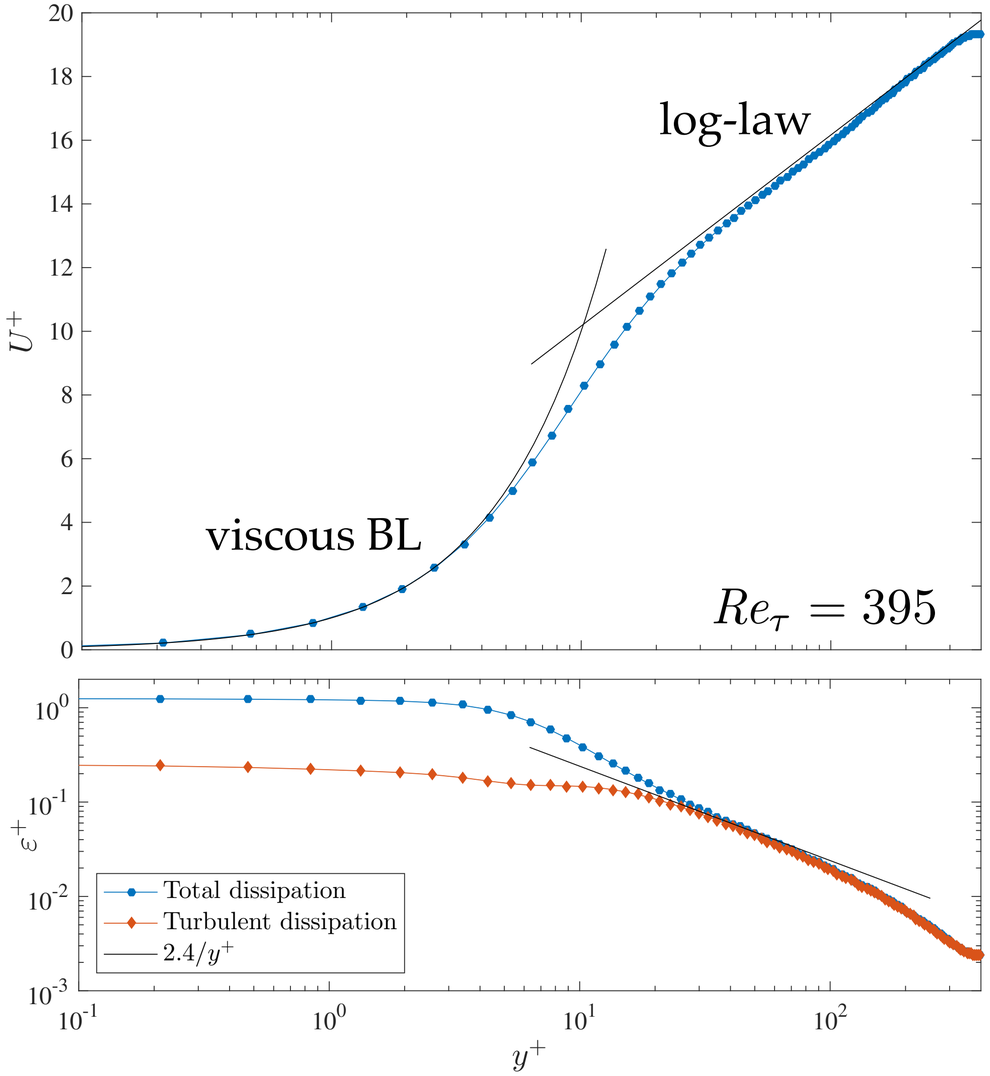
Case II: Turbulent channel flow

Direct numerical simulations

Unforced Navier–Stokes
in channel geometry



Fourier-Chebyshev solver
+ passive particles



Friction velocity:

$$u_\tau = \sqrt{\nu \langle \partial_y u_x \rangle_{\text{wall}}}$$

Dissipation rate:

$$\varepsilon(y) = \frac{\nu}{2} \left\langle \| \nabla \boldsymbol{u} + \nabla \boldsymbol{u}^\top \|^2 \right\rangle$$

Viscous scales:

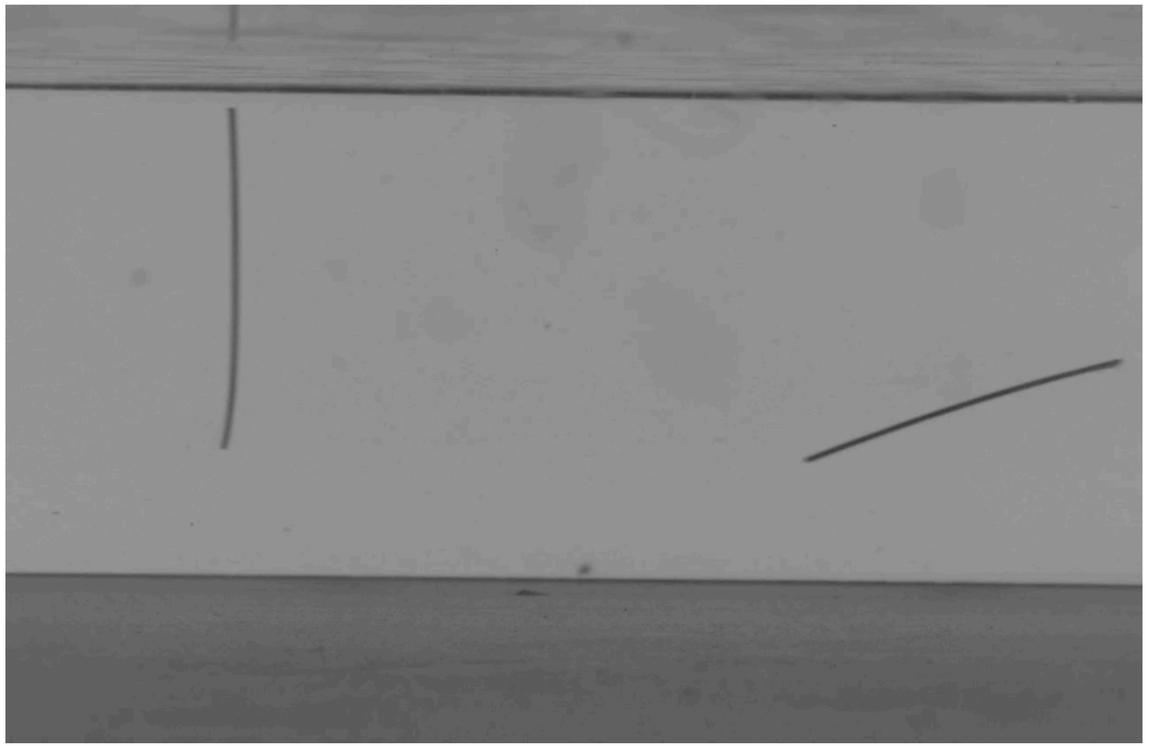
$$\delta = \nu / u_\tau, \ \tau_\nu = \nu / u_\tau^2$$

Reynolds number:

$$Re_\tau = \frac{u_\tau \, h}{\nu} \approx 180 - 800$$

Connection to experiments

Turbulent channel flow @InPhyNi (Christophe Brouzet)



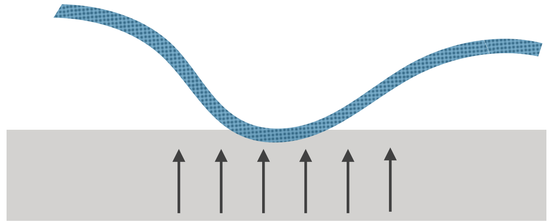
3D reconstruction and tracking of long fibres ⇒ De-Fa Sun's PhD

Integration of fibres

Flexibility measured by $\mathcal{F}^+ = \dfrac{4\pi\,\mu\,\ell^4}{\tau_\nu E\,\sigma\,\log\lambda}$

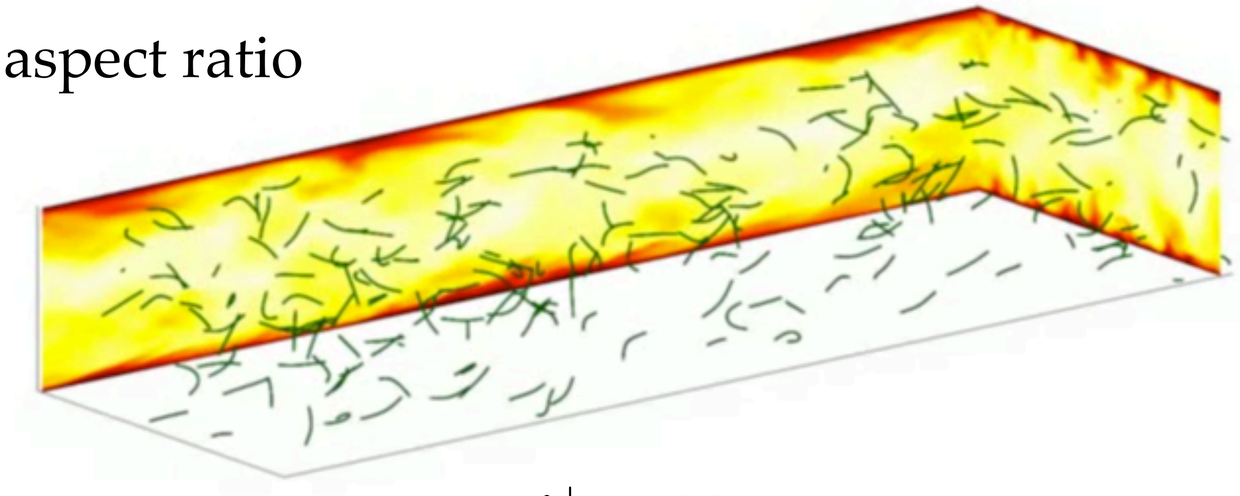
Interaction with walls: **soft boundaries**

$u_y = \gamma\, y$ when $y < 0$ with γ a large parameter



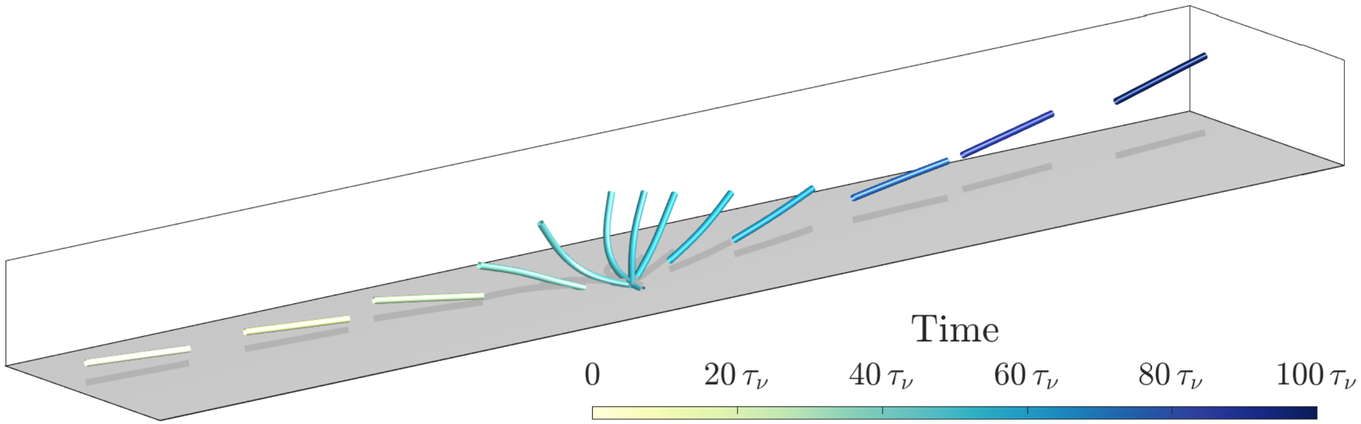
Different lengths but equal elasticity and aspect ratio

$\ell^+ = \ell/\delta_\nu$

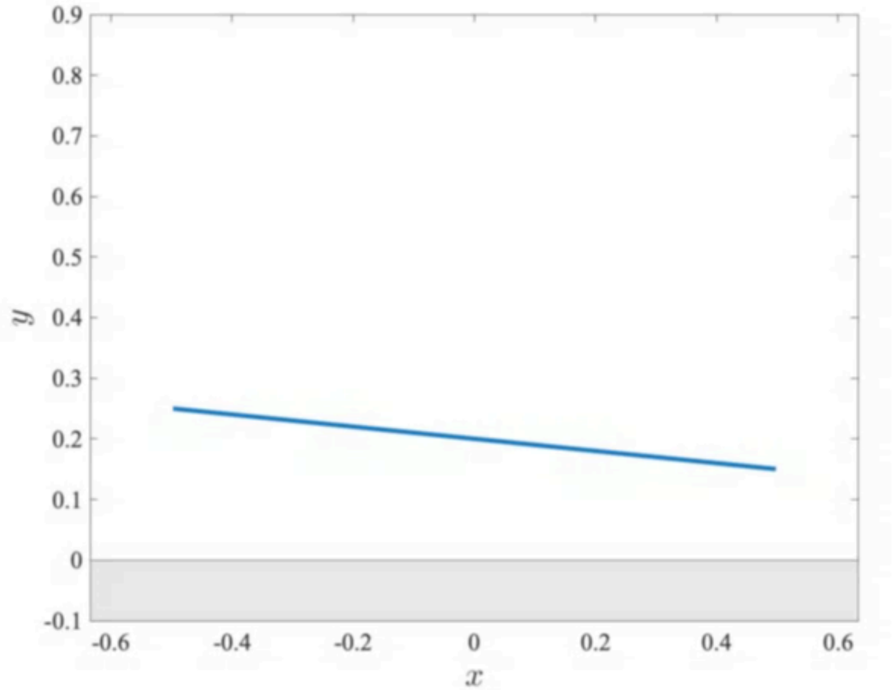


$\ell^+ = 90$

Pole-vault effect upon tumbling



When bouncing, fibres are propelled away from the walls

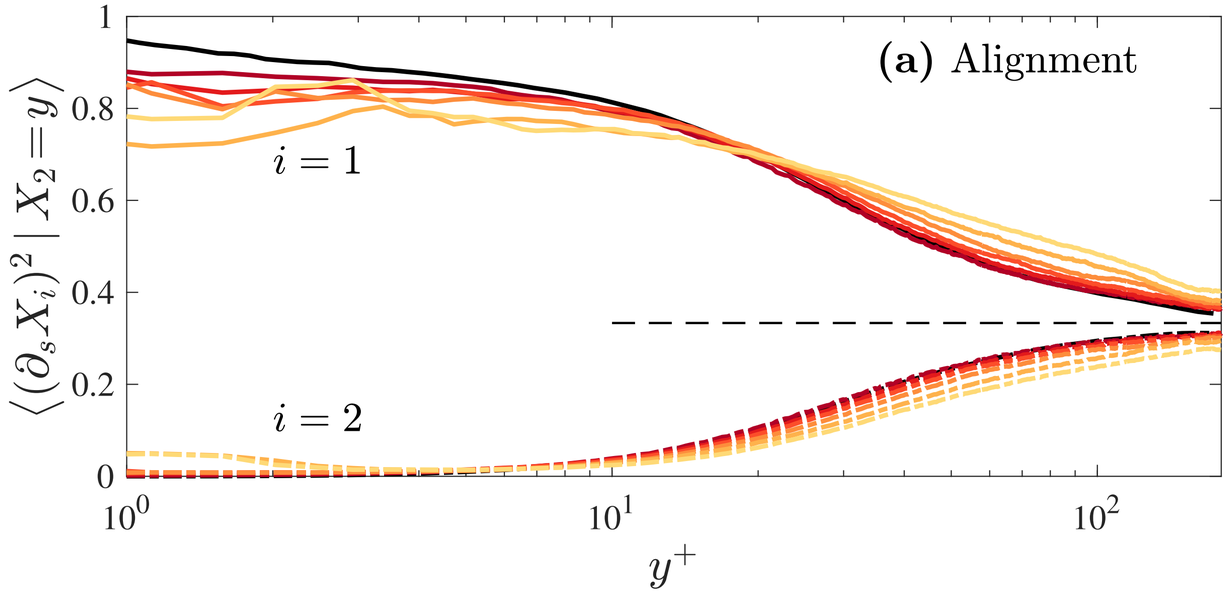


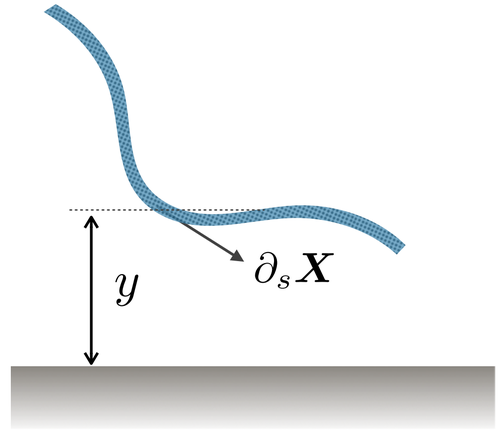
Reproducible effect in a pure shear flow

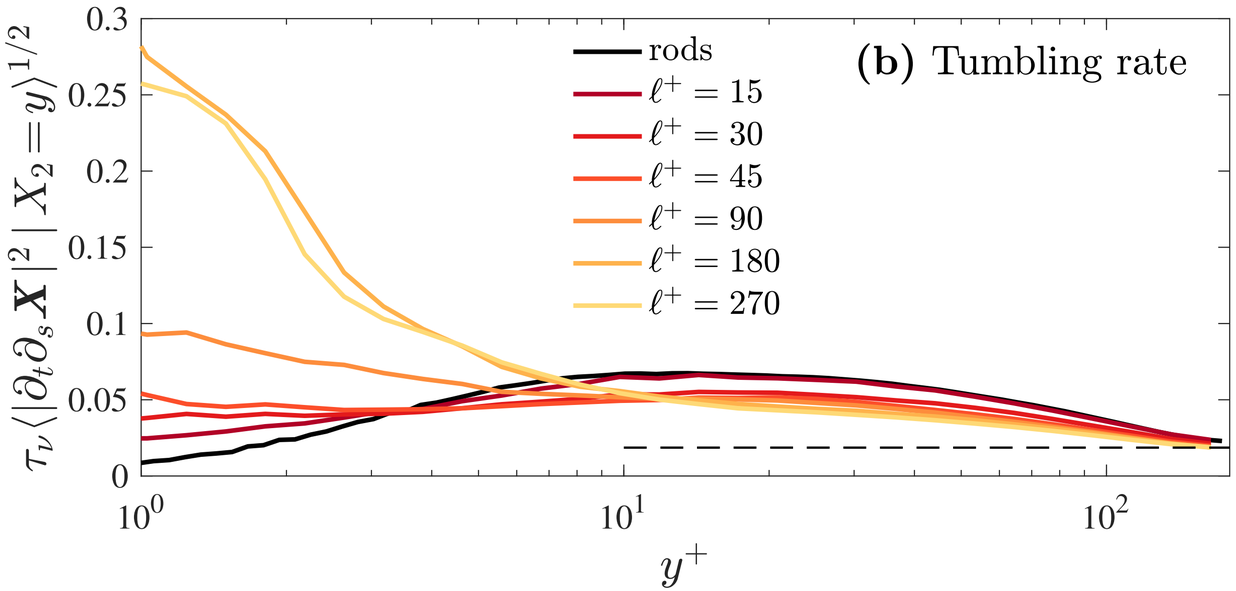
$$\boldsymbol{u} = \begin{pmatrix} \sigma\, y \\ 0 \end{pmatrix}$$

⇒ Responsible for a net migration toward the flow's bulk

Near-wall alignment and tumbling

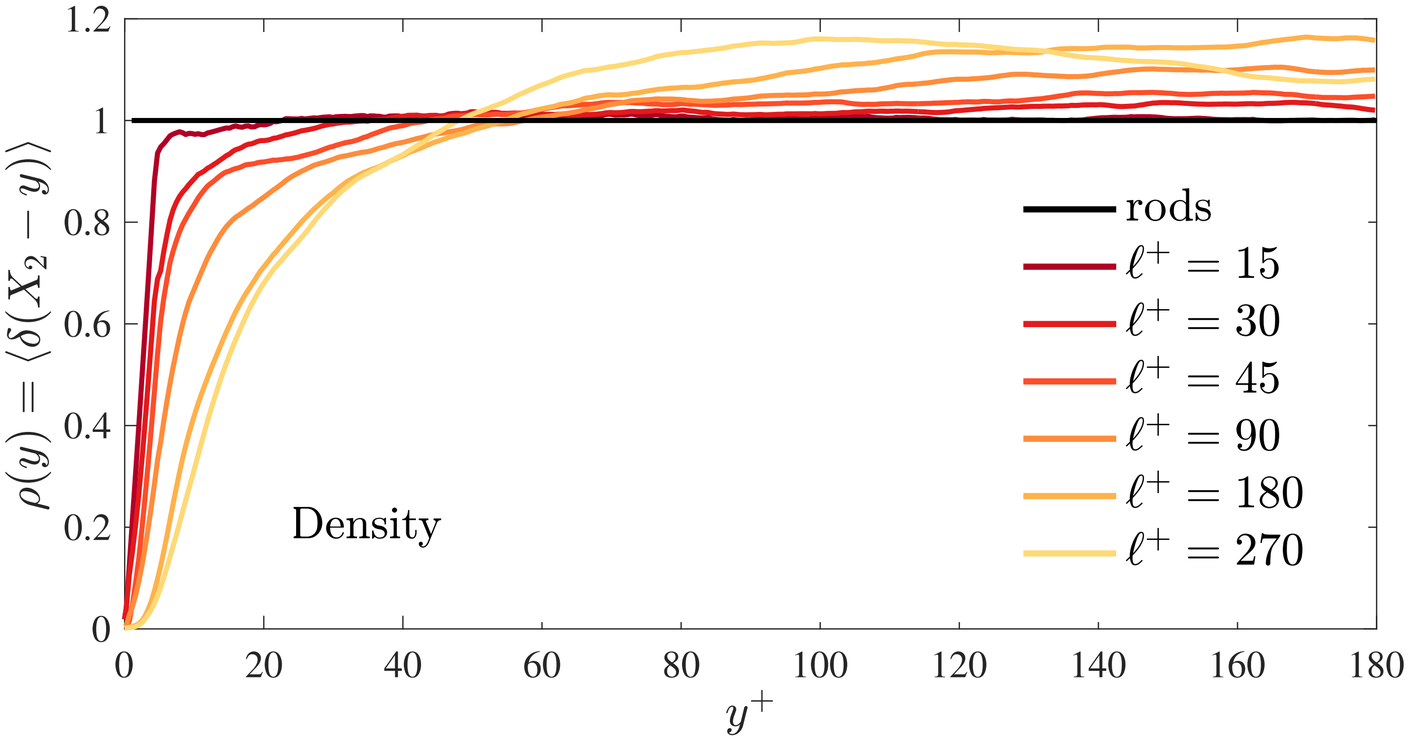




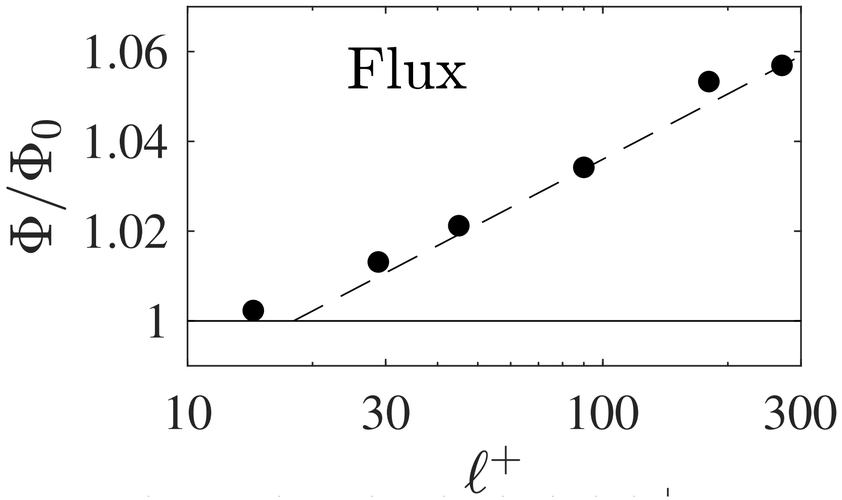


- Strong stream-wise alignment near walls
- Violent deformations of longest fibres
  ⇒ contacts with walls?

Consequences on transport



Over-concentration in the channel bulk

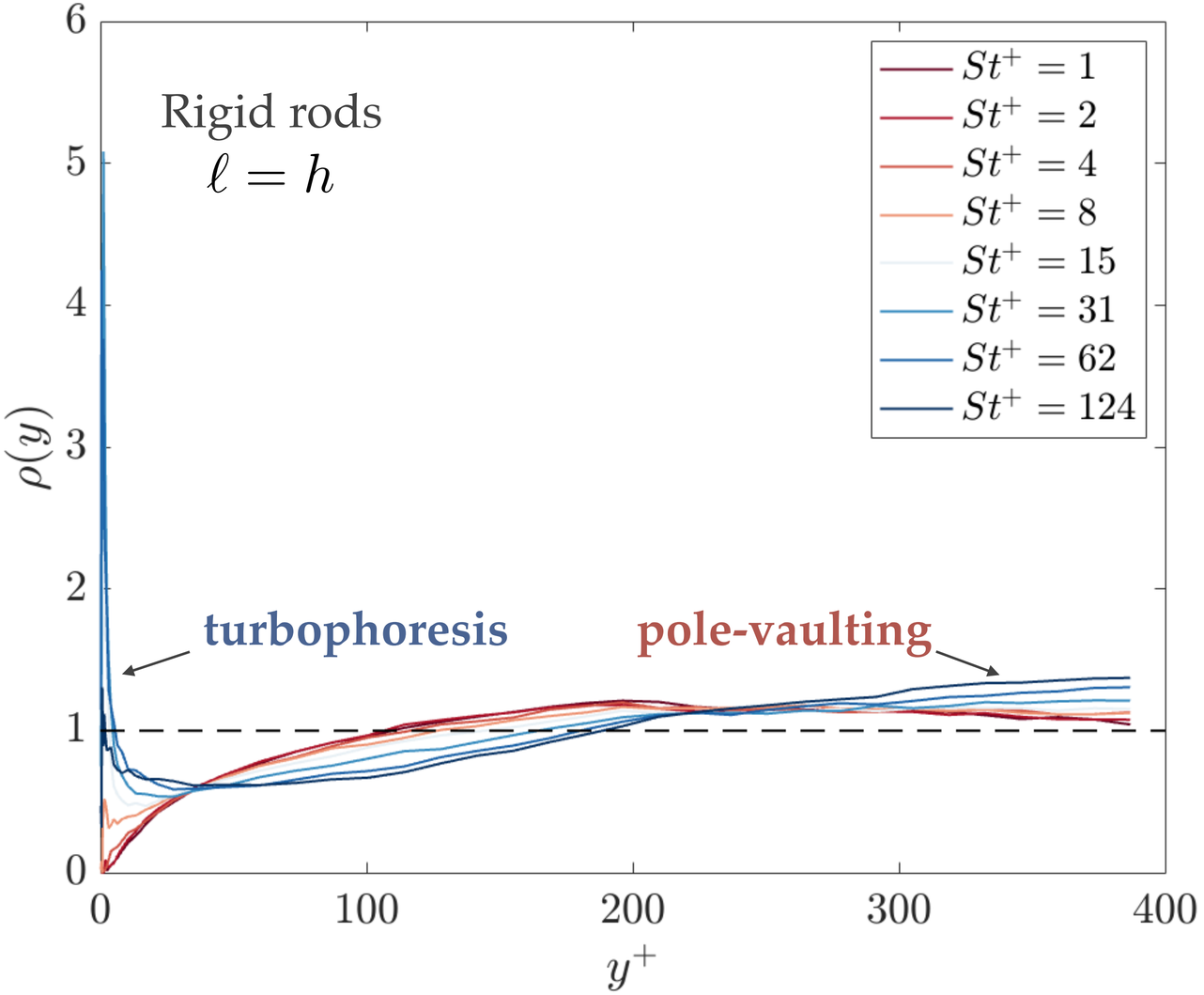


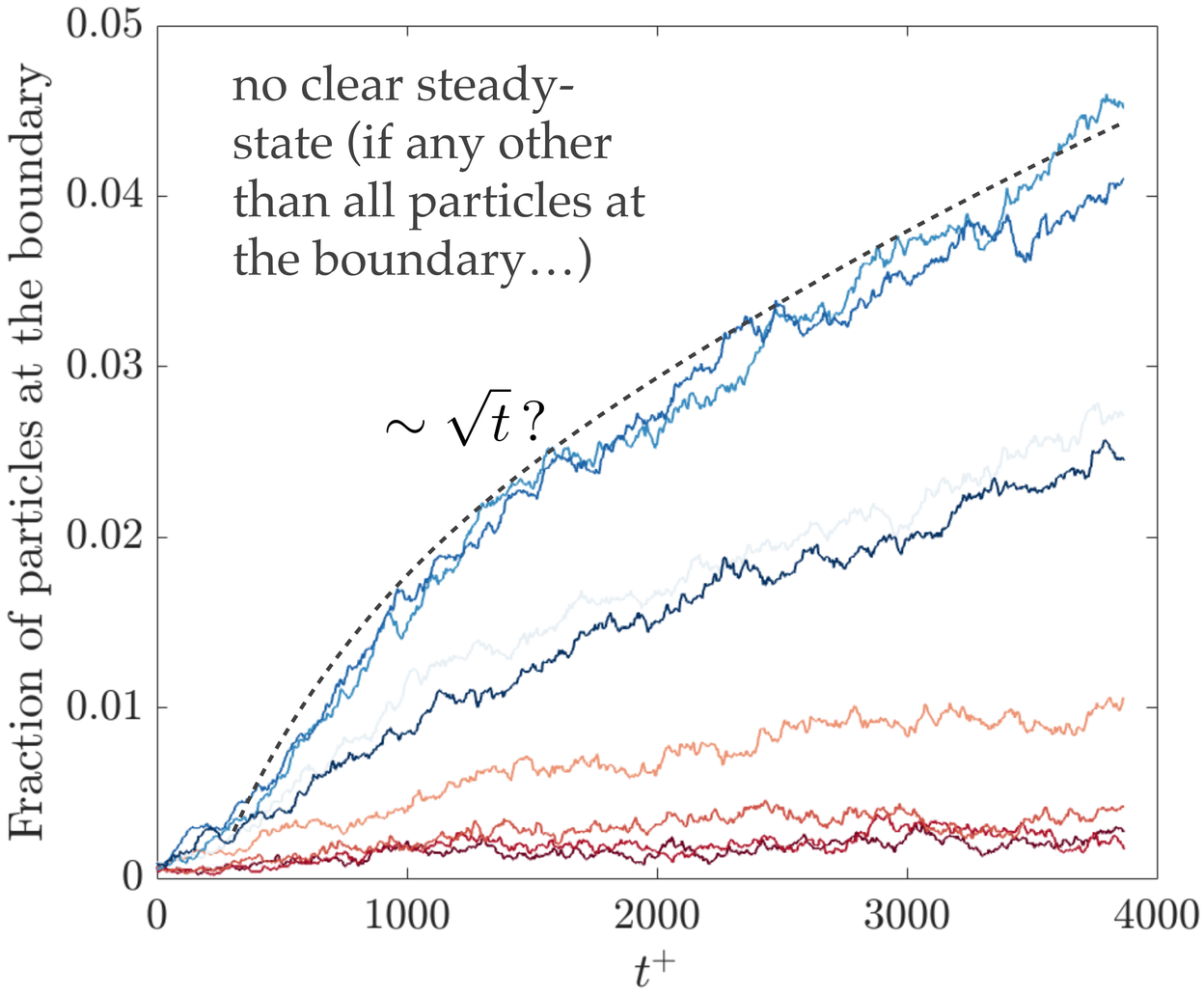
Enhancement of fibres' flux compared to fluid transport

(Bec, Brouzet & Henry 2024)

Are such ideal settings reproducible in experiments?

Effects of inertia





Ongoing comparisons with experiments…

Conclusions

Key results

- Macroscopic elongated particles align with an **inertial-range direction of stretching** (a.k.a. longest elongation of an advected tetrahedron)
- Deformability enhances transport (velocity amplitude). **Surfing** effect?
- Coupling between rotation and translation otherwise unclear…
- When tumbling, long fibers tend to **migrate away from boundaries** (“pole-vaulting”), enhancing their transport. However, this might be hindered by **inertia/turbophoresis**.

Perspectives

- Consider combined effect of inertia and flexibility. Turbulent settling?
- Change the fibers’ equilibrium shape (e.g. curved arcs, helices, etc).
- Effect of refined interactions with boundaries (friction vs. gripping)
- Two-way coupling: does the alignment with boundaries laminarize the flow?

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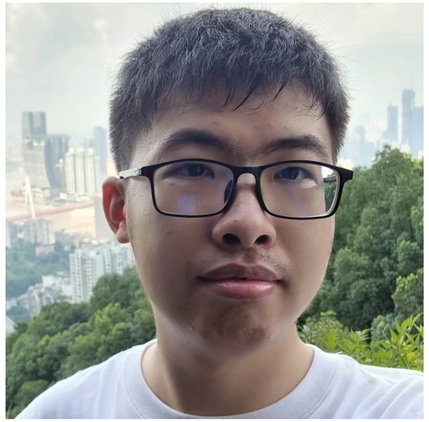

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