THE SURPRISING FATE OF TURBULENCE IN FLUIDS WITH *ODD VISCOSITY*

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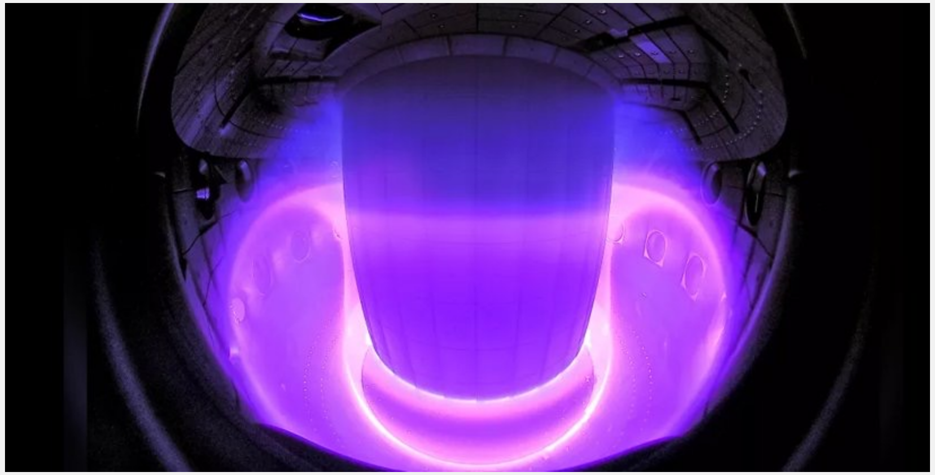
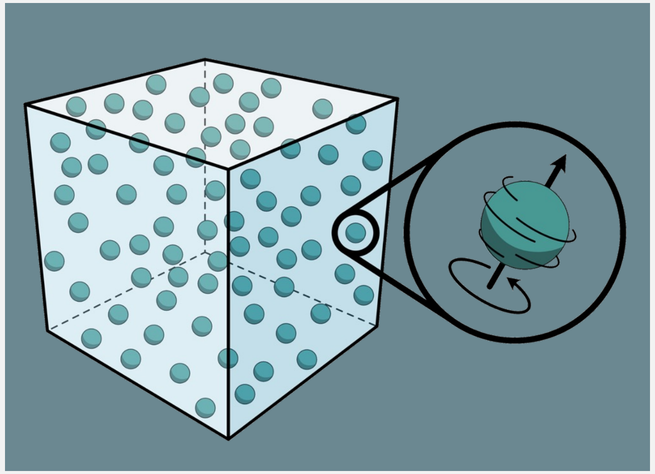
Chiral particles and Odd fluids, Jan 12, 2026

ODD VISCOSITY

Source: polytechnique-insights.com



Source: New Scientist



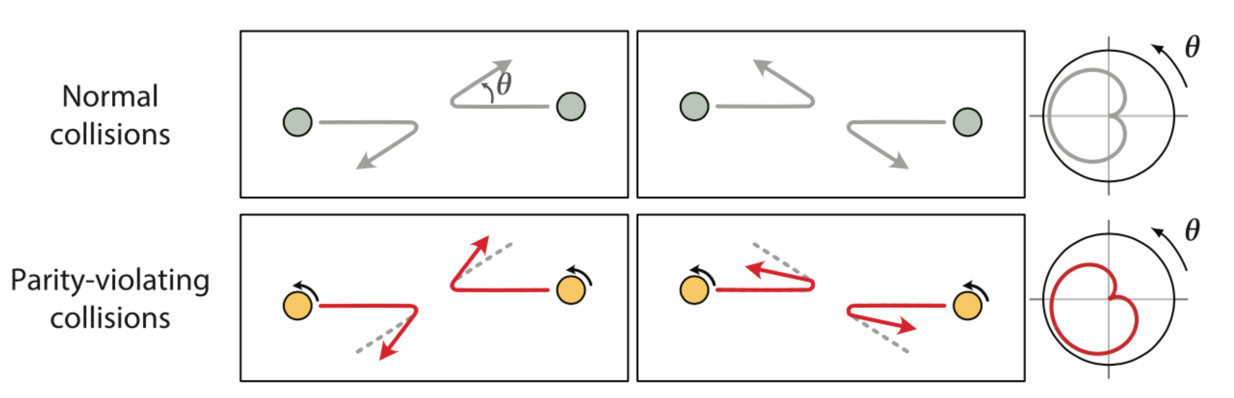
de Wit et al. (2024)

WHAT IS ODD VISCOSITY?

- General viscosity

$$\tau_{ij} = \eta_{ijkl}\, \partial_k u_l$$

- Odd when: $\eta_{ijkl} \neq \eta_{klij}$
- Odd viscosity acts perpendicular to the fluid velocity: *no dissipation*
- Also called Hall or gyro viscosity



Adopted from Fruchart et al. 2023

What does the turbulent dynamics look like?

WHAT KIND OF ODD VISCOSITY?

- Cylindrical symmetry yields term in (incompressible) Navier-Stokes

$$D_t \boldsymbol{v} = -\boldsymbol{\nabla} P + \nu \Delta \boldsymbol{v} + \boxed{\nu_{\mathrm{odd}} \boldsymbol{e}_z \times \Delta \boldsymbol{v}} + \boldsymbol{f}(t, \boldsymbol{x})$$

- Very similar to Coriolis force, but with extra Laplacian
  - *Its action is restricted to high wavenumbers*

ODD WAVES

- Linear solutions of Navier-Stokes

$$\cancelto{\partial_t}{D_t}\boldsymbol{v} = -\boldsymbol{\nabla} P + \cancelto{0}{\nu \Delta \boldsymbol{v}} + \nu_{\mathrm{odd}} \boldsymbol{e}_z \times \Delta \boldsymbol{v} + \cancelto{0}{\boldsymbol{f}(t, \boldsymbol{x})}$$

- Gives rise to *odd waves*:

$$\omega(\boldsymbol{k}) = \pm \nu_{\mathrm{odd}} k_z |k|$$

ODD WAVES

- Evolution of kinetic energy in Fourier space:

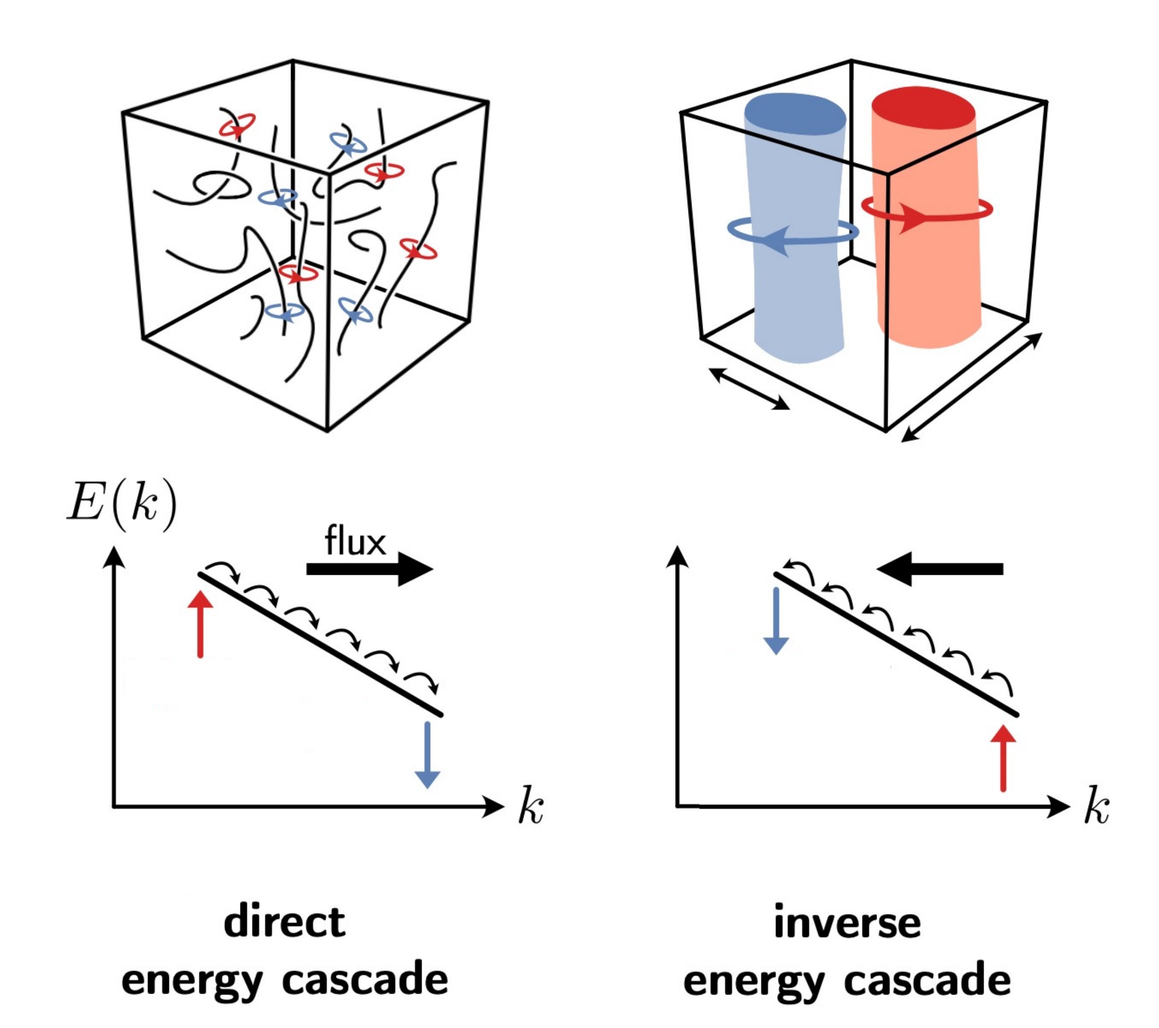
$$\frac{\partial|\hat{\boldsymbol{u}}(\boldsymbol{k})|^2}{\partial t} = \overbrace{-i \sum_{\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{k}} \underbrace{\left[\hat{\boldsymbol{u}}(\boldsymbol{k})^* \cdot \boldsymbol{P}(\boldsymbol{k}) \cdot \hat{\boldsymbol{u}}(\boldsymbol{p}) \cdot \boldsymbol{q}\hat{\boldsymbol{u}}(\boldsymbol{q})\right]}_{\propto \exp(i\,[\omega(\boldsymbol{k})+\omega(\boldsymbol{p})+\omega(\boldsymbol{q})]\,t)}}^{\text{transport}} - \overbrace{\nu k^2 |\hat{\boldsymbol{u}}(\boldsymbol{k})|^2}^{\text{dissipation}} + \overbrace{\hat{\boldsymbol{u}}(\boldsymbol{k})^* \cdot \hat{f}(\boldsymbol{k})}^{\text{injection}},$$

- *Odd waves completely change the dynamics!* $\omega(\boldsymbol{k}) = \pm \nu_{\mathrm{odd}} k_z |k|$
  - They impede the 3D energy transfer with $k_z \neq 0$
  - Two-dimensionalisation: Taylor-Proudman theorem

TURBULENT CASCADES

If the odd viscosity constraint is **weak**:

- 3D turbulence
- Direct cascade



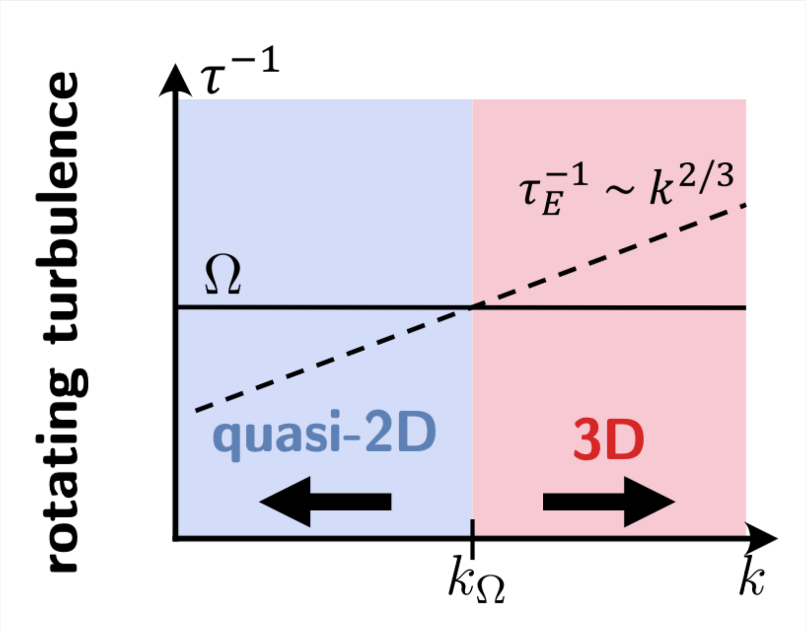
If the odd viscosity constraint is **strong**:

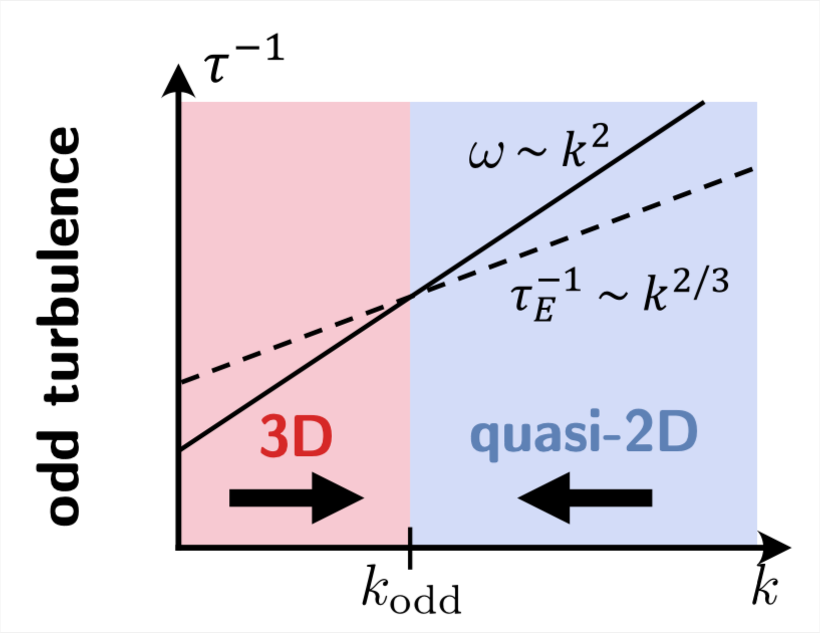
- Quasi-2D turbulence
- Inverse cascade

We need to compare the timescale of the odd waves with the Eddy turnover time!

TURBULENT CASCADES

Compare the (inertial or odd) wave frequency with the Eddy turnover frequency

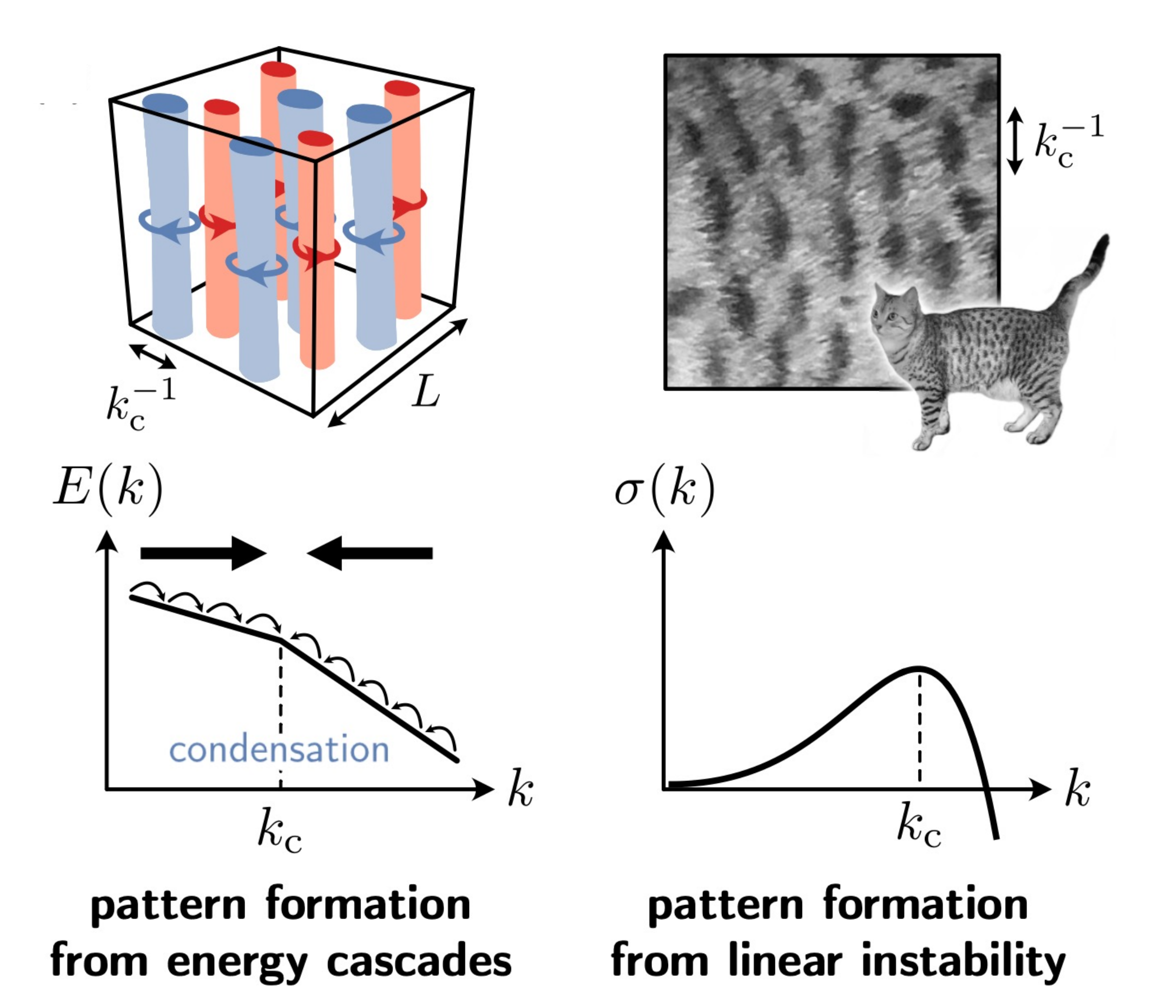




NON-LINEAR PATTERN FORMATION

Converging fluxes
⇒ non-linear pattern formation!

Further reading:
de Wit et al. (2024) Nature 627

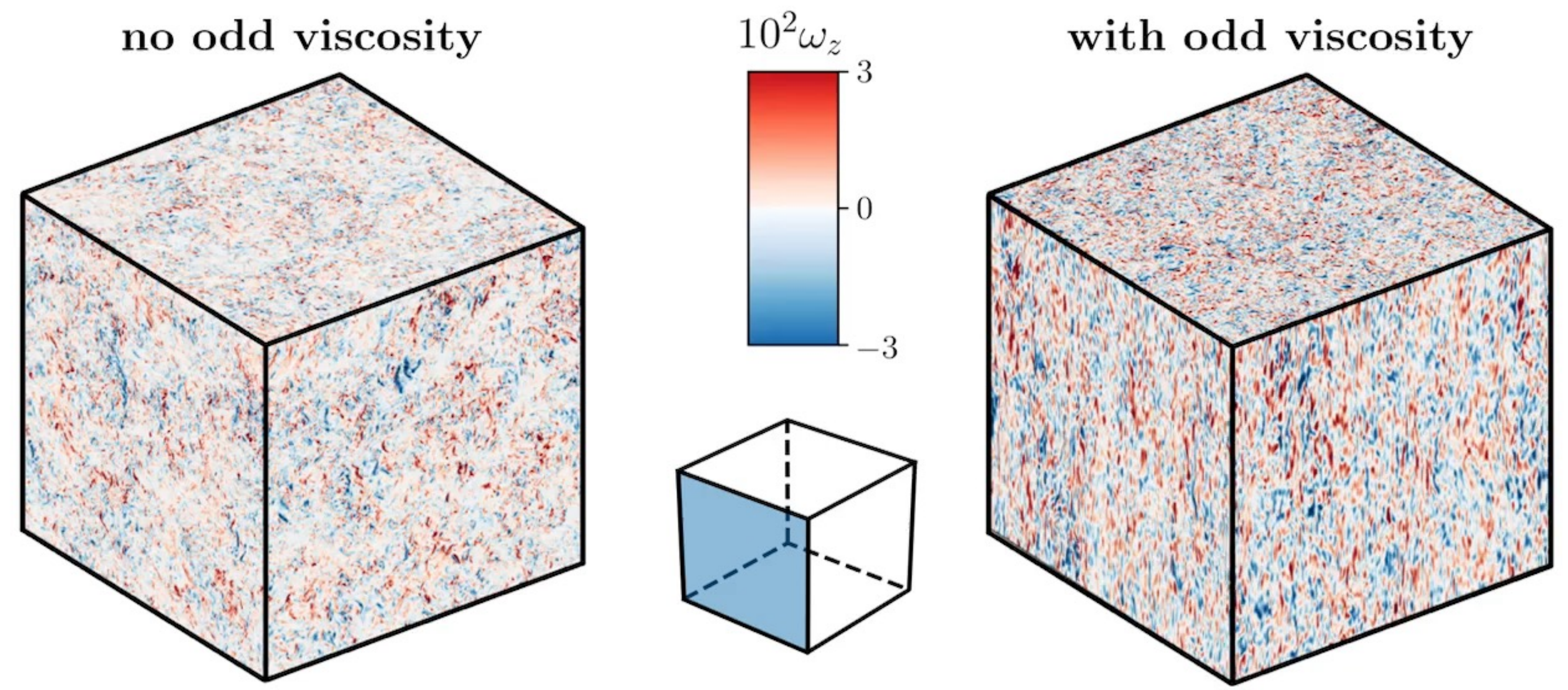


SCALING THEORY OF ODD TURBULENCE

- We can derive scaling relations for:
  - From comparison of timescales:
    - The start of condensation $k_{\text{odd}} \propto \epsilon^{1/4} {\nu_{\text{odd}}}^{-3/4}$
  - From dimensional argument:
    - The energetic amplification $E(k)/E_o(k) \propto (k/k_{\text{odd}})^{2/3}$ for $k > k_{\text{odd}}$
  - From balance with dissipation:
    - The peak condensation $k_c \propto \epsilon^{1/4} \nu^{-1/2} {\nu_{\text{odd}}}^{-1/4}$

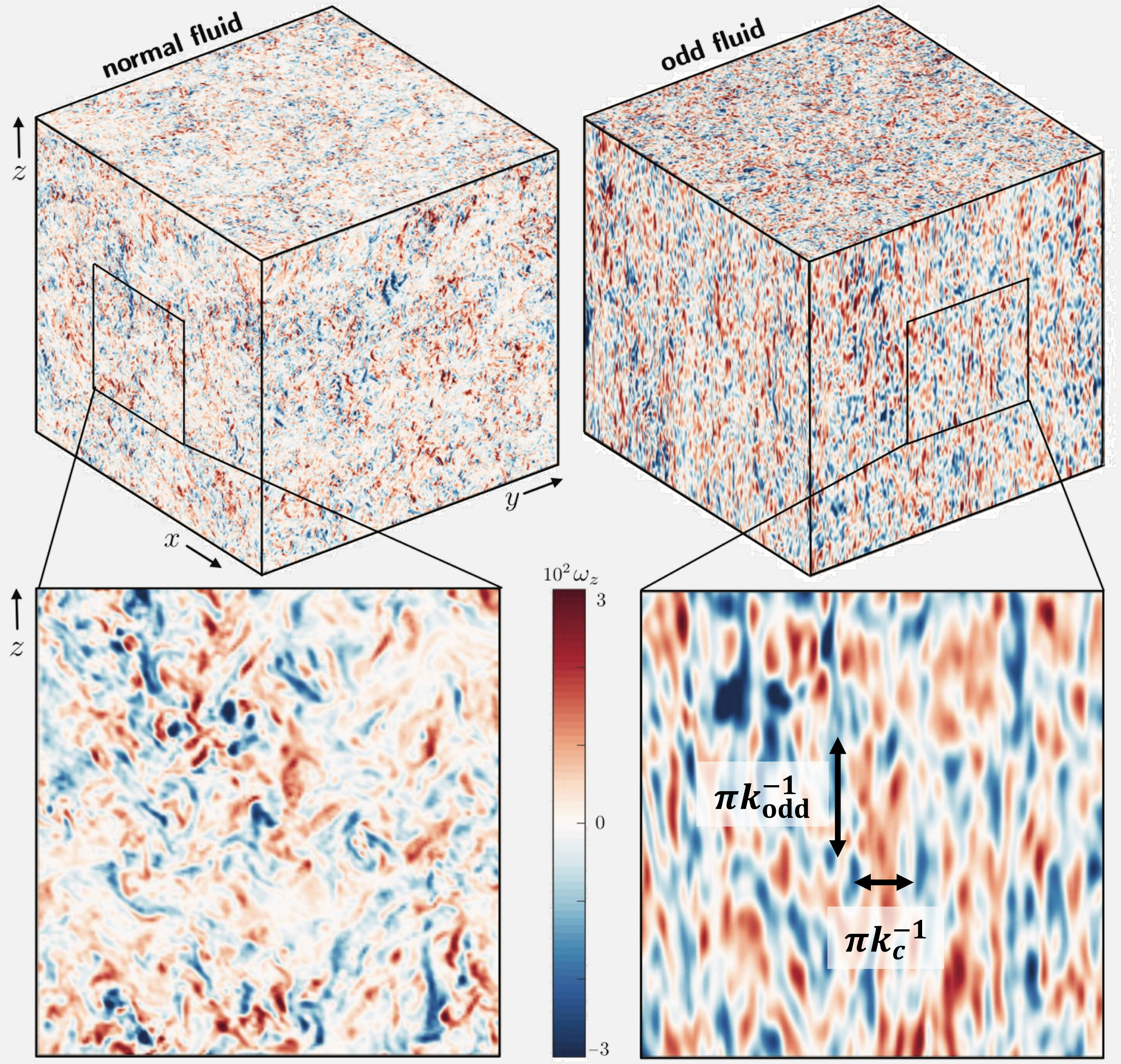
Let's test it!

NUMERICAL RESULTS



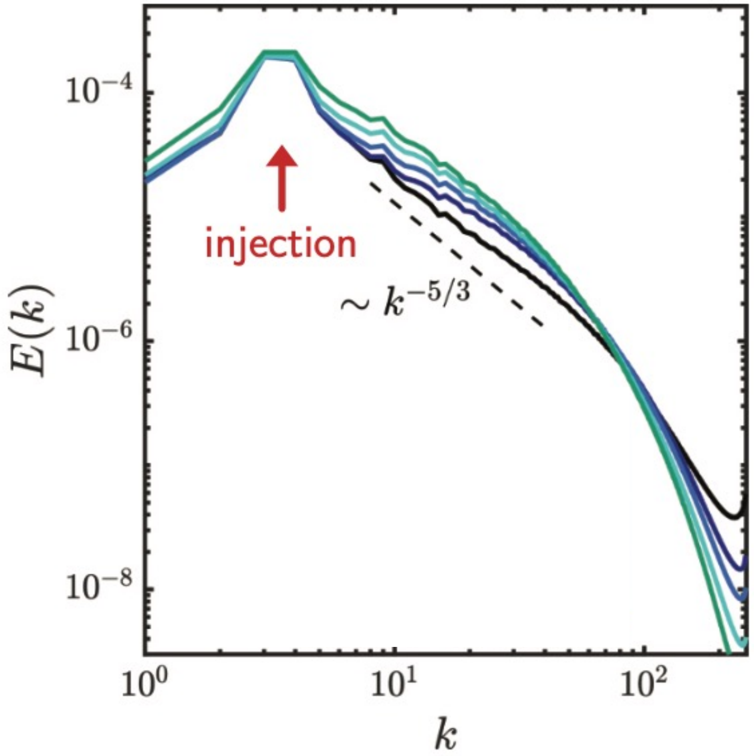
NUMERICAL RESULTS

Vertically oriented features at a preferred wavelength!



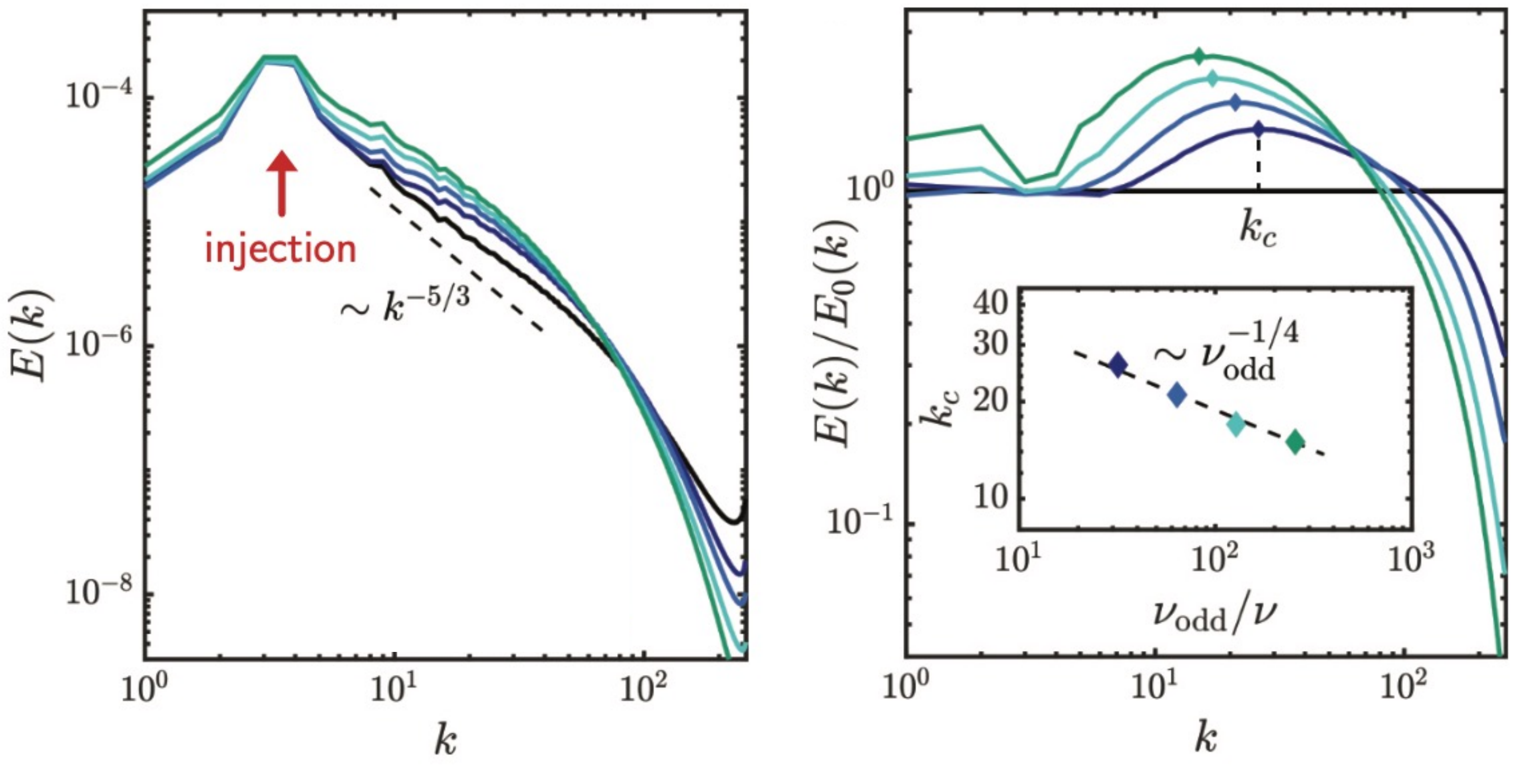
NUMERICAL RESULTS: FORWARD CASCADE

Energy condensation at intermediate scales



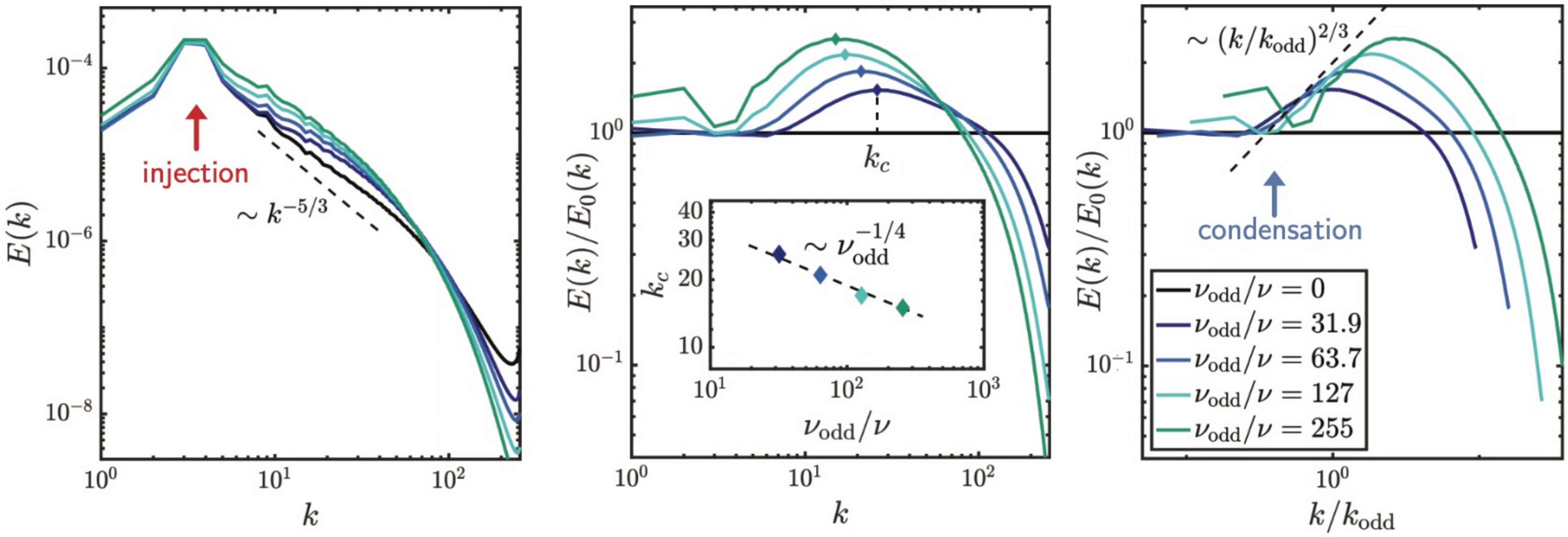
NUMERICAL RESULTS: FORWARD CASCADE

Compensate by the normal spectrum



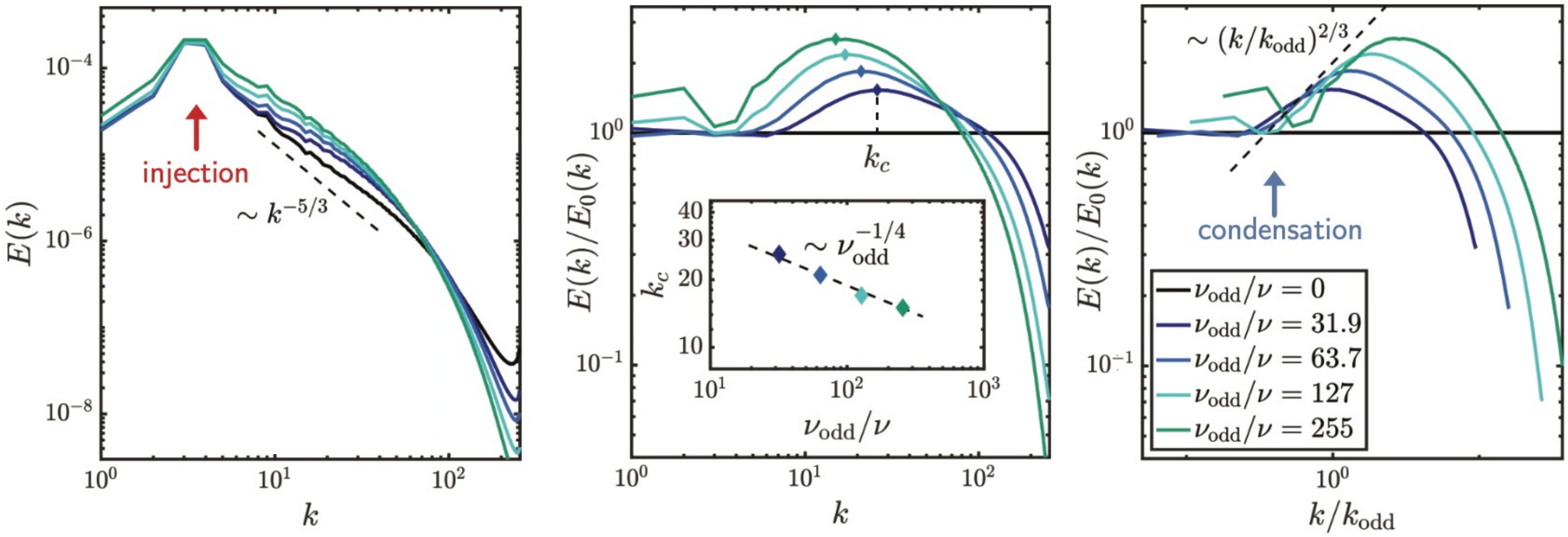
NUMERICAL RESULTS: FORWARD CASCADE

Rescale with the odd wavelength

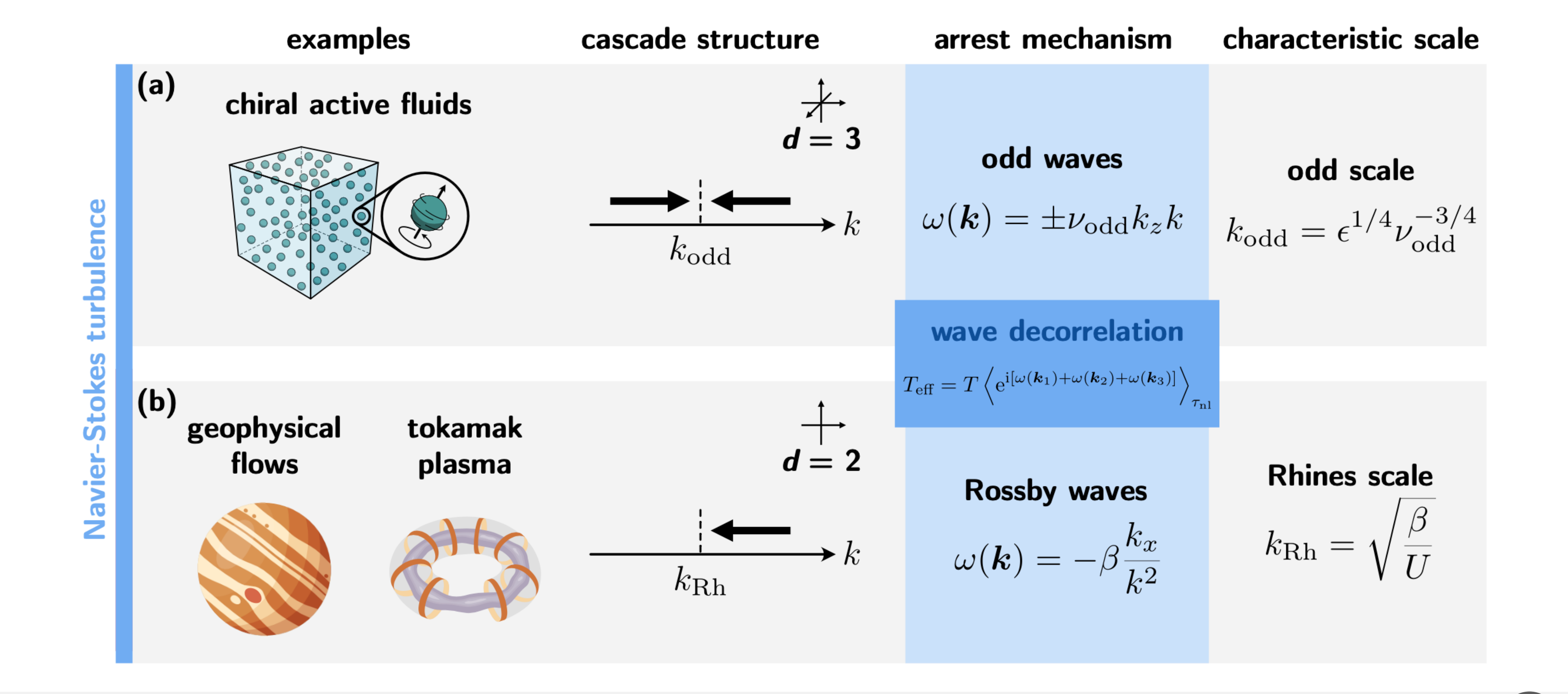


NUMERICAL RESULTS: FORWARD CASCADE

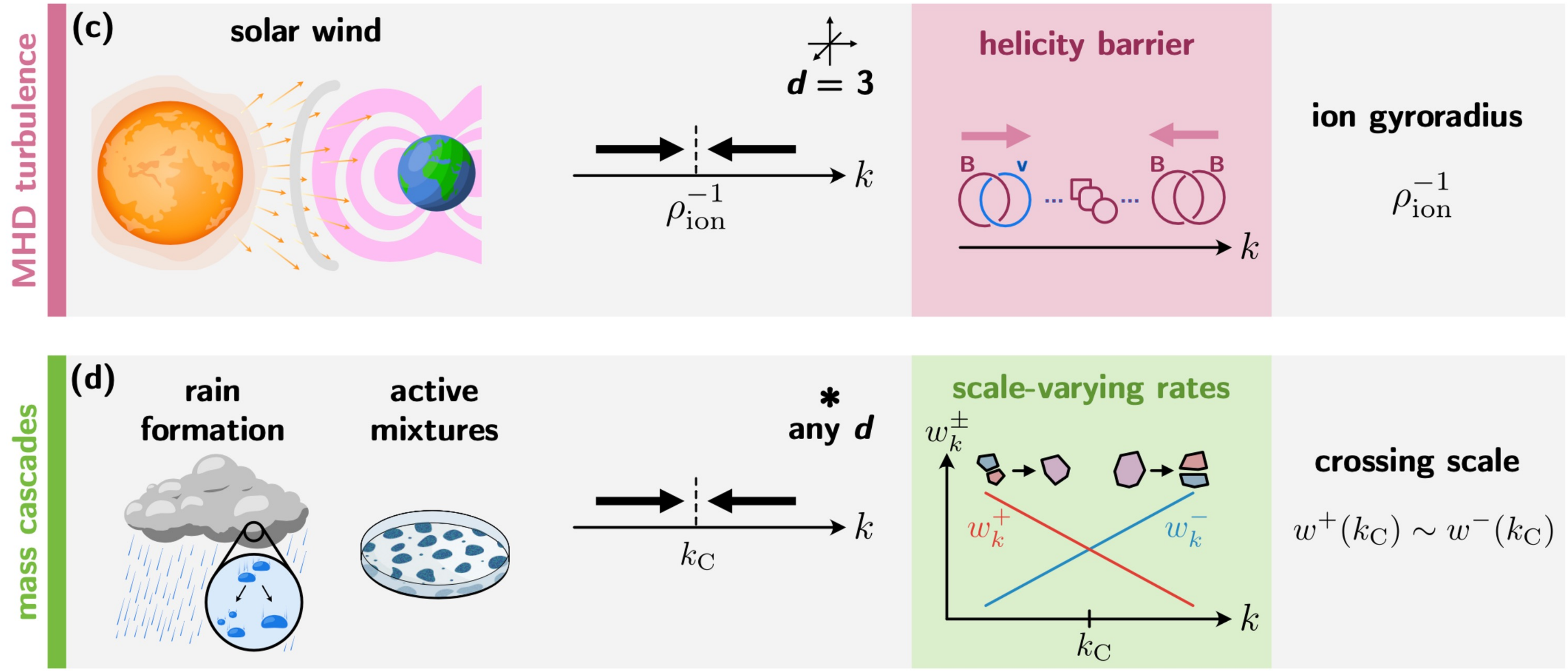
Scaling relations are retrieved over the accessible dynamic range!



CASCADE-INDUCED PATTERN FORMATION: BEYOND ODD VISCOSITY



CASCADE-INDUCED PATTERN FORMATION: BEYOND ODD VISCOSITY



CASCADE-INDUCED PATTERN FORMATION: BEYOND ODD VISCOSITY

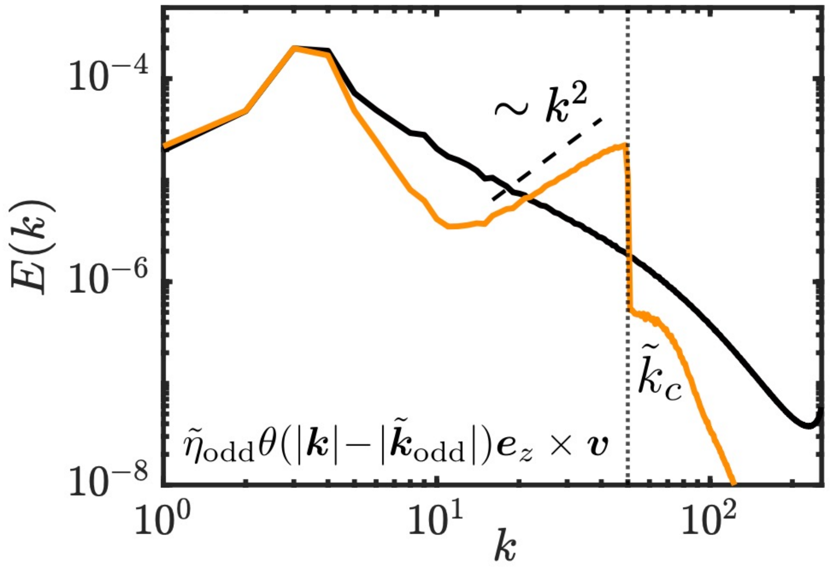
- Odd fluids are a model for more complicated systems that show cascade-induced pattern formation
- Generalization: odd hyperviscosity $\tilde{\eta}_{\mathrm{odd}} \boldsymbol{e}_z \times \Delta^p \boldsymbol{v}$
- Scaling relations:

$$\tilde{\omega}(\boldsymbol{k}) = \pm \tilde{\nu}_{\mathrm{odd}} k_z |k|^{2p-1} ,$$

$$\tilde{k}_{\mathrm{odd}} = \epsilon^{1/(6p-2)} \tilde{\nu}_{\mathrm{odd}}^{-3/(6p-2)} ,$$

$$\tilde{E}(k) \sim \epsilon^{1/2} \tilde{\nu}_{\mathrm{odd}}^{1/2} k^{p-2} \qquad (k \gg \tilde{k}_{\mathrm{odd}}),$$

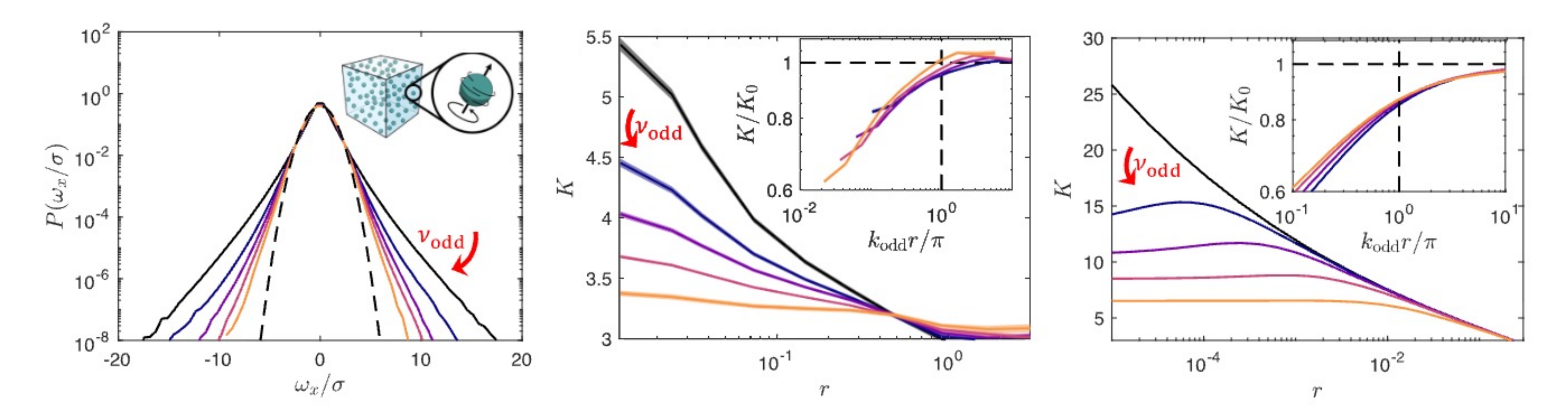
$$\tilde{k}_c \sim \left(\epsilon^{1/2} \nu^{-1} \tilde{\nu}_{\mathrm{odd}}^{-1/2} \right)^{1/(p+1)} .$$



INTERMITTENCY IN ODD FLUIDS

What about intermittency?

INTERMITTENCY IN ODD FLUIDS

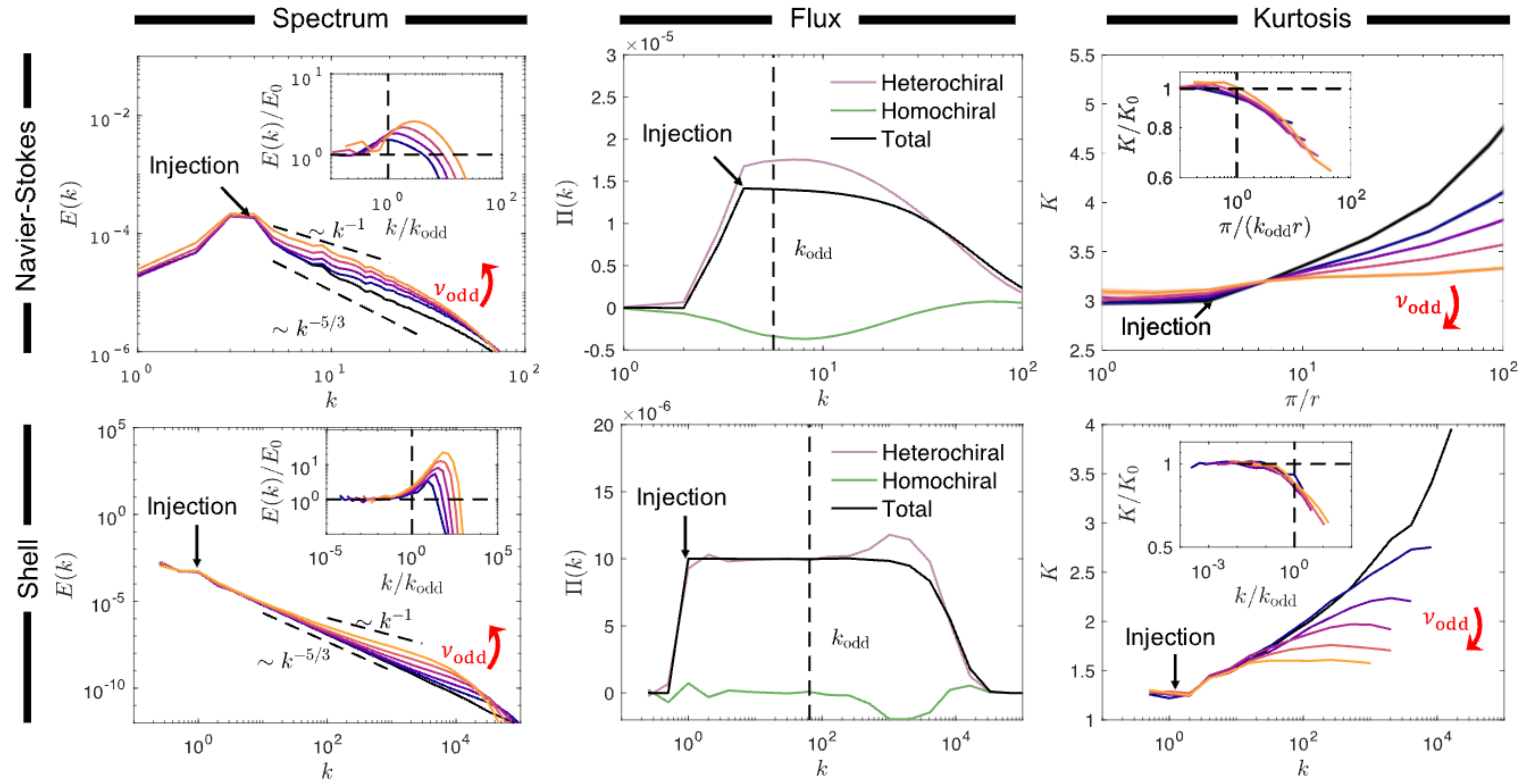


Odd viscosity suppresses intermittency!

Self-similarity is restored for $k > k_{\text{odd}}$

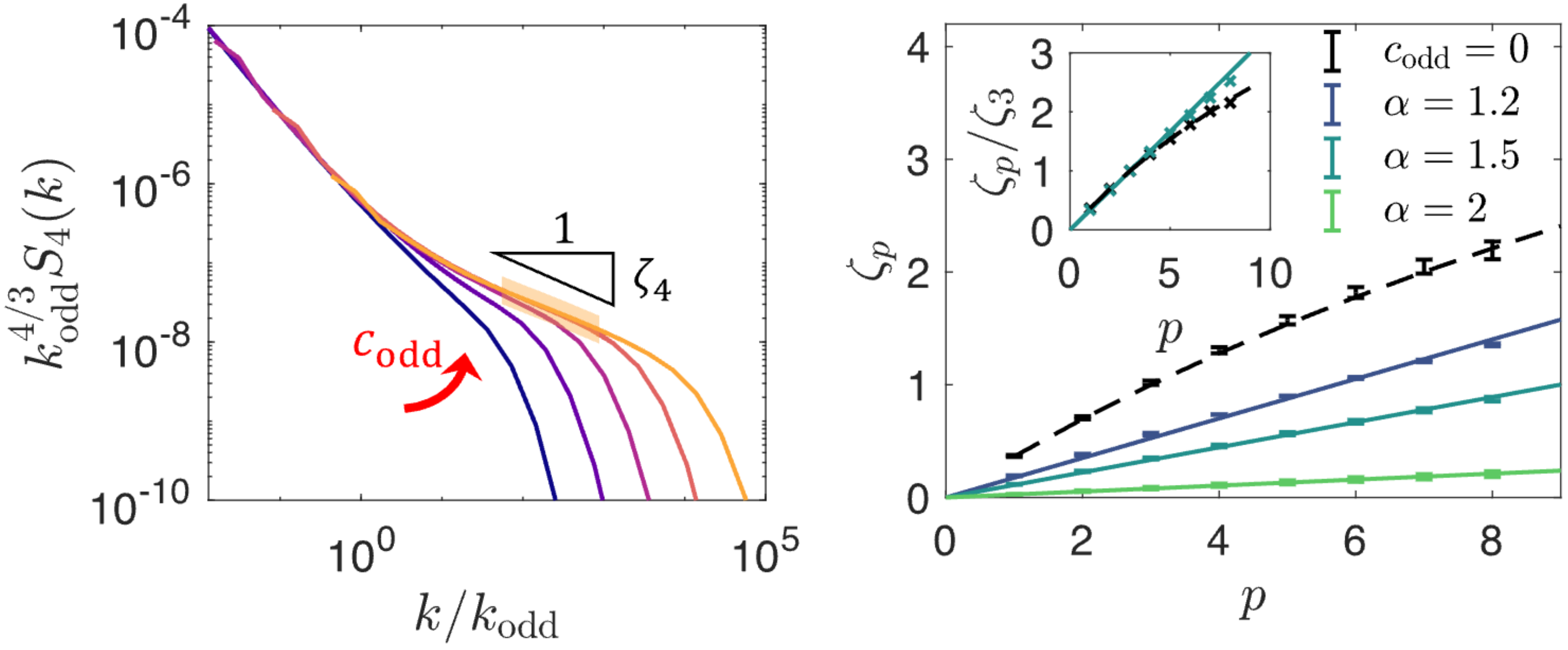
Why? Breaking of λ^h invariance of Navier-Stokes

INTERMITTENCY IN ODD FLUIDS



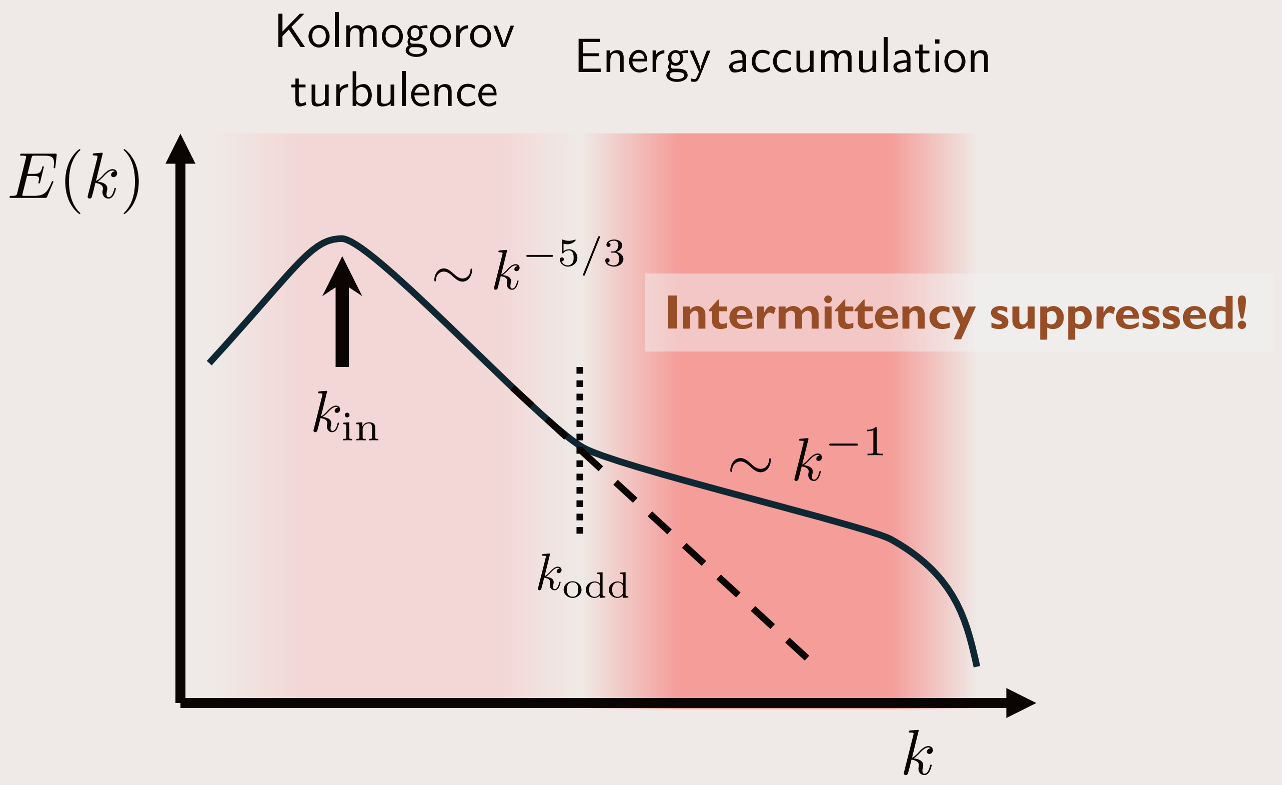
INTERMITTENCY IN ODD FLUIDS

Self-similarity confirmed by scaling exponents of shell model!

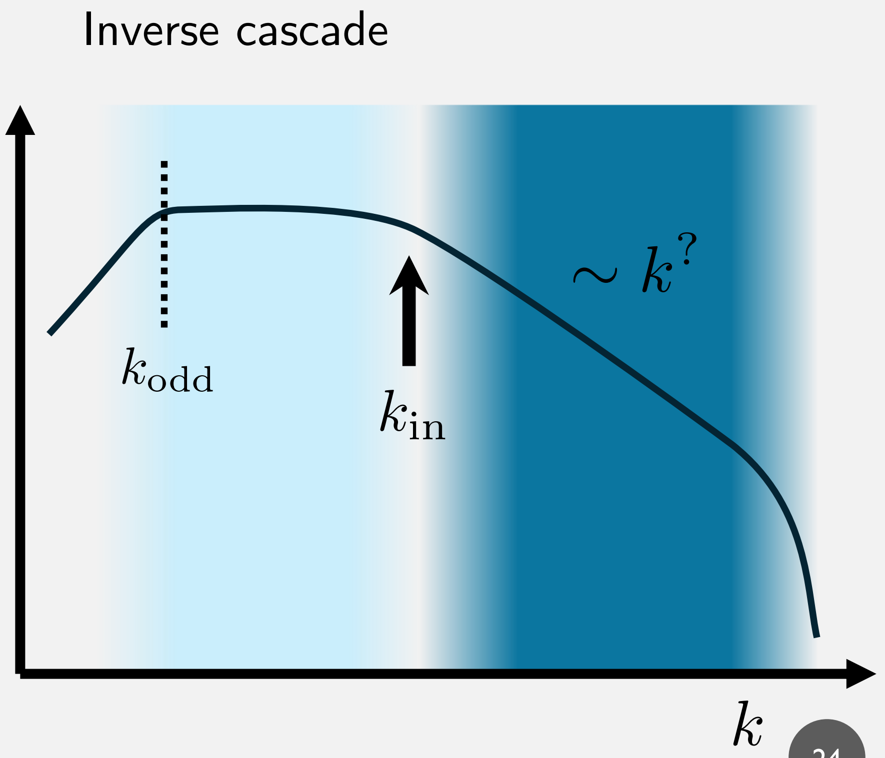


TURBULENT CASCADES

LARGE-SCALE FORCING

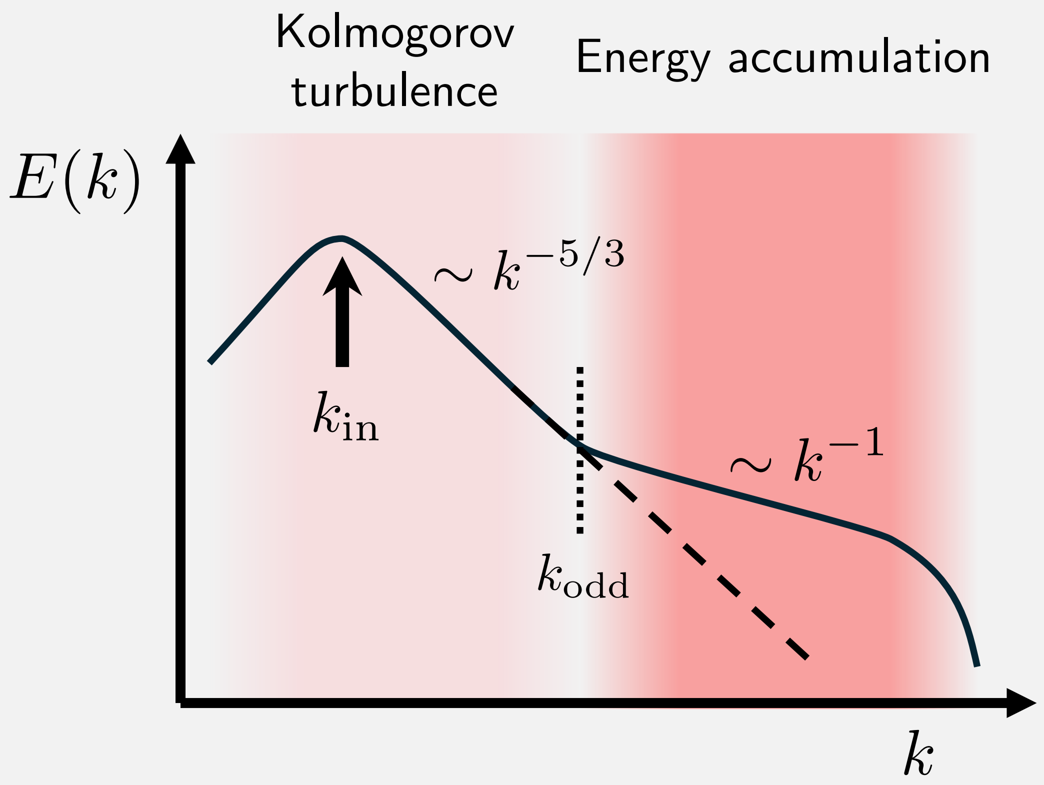


SMALL-SCALE FORCING

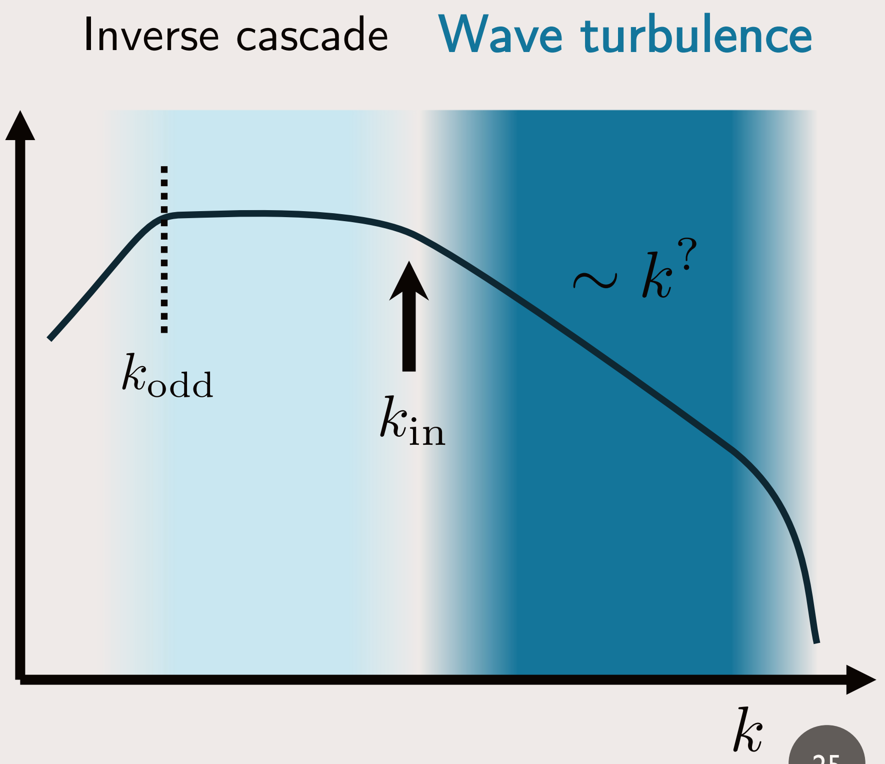


TURBULENT CASCADES

LARGE-SCALE FORCING



SMALL-SCALE FORCING

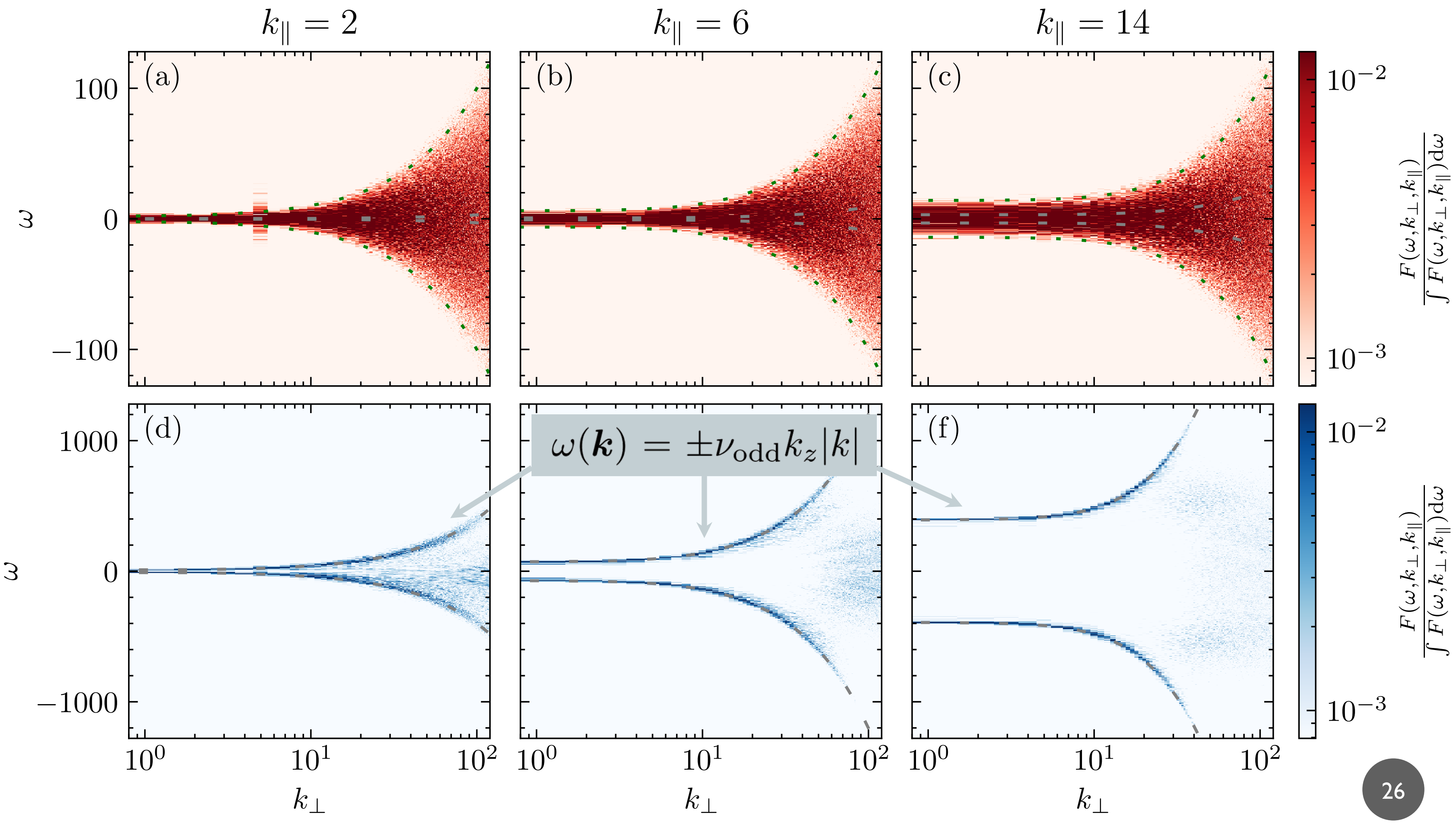


ODD WAVE TURBULENCE

In collaboration with Sebastien Galtier

LARGE-SCALE FORCING

SMALL-SCALE FORCING



ODD WAVE TURBULENCE

- Weak wave turbulence theory:
  - Kinetic equation
  - Exact solution (Kolmogorov-Zakharov spectrum):

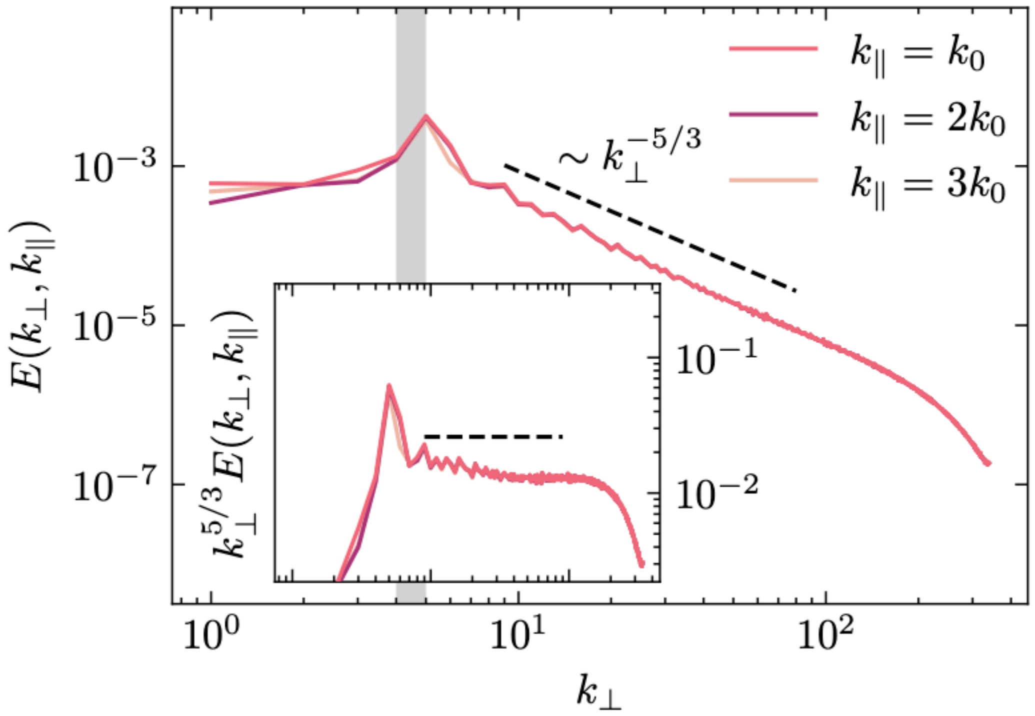
$$E(k_\perp, k_\parallel) \sim \sqrt{\varepsilon \nu_{\mathrm{odd}}} k_\perp^{-3/2} k_\parallel^{-1/2}$$

  - Energy cascade is anisotropic and forward

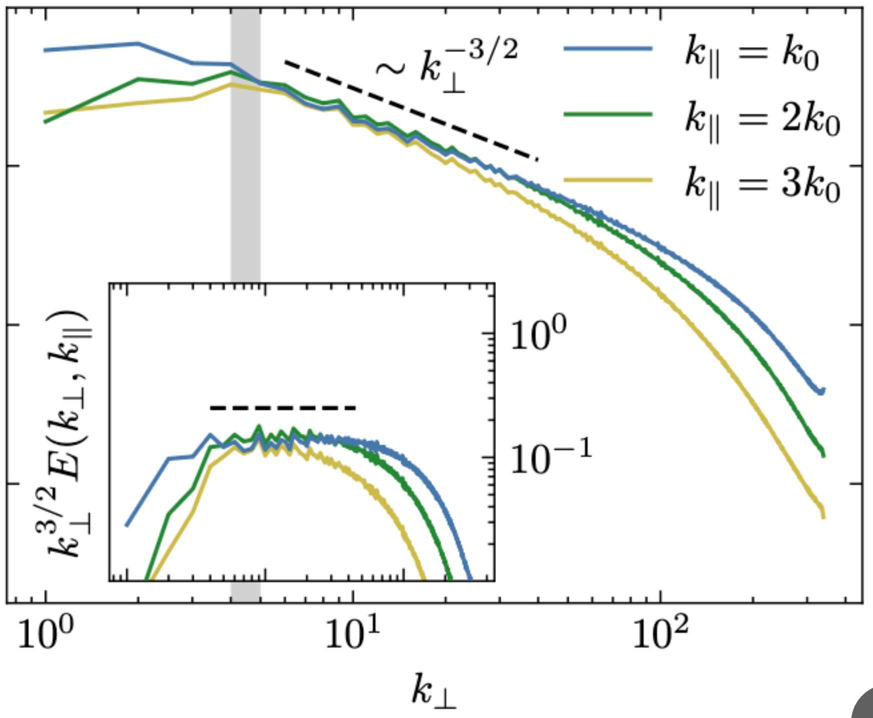
ODD WAVE TURBULENCE

Spectral scaling can be tested in DNS!

STRONG TURBULENCE



WEAK TURBULENCE

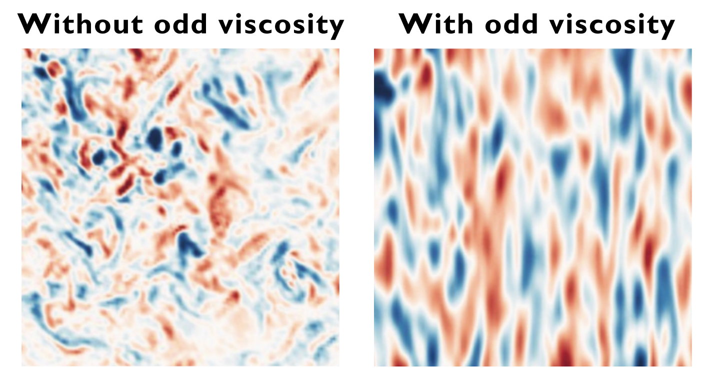


SUMMARY

Odd viscosity gives rise to *odd waves*, which completely changes the dynamics

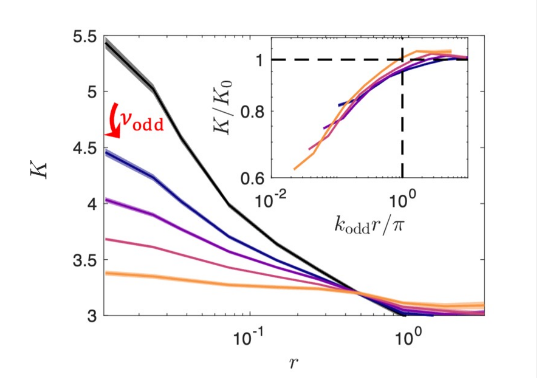
Non-dissipative arrest of turbulent cascades: pattern formation

de Wit et al. (2024) *Pattern formation by turbulent cascades*, Nature **627** (8004), 515-521



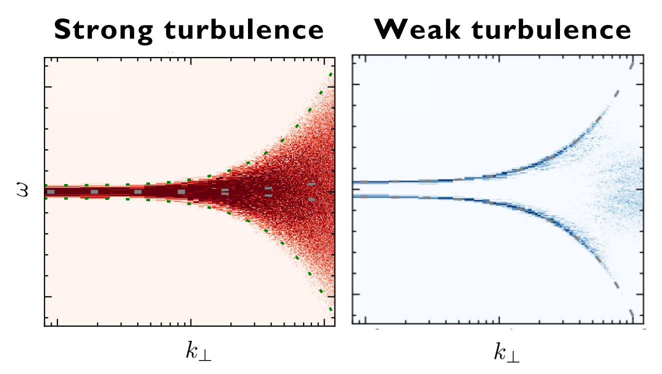
Suppression of intermittency

Chen et al. (2024) *Odd viscosity suppresses intermittency in direct turbulent cascades*, Phys. Rev. Lett. **133** (14), 14402



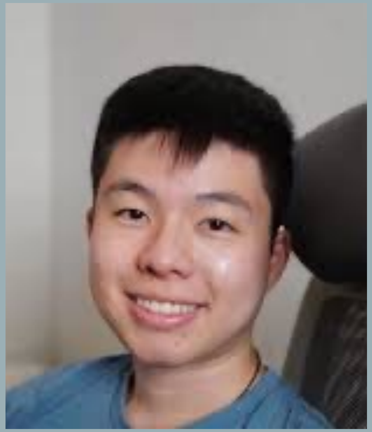
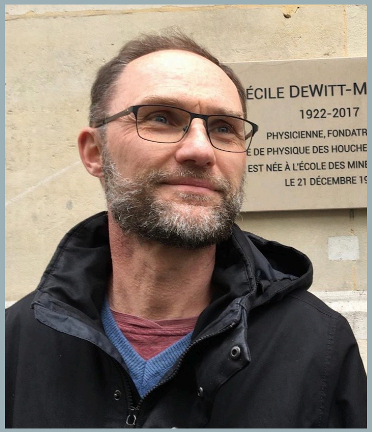
Odd wave turbulence

de Wit et al. (2025) *Non-Hermitian wave turbulence*, arXiv:2504.15403



THANK YOU!



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