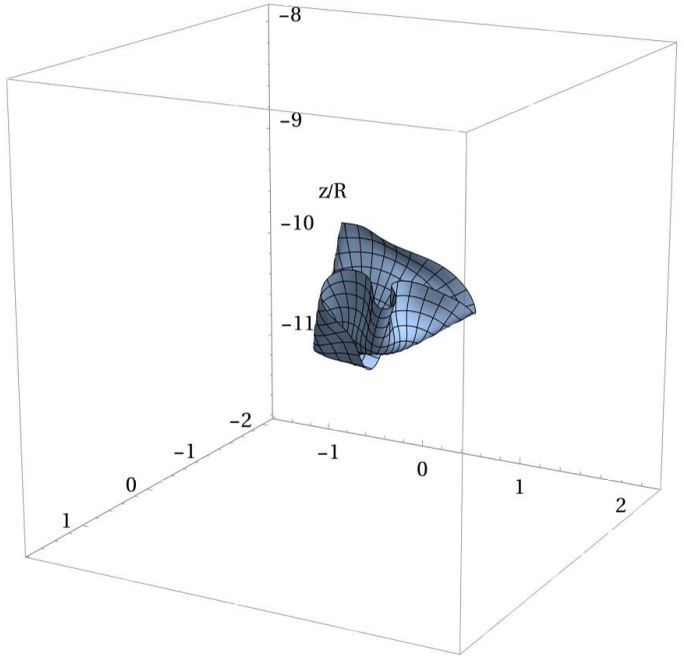
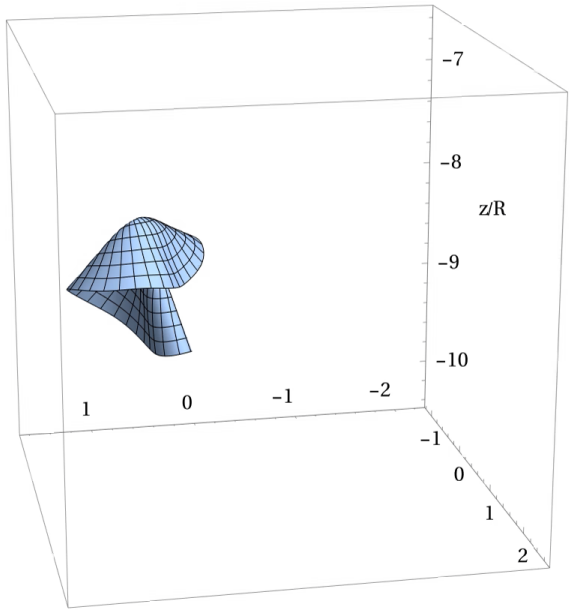
Inertialess sedimentation of rigid and elastic sheets

Tymoteusz Miara, Christian Vaquero-Stainer, Draga Pihler-Puzović, Matthias Heil and Anne Juel

Manchester Centre for Nonlinear Dynamics, University of Manchester

Light-scattering reconstruction of transparent shapes using neural networks.
Miara et al. Under review





Chiral particles and odd fluids, Eindhoven, January 2026

MANCHESTER 1824
The University of Manchester

Inertialess sedimentation of rigid and elastic sheets

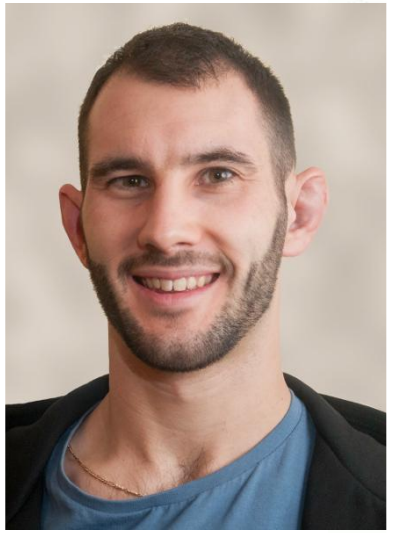
Experiments

Numerics

Tymoteusz Miara

Anne Juel

Draga Pihler-Puzović



Christian Vaquero-Stainer

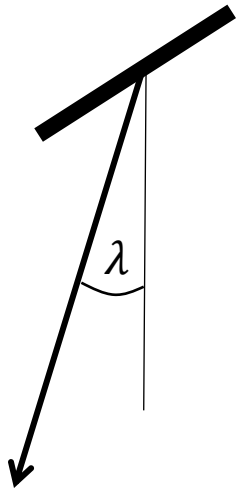
Matthias Heil

Sedimentation of particles: $Re = 0$ vs inertia

Sphere
(Stokes)

Stokes (1851)

Disk or Rod
(Stokes)



Brenner, *Chem. Eng. Sci.* **18** (1963), 1

Coin in water
(Re $\sim 10^3$)

Heisinger et al., *JFM 742* (2014), pp. 243–253

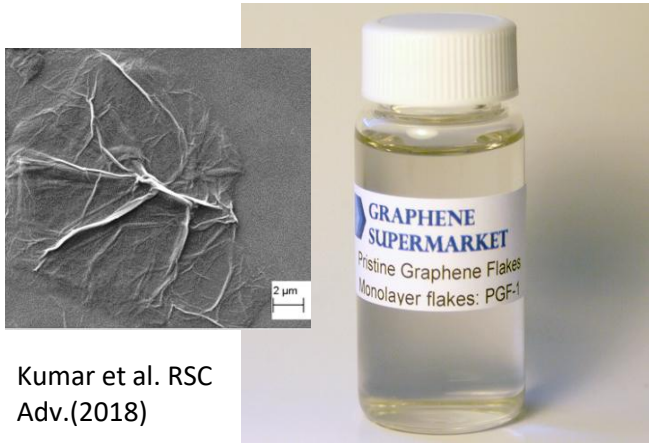
Particle shape determines sedimentation at $Re \to 0$

- For a rigid particle in unbounded flow,

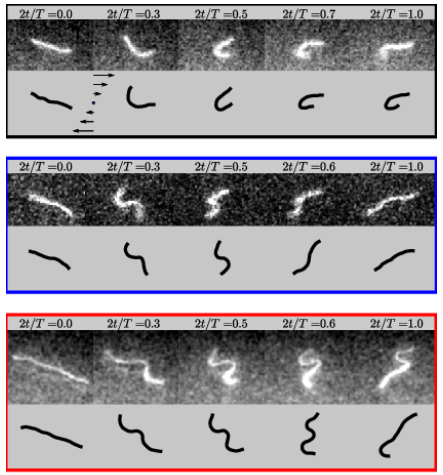
$$\frac{1}{\mu}\begin{pmatrix} \boldsymbol{K} & \boldsymbol{C}^{\boldsymbol{T}} \\ \boldsymbol{C} & \boldsymbol{\Omega} \end{pmatrix}\begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{T} \end{pmatrix} = \begin{pmatrix} \boldsymbol{U_M} \\ \boldsymbol{\omega} \end{pmatrix}$$

  determines particle motion.

- Shape information contained in ***mobility matrix***:
  - Form of matrix imposed by symmetries or centre of mass relative to hydrodynamic centre (Witten, Mehlig, Voth, …)
  - but no general way of inferring its terms…
- New framework needed to infer motion given an arbitrary particle at $Re \to 0$.

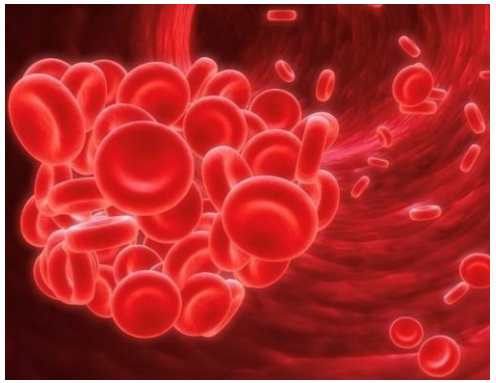


Kumar et al. RSC Adv.(2018)



Bonacci et al. JFM 2022

watergrades.com



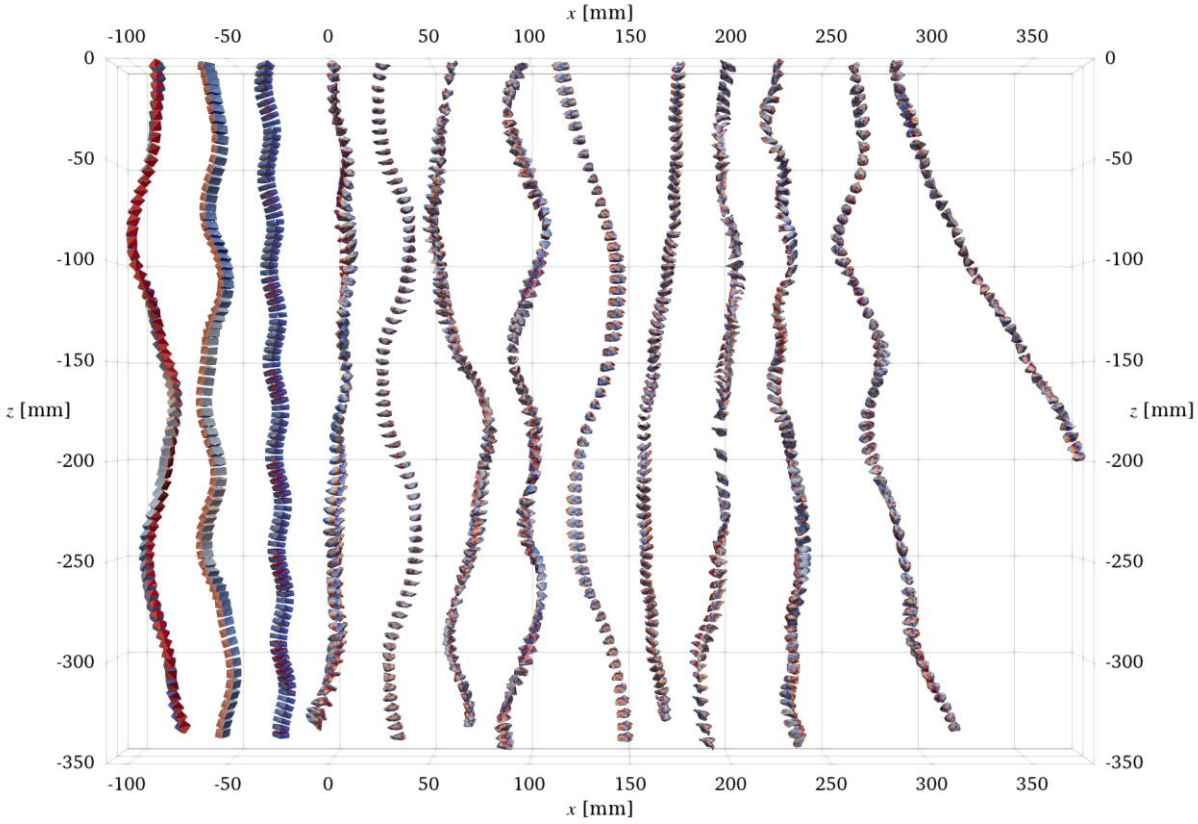
Wintrobe et al. Am. J. Med, Sci. (1935)

Handling of micro/nanoparticles in liquid phase , e.g., graphene flakes, actin filaments.

Application: Separation of recycled plastics

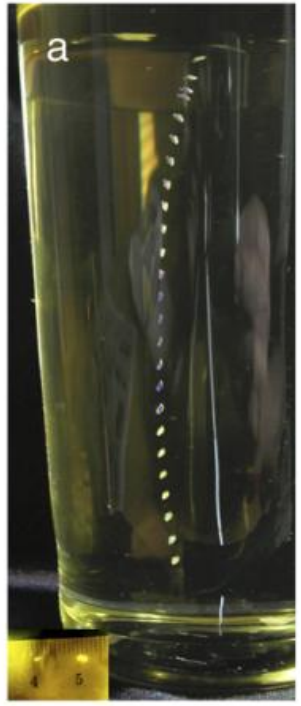
- Plastic waste recycling company.
- Separation with only slight density differences.
- Technology requires a thorough understanding and precise measurement of particle settling.

- Low Re
- Significant effect of shape on translation/rotation coupling.



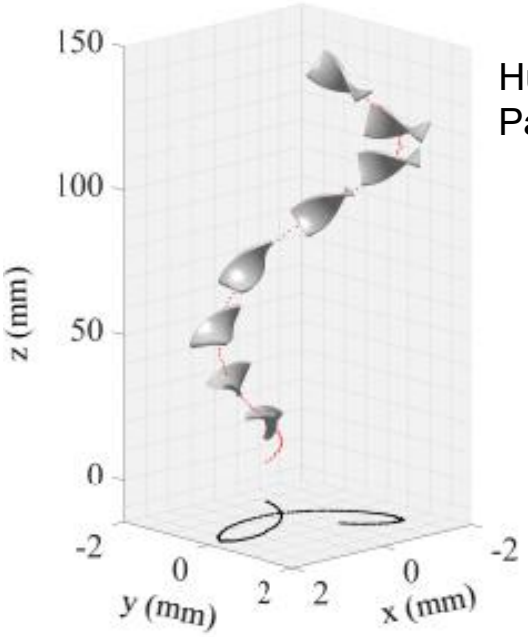
Eberhard, Lundgaard, Walther, Meyer 2024
Particle Tracking of Orientation and Velocity of Arbitrary Shape (doi:10.55037/lxlaser.21st.164)

Fluttering of rigid chiral particles at $Re = 0$



Witten & Diamant (2020)

Helix



Huseby et al. (2025)
Palusa et al. PRF (2018)

Propellers

Doi & Makino (2005)

- Periodic reorientation due to translation-rotation coupling.

$$\frac{1}{\mu}\begin{pmatrix} M_{11} & 0 & 0 & 0 & M_{51} & 0 \\ 0 & M_{22} & 0 & M_{42} & 0 & 0 \\ 0 & 0 & M_{33} & 0 & 0 & 0 \\ 0 & M_{42} & 0 & M_{44} & 0 & 0 \\ M_{51} & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{pmatrix}$$

- Chiral trajectories with handedness determined by the particle shape.

1. Can an **achiral** particle flutter at $Re \to 0$?

Sedimentation of flexible particles at $Re \to 0$

- Balance between gravity, viscous drag and elastic forces.

- Fluid-structure interaction parameter:

$$E_G = \frac{\text{Gravitational force}}{\text{Elastic bending forces}}$$

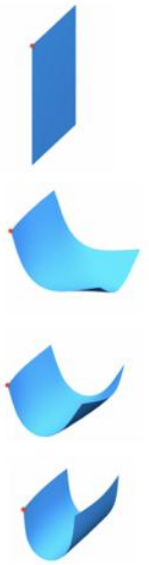
- Quasi-static, in effect a different mobility matrix for each shape.

Flexible fibre

Marchetti et al., *Phys. Rev. Fluids* **3**, (2018) 104102

Experiments + Numerics

Elastic sheet



Yu & Graham, *Phys. Rev. Fluids* **9**, (2024) 054104

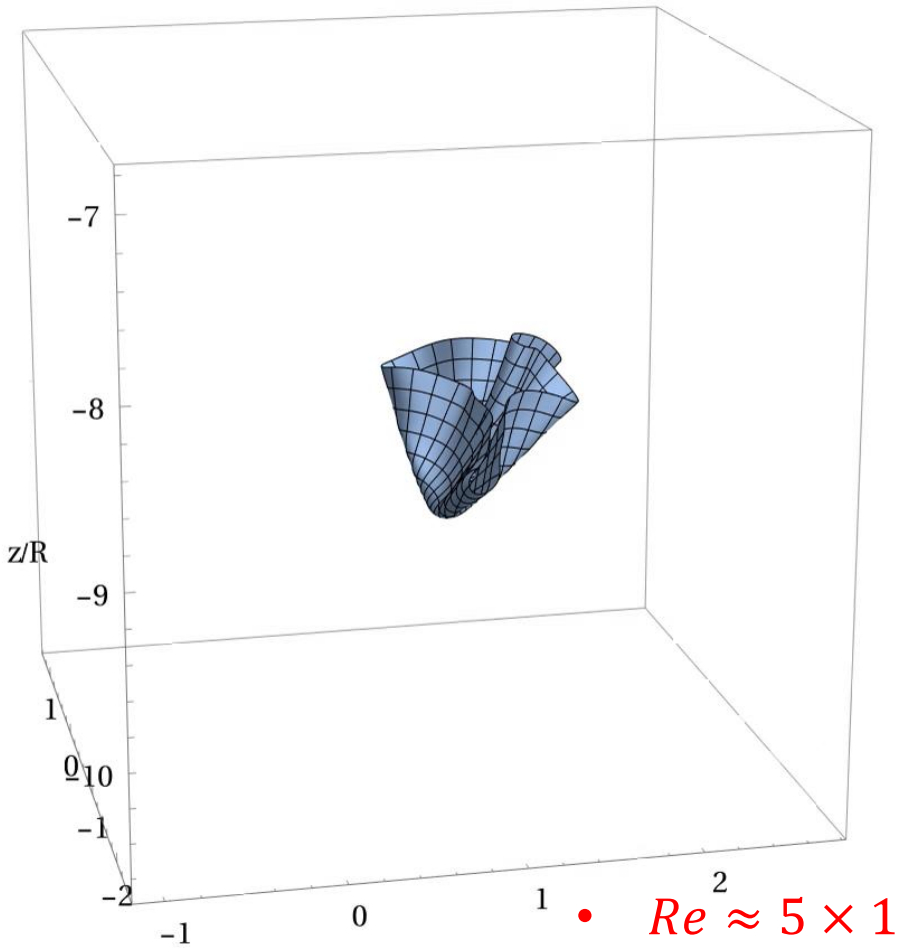
Numerics

$Re = 0$ 'bead-spring' model

Stable steady state.

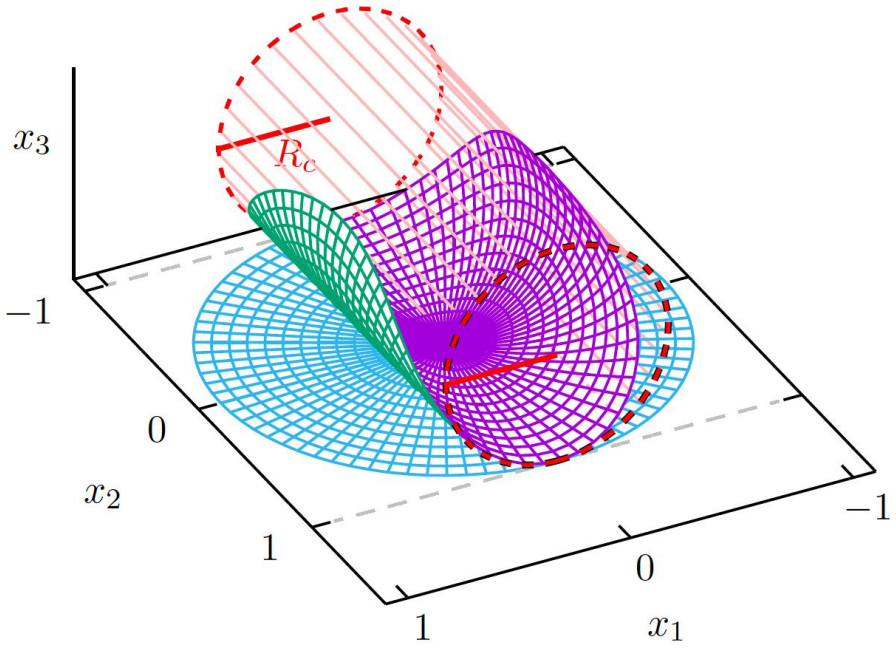
2. Can elastic sheets exhibit flutter behaviour?

Crumpled disk relaxes into U-shape (or ‘pringle’)



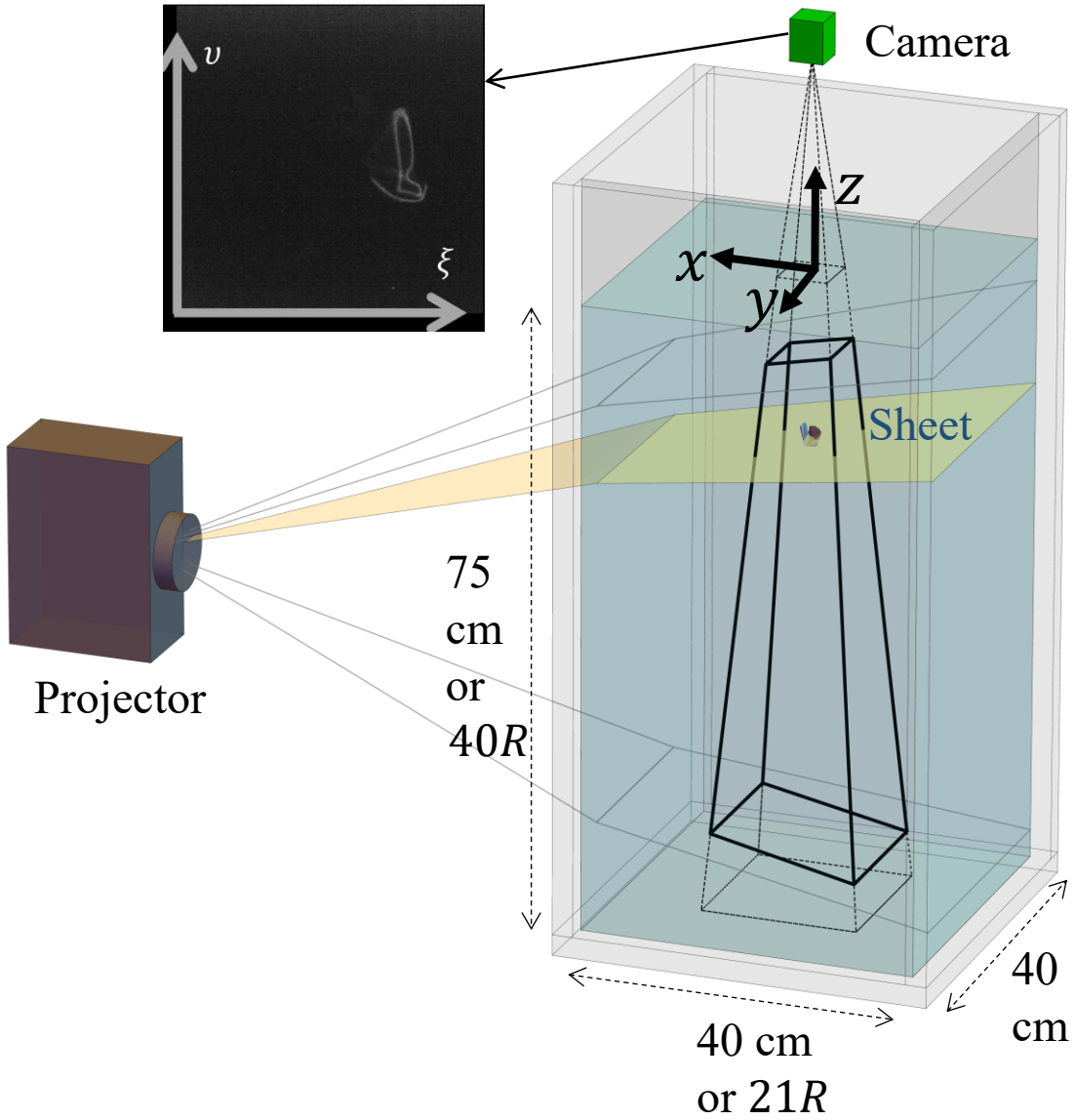
- $Re \approx 5 \times 10^{-6}$
- $E_g \approx 83$ (similar to graphene)

- Rigid particle
- Doubly symmetric U-shaped disk



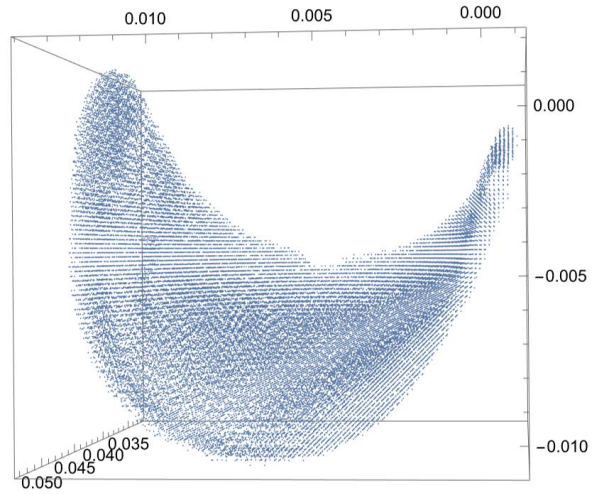
Non-chiral!

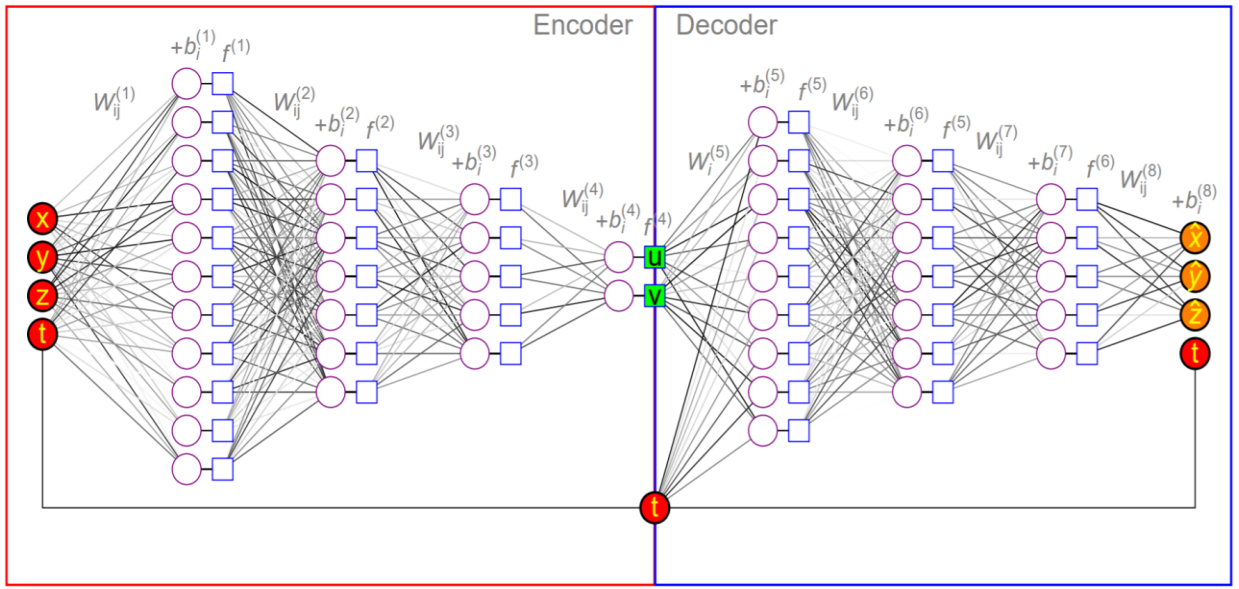
Experimental set-up



Elastic PDMS disk (soaked in silicone oil):

- $R = 19.0 \pm 0.05\text{mm}$
- $h = 52.7 \pm 2\mu\text{m}$
- $E = 8.8 \times 10^5$ Pa
- $\overline{U} = 0.27$ mm/s

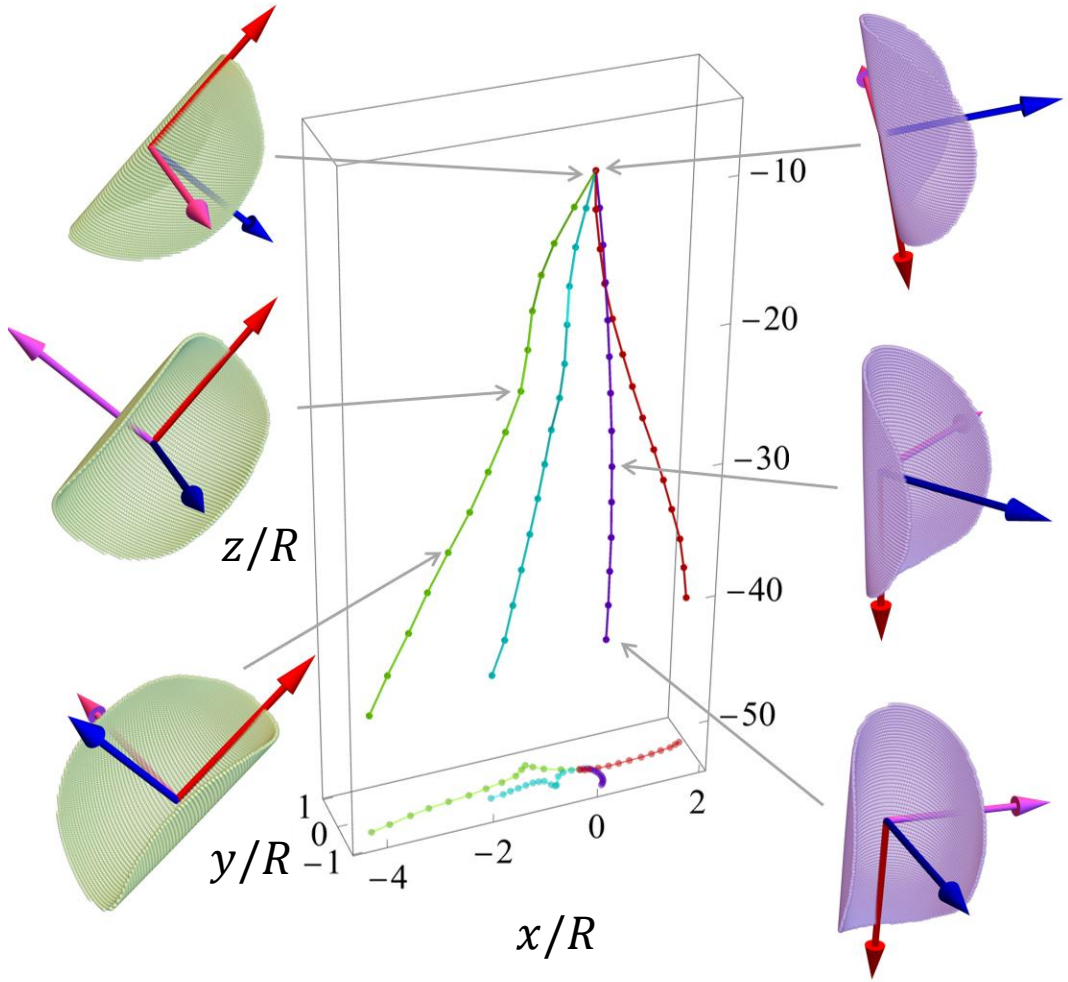


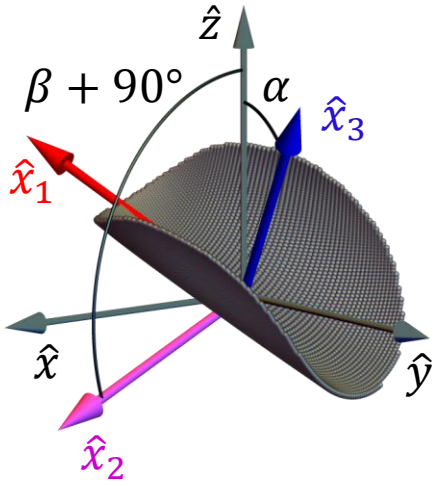


Uncertainties: ~2% sheet's boundary
~3% in curvature

Miara, Pihler-Puzović, Heil & Juel Under review (arXiv:2311.02970)

Fixing the shape: motion of rigid U-shaped disk





Fixing the shape: body-fitted coordinates

1. Translation of the CoM is not dynamically-relevant
2. Rotations around vertical irrelevant => inclination described by two coordinates

⇒ Only 2 orientational degrees of freedom, no chaos

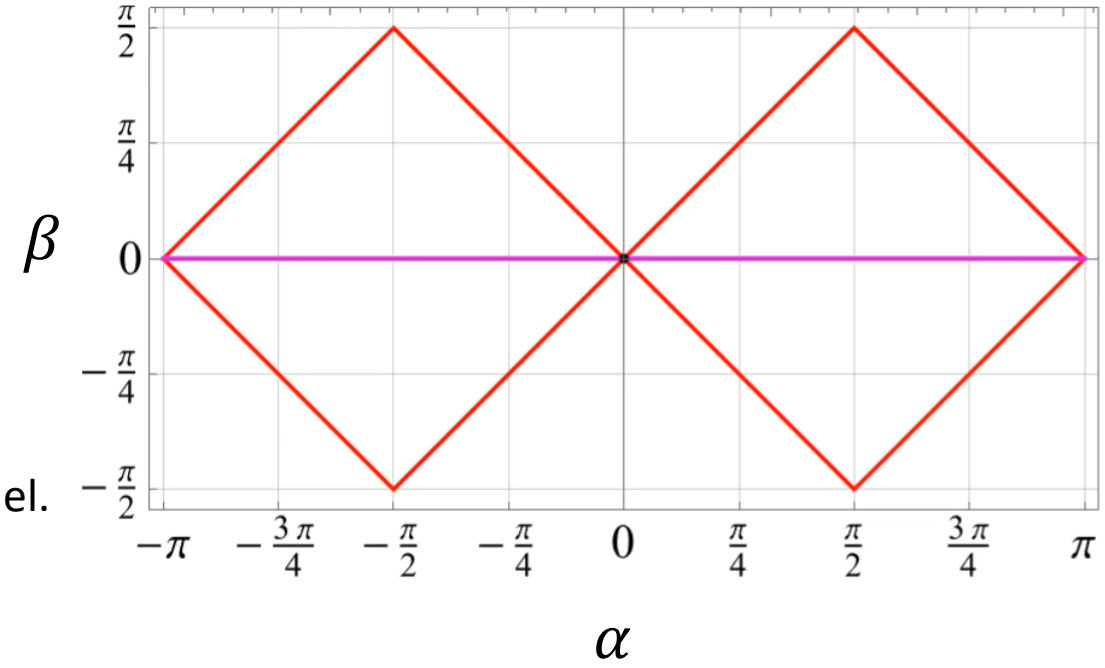
Principal Axes:

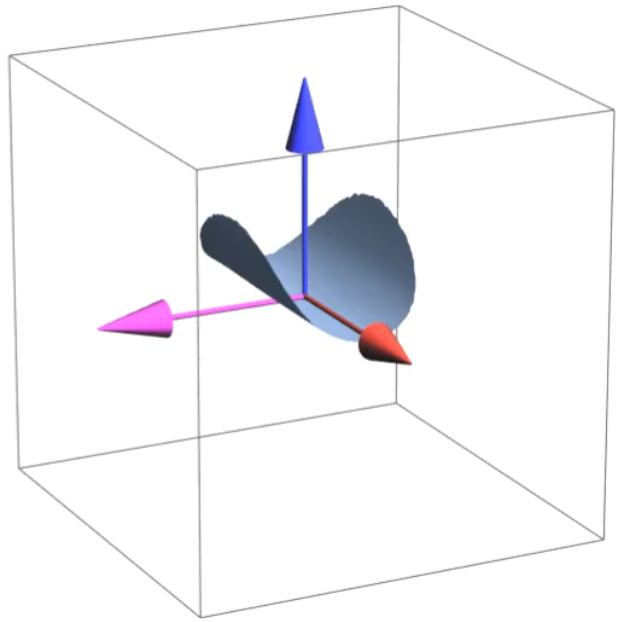
Roll axis $\hat{x}_1$

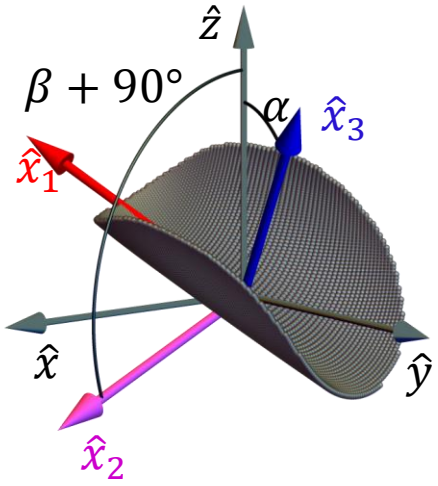
Pitch axis $\hat{x}_2$

Normal $\hat{x}_3$

Miara, Vaquero-Stainer, Pihler-Puzović, Heil & Juel. Comms. Phys. 2024







Reorientation of the rigid U-shaped disk

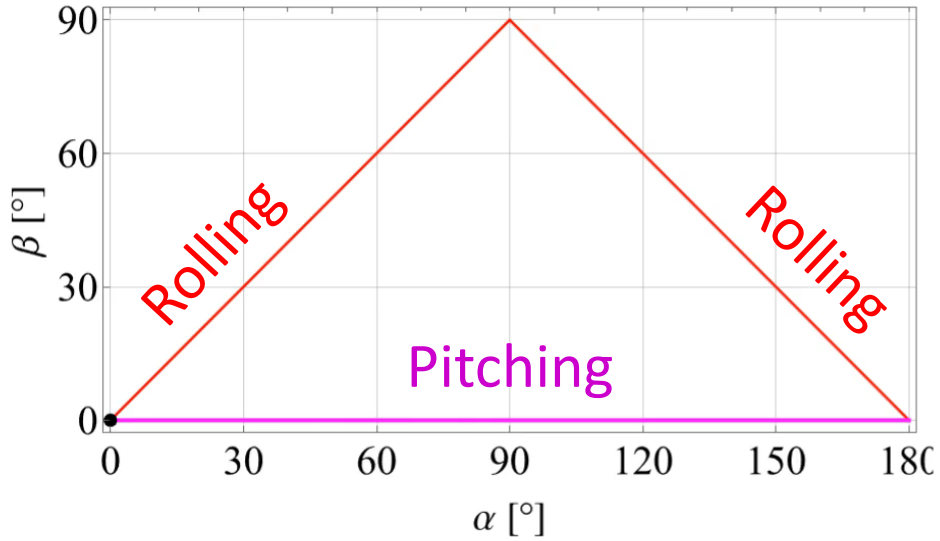
Two planes of symmetry!

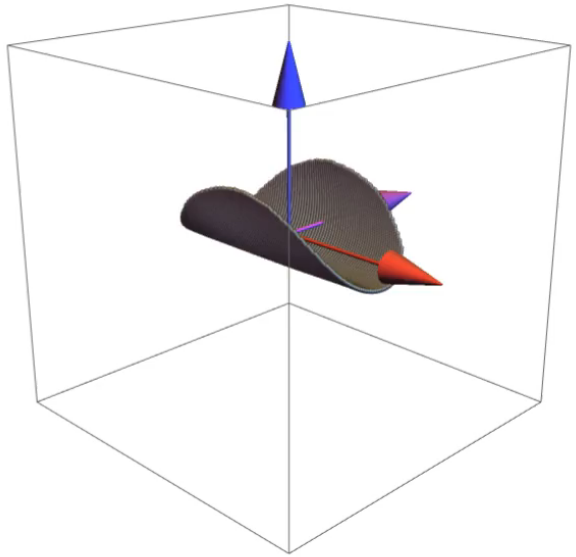
Principal Axes:

Roll axis $\hat{x}_1$

Pitch axis $\hat{x}_2$

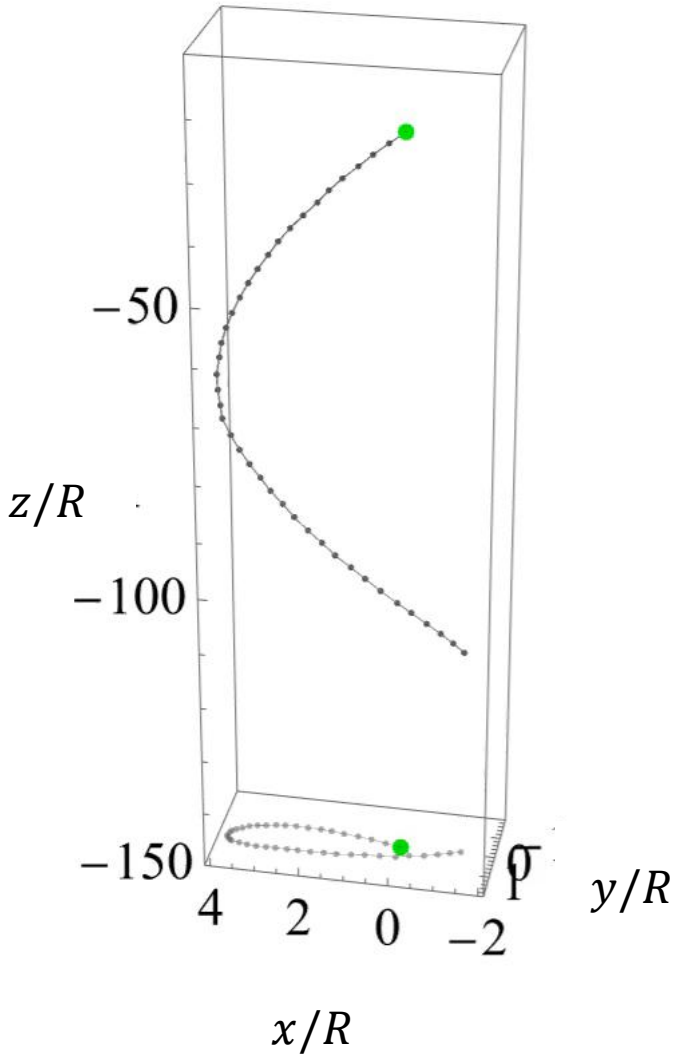
Normal $\hat{x}_3$

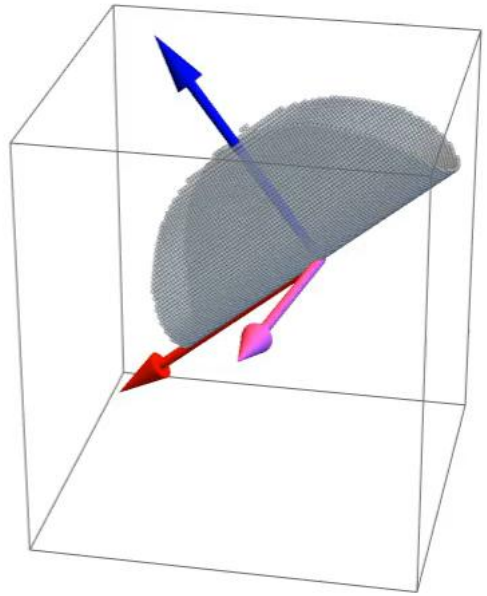




See also Joshi & Govindarajan PRL 2025
for theoretical predictions of sedimentation of bodies with two planes of symmetry.

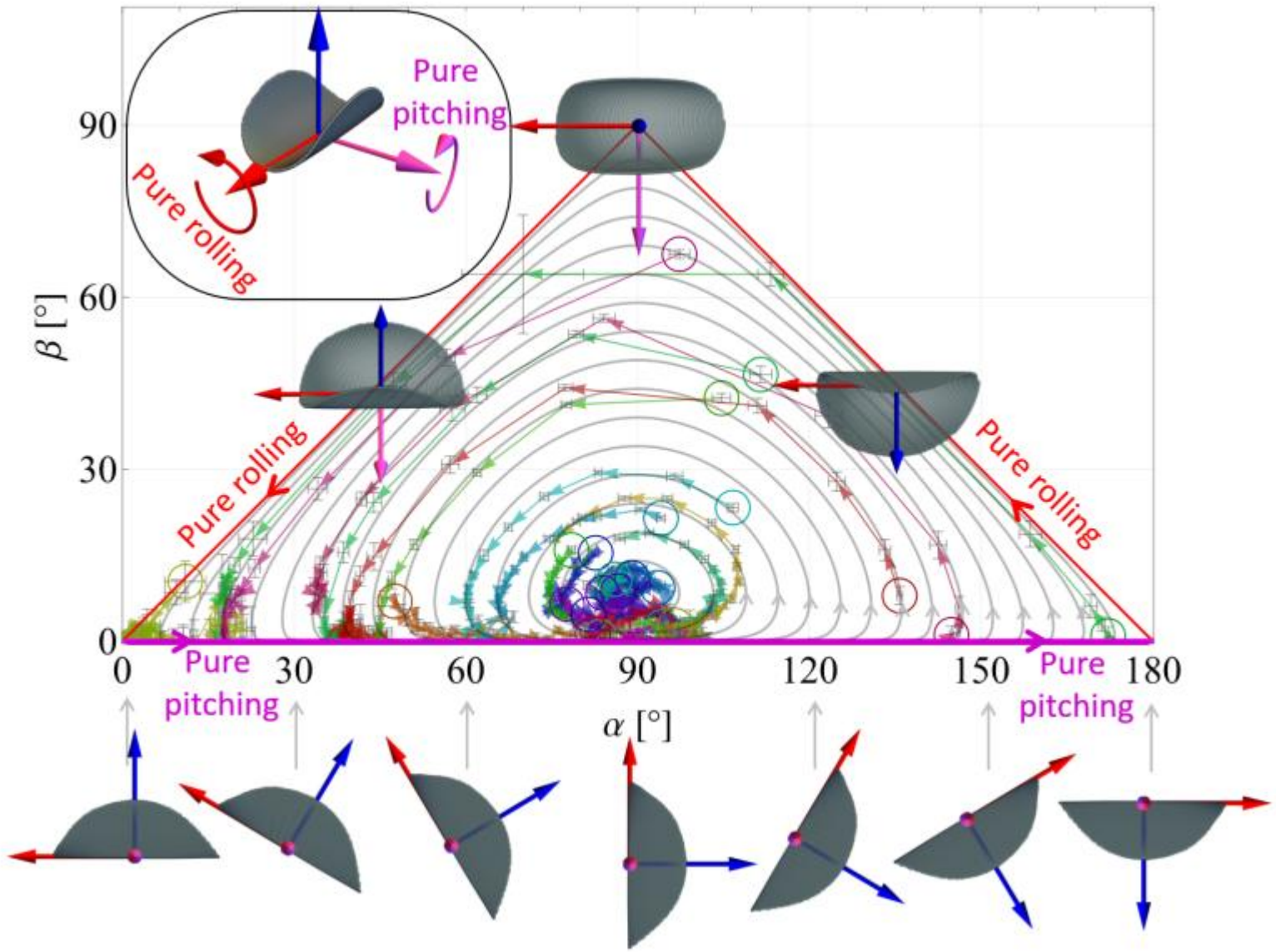
Motion of rigid U-bent disk in the experiment





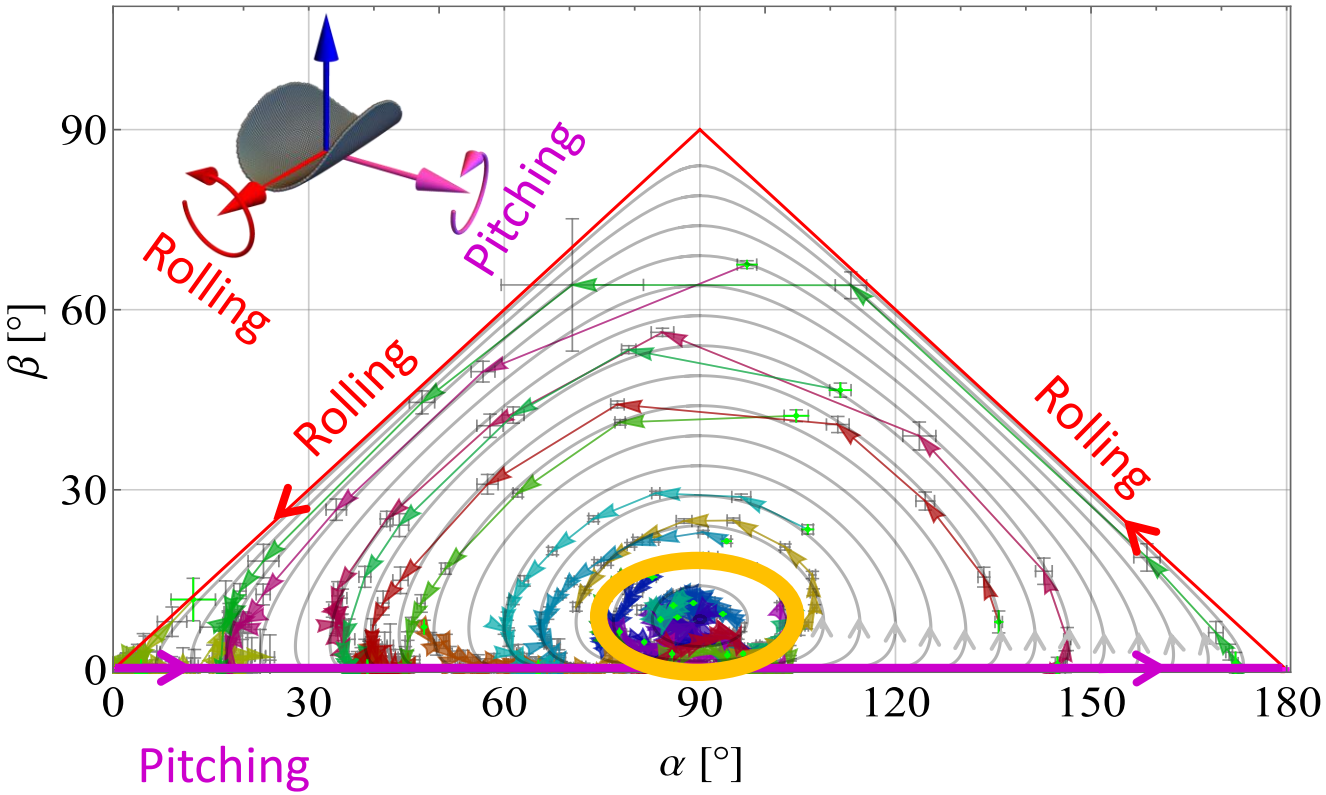
The sheet travels ~45 diameters

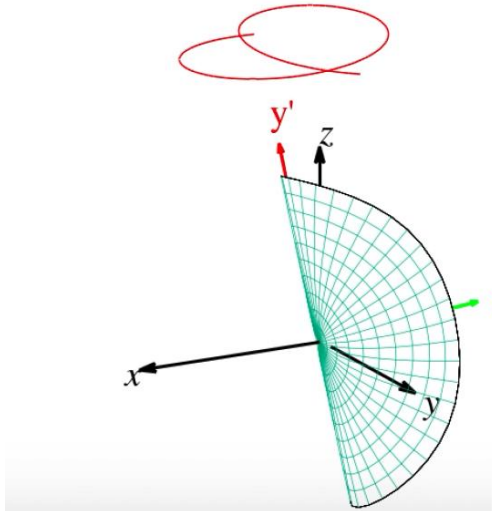
Periodic reorientation of rigid U-bent disk

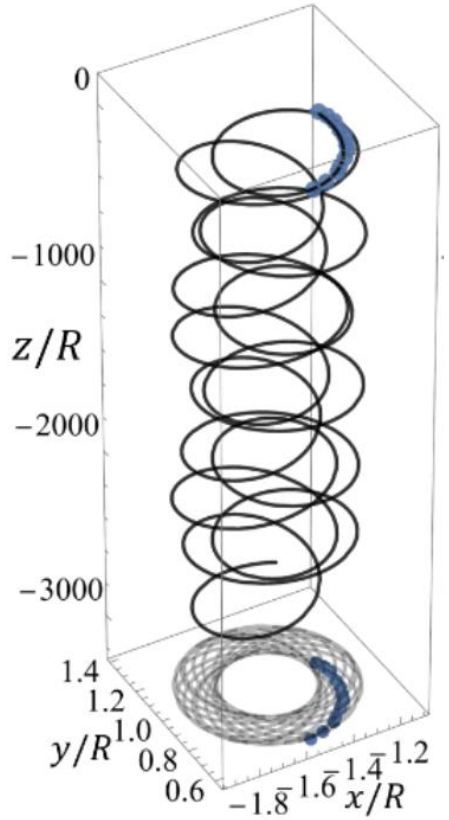


Miara, Vaquero-Stainer, Pihler-Puzović, Heil & Juel
Comms. Phys. 2024

Motion from fitted mobility matrix

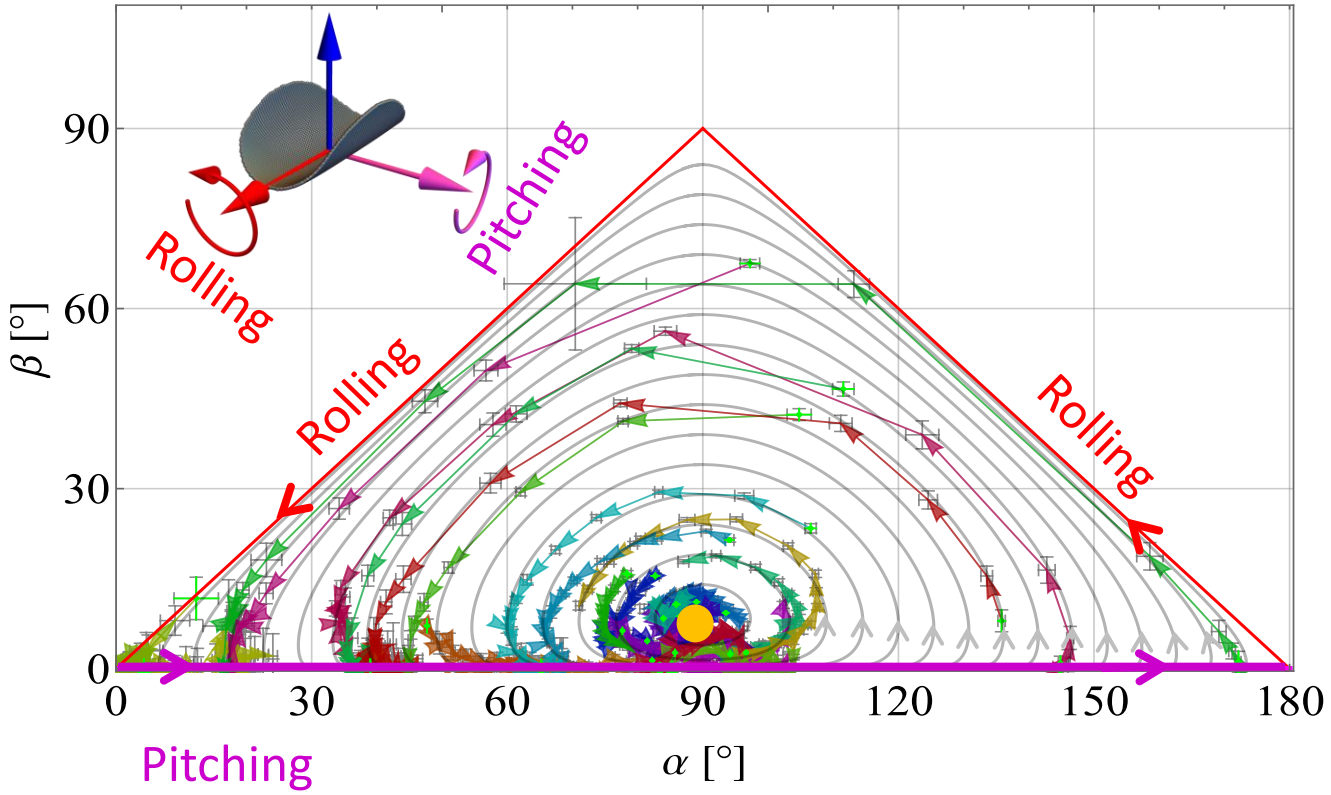


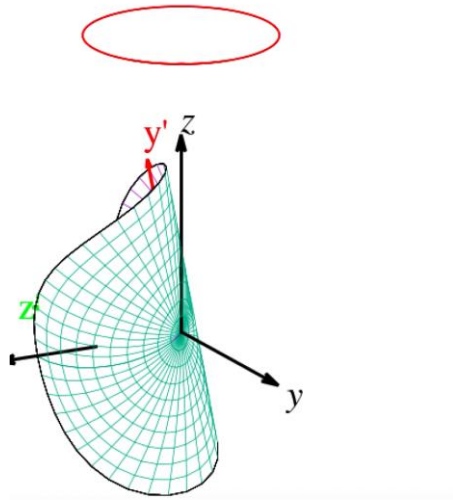


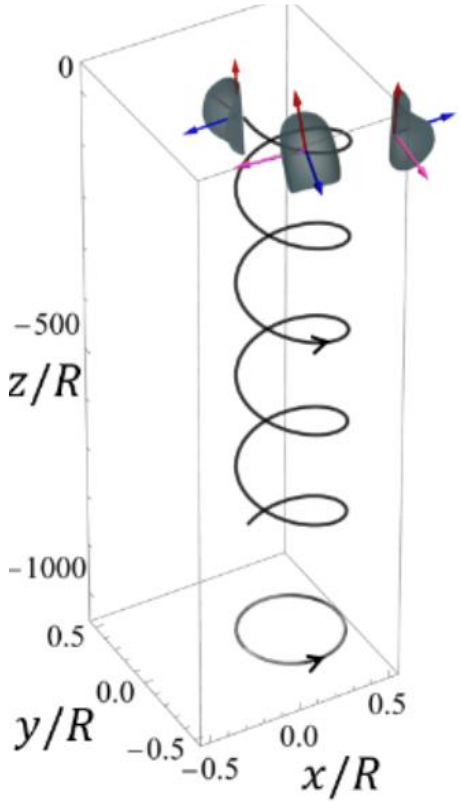


- No steady state in laboratory frame!

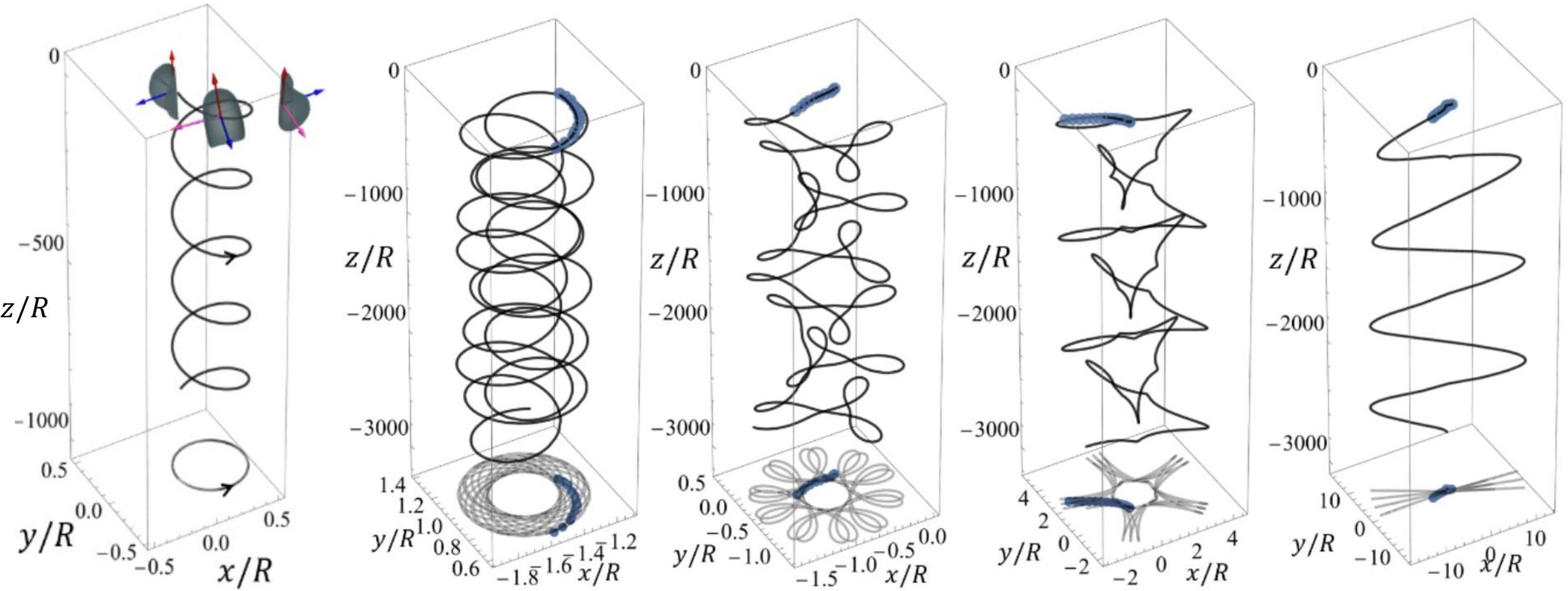
Motion from fitted mobility matrix







Changing initial conditions



away from dynamical centre

- Chiral sedimentation with handedness determined by initial orientation.

Miara et al. 2024 *Comm. Phys.*

Finding the mobility matrix

From experiments:

- Guess form of the mobility matrix and integrate equations to predict CoM trajectories.
- Perform least-squares fit to experimental CoM trajectories.
- Iterate until the actual components of the mobility matrix are obtained.

From numerics:

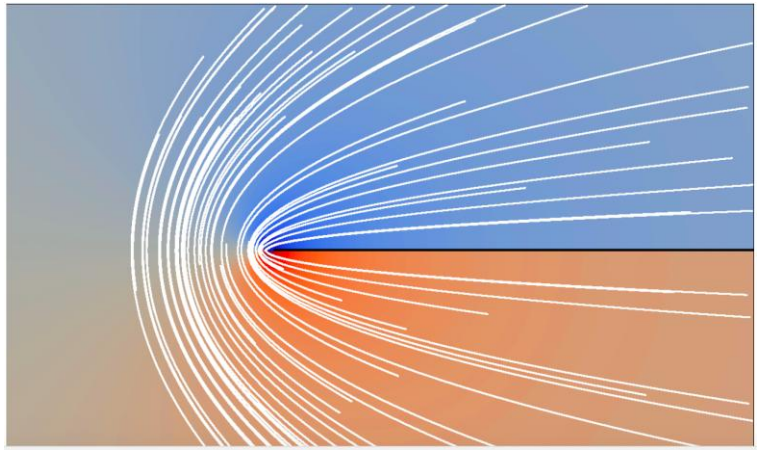
- The mobility matrix is assembled directly (it's a function of geometry).
- It's a 6 x 6 mobility matrix: requires 6 full FEM solves to assemble.

➢ Vaquero-Stainer, Miara et al. JFM 2024
U-shaped disks in Stokes flow: chiral sedimentation of a non-chiral particle.

➢ Vaquero-Stainer, Miara et al. PRF 2026
From fluttering to drifting in achiral particles.

Direct numerical simulations

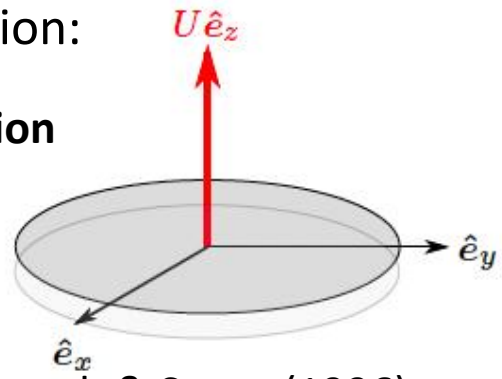
- Known singularities in pressure and velocity gradient in flow around sharp edge.
- 30 % of total drag from outmost 5% of the radius.



- Assume form of singularity is known.
- Subtract singularity expression in vicinity of edge.
- Use augmented FEM to compute regular solutions and the parameters associated with the singularity.

- Validation against analytical expression:

Broadside translation



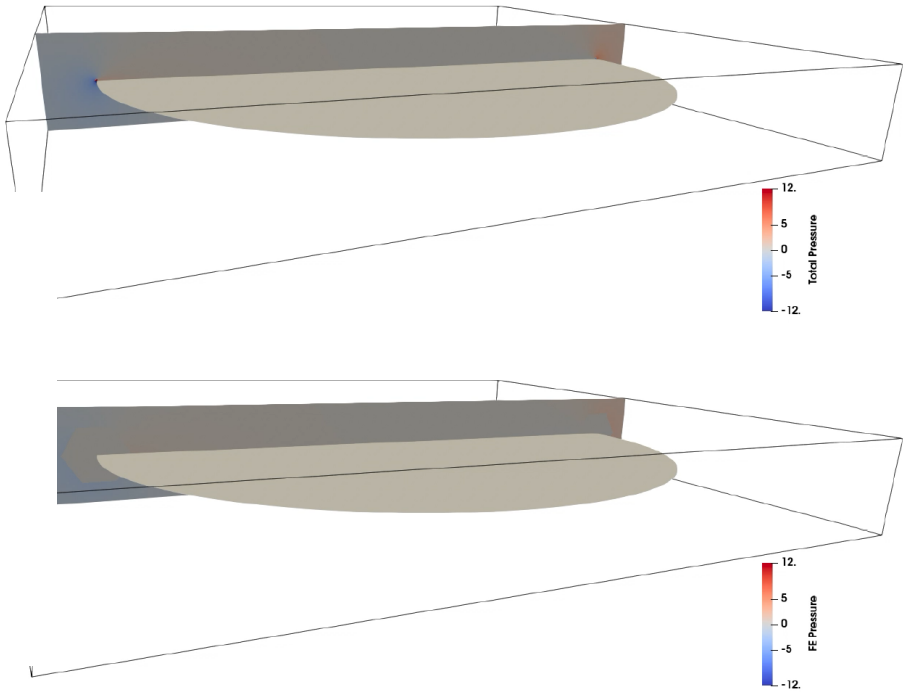
Tanzosh & Stone (1996)

oomph-lib

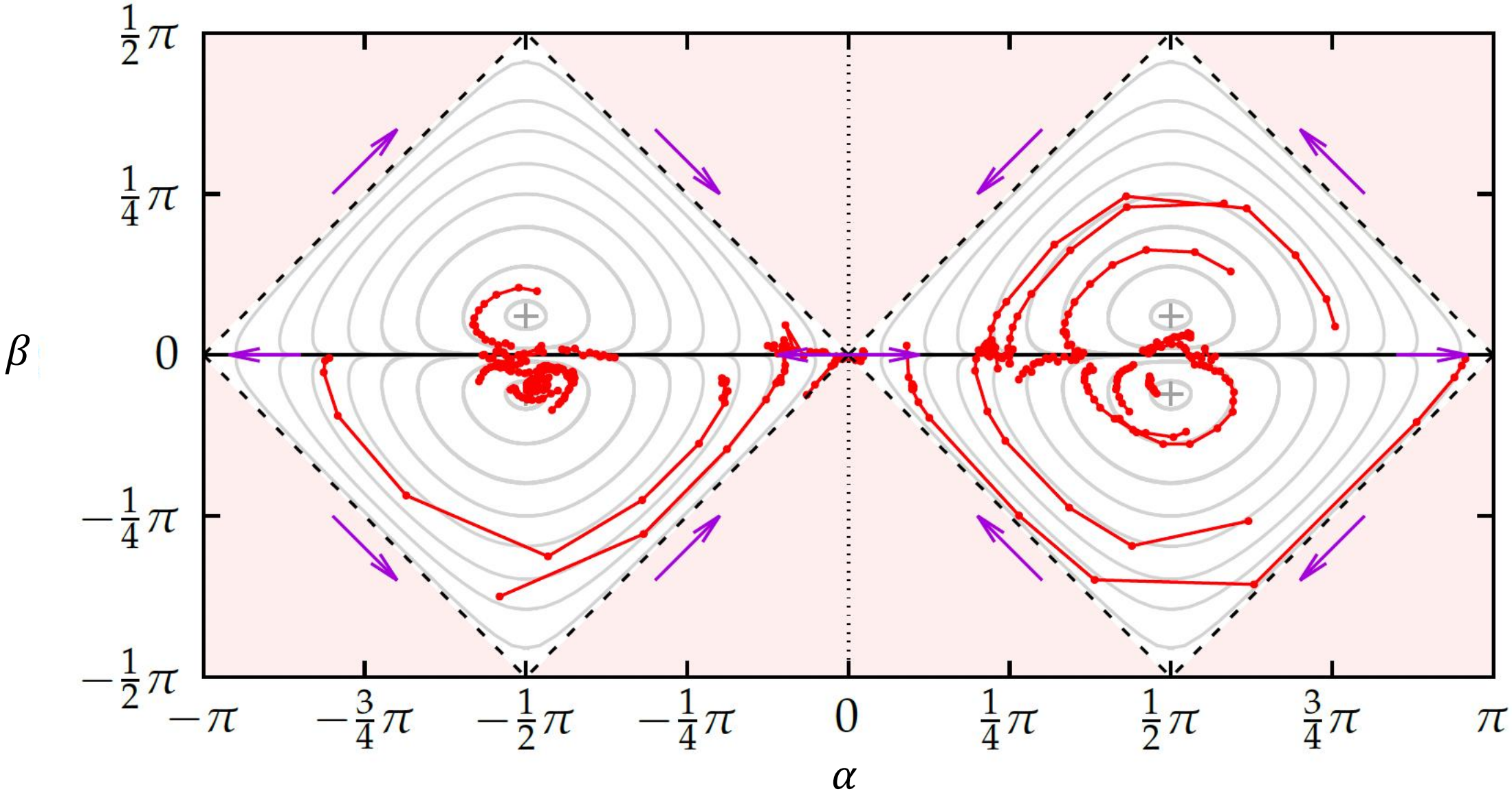
Heil & Hazel (2006)

Numerical Results

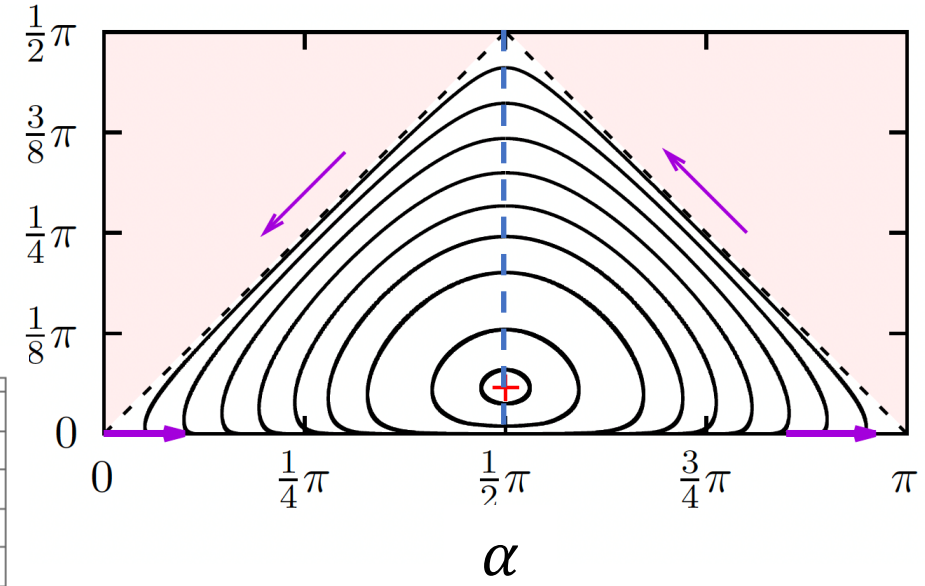
Quasi-static

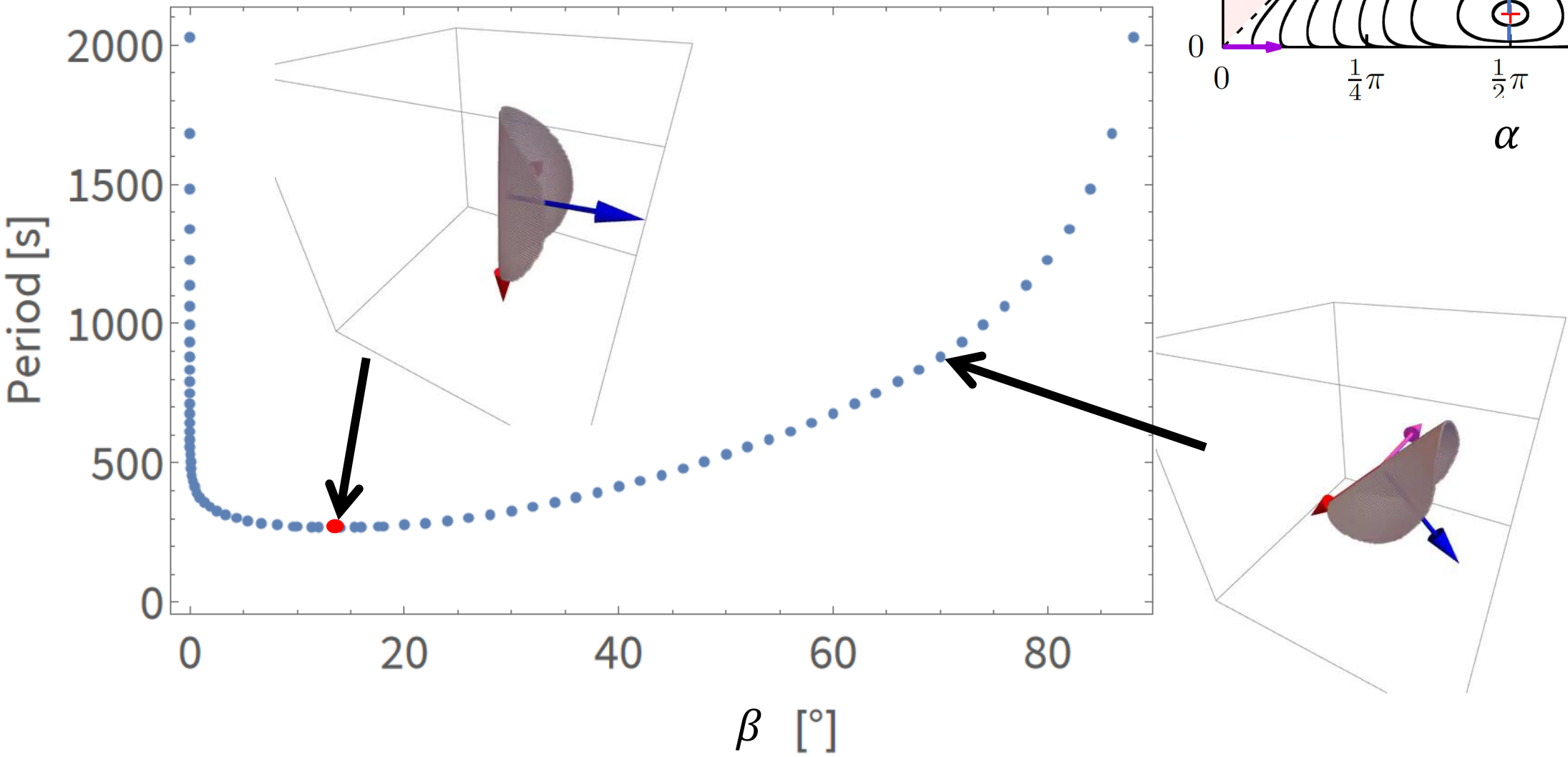


Comparison: numerics vs experiments

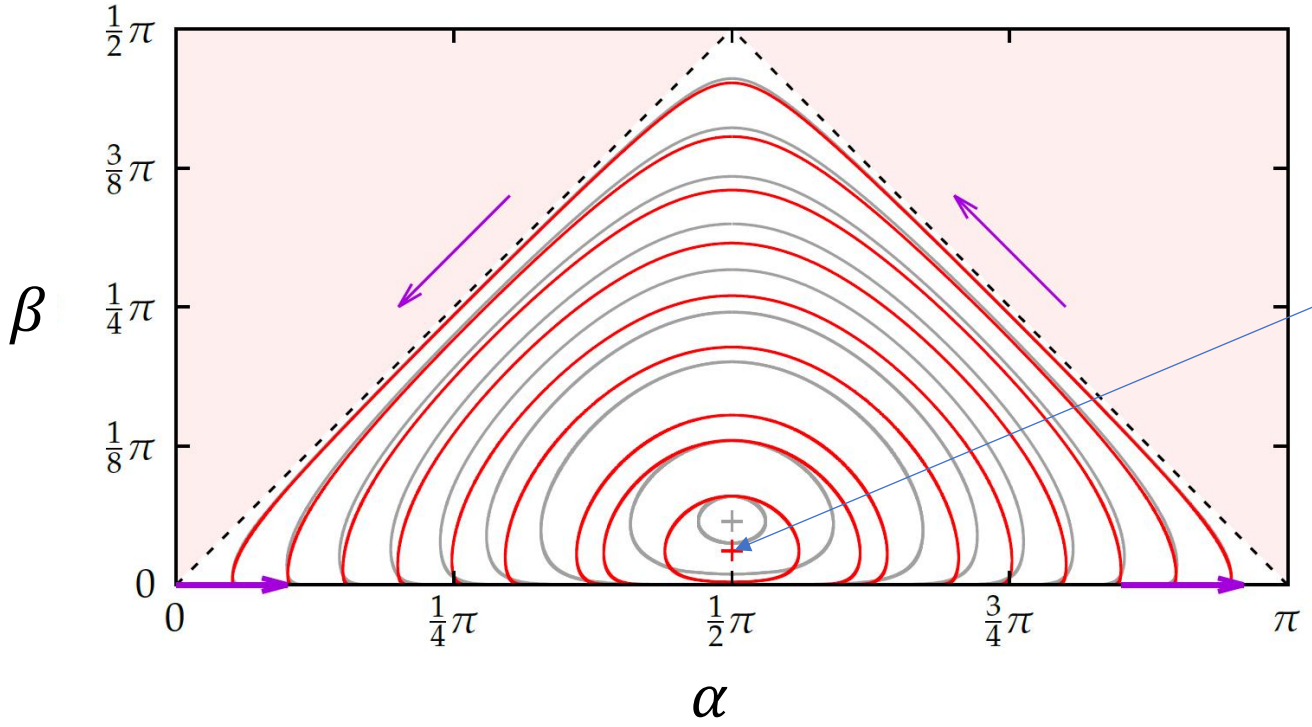


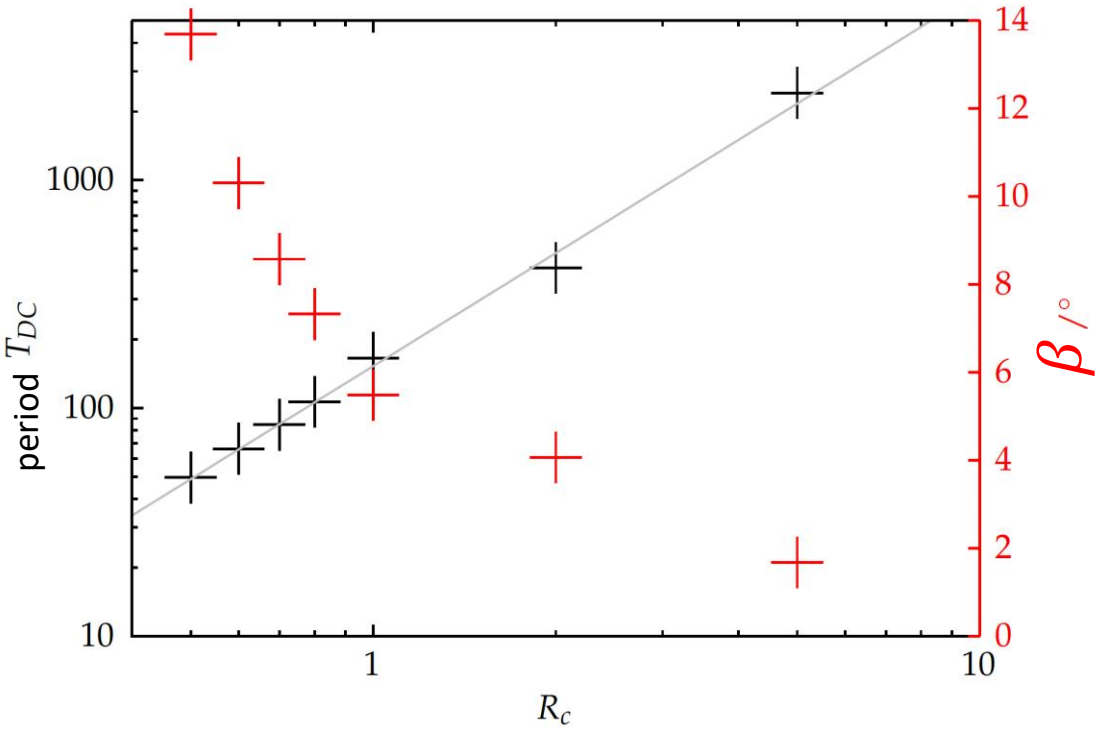
Periods


$\frac{1}{2}\pi$
$\frac{3}{8}\pi$
$\frac{1}{4}\pi$
$\frac{1}{8}\pi$
0
β
0
$\frac{1}{4}\pi$
$\frac{1}{2}\pi$
$\frac{3}{4}\pi$
π
α


2000
1500
1000
500
0
Period [s]
0
20
40
60
80
β [°]

Towards a flat disk

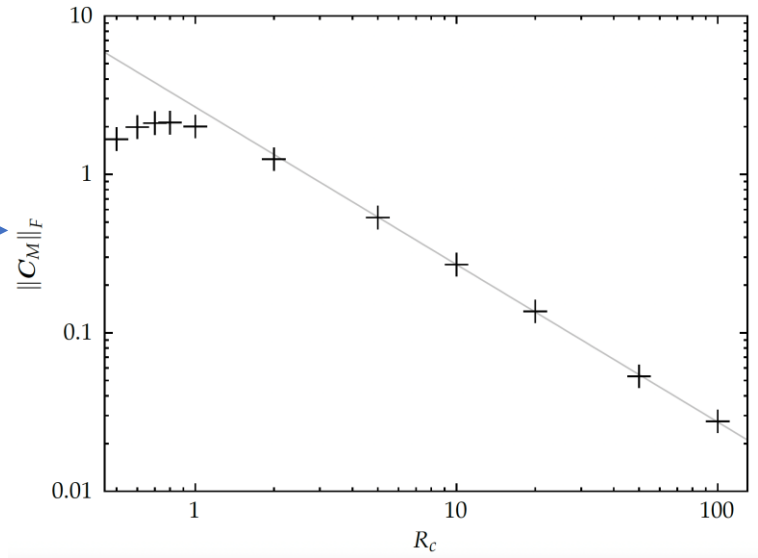




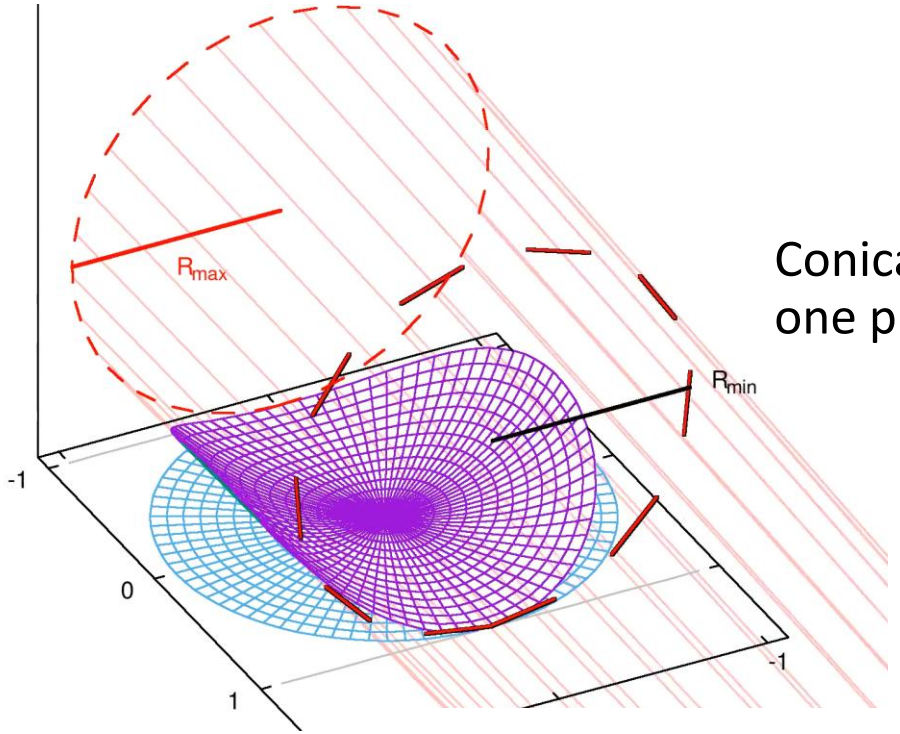
$$\begin{pmatrix} \boldsymbol{F} \\ \boldsymbol{T} \end{pmatrix} = \begin{pmatrix} \boldsymbol{K} & \boldsymbol{C}^{\dagger} \\ \boldsymbol{C} & \boldsymbol{\Omega} \end{pmatrix} \begin{pmatrix} \boldsymbol{U} \\ \boldsymbol{\Omega} \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} 0 & C_{12} & 0 \\ C_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Frobenius norm:

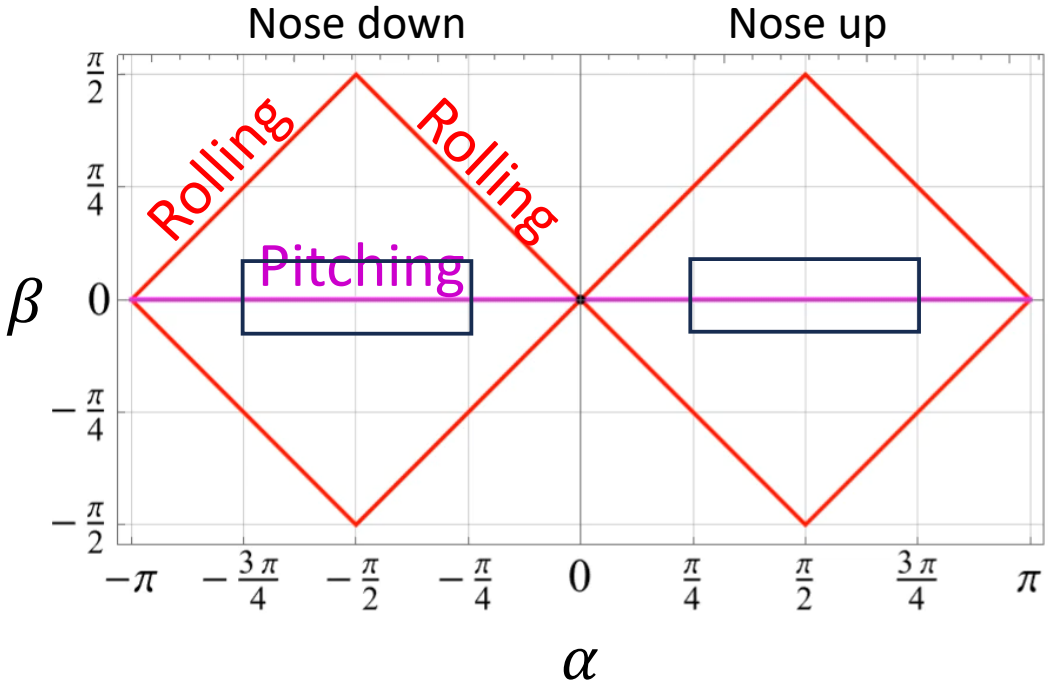
$$||C_M||_F = \sqrt{C_{12}^2 + C_{21}^2}$$

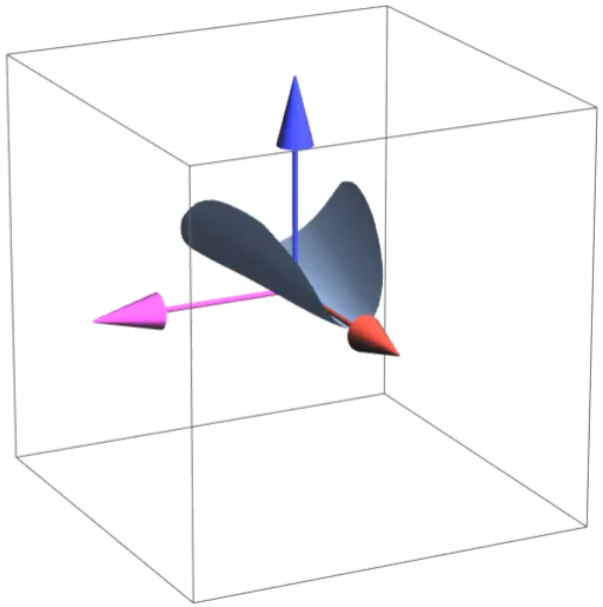


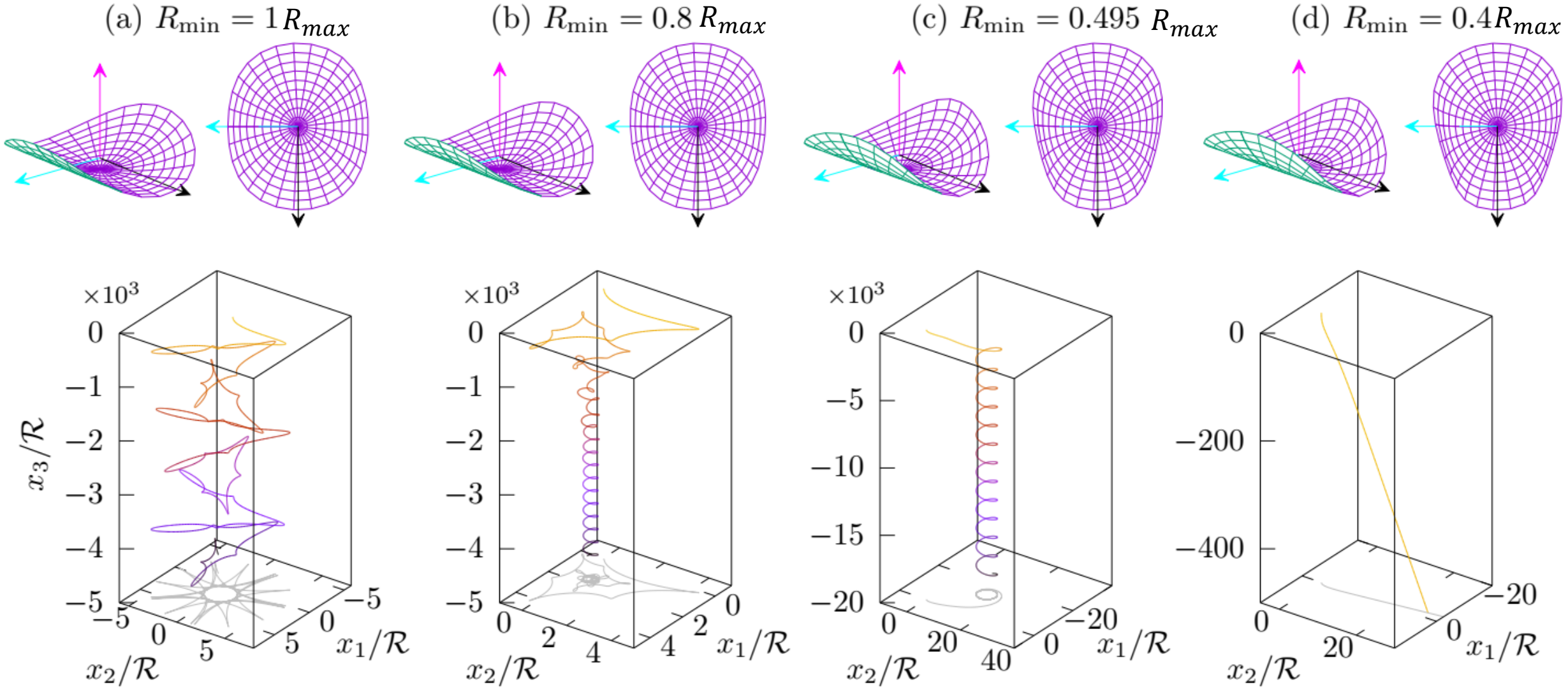
Breaking the fore-aft symmetry



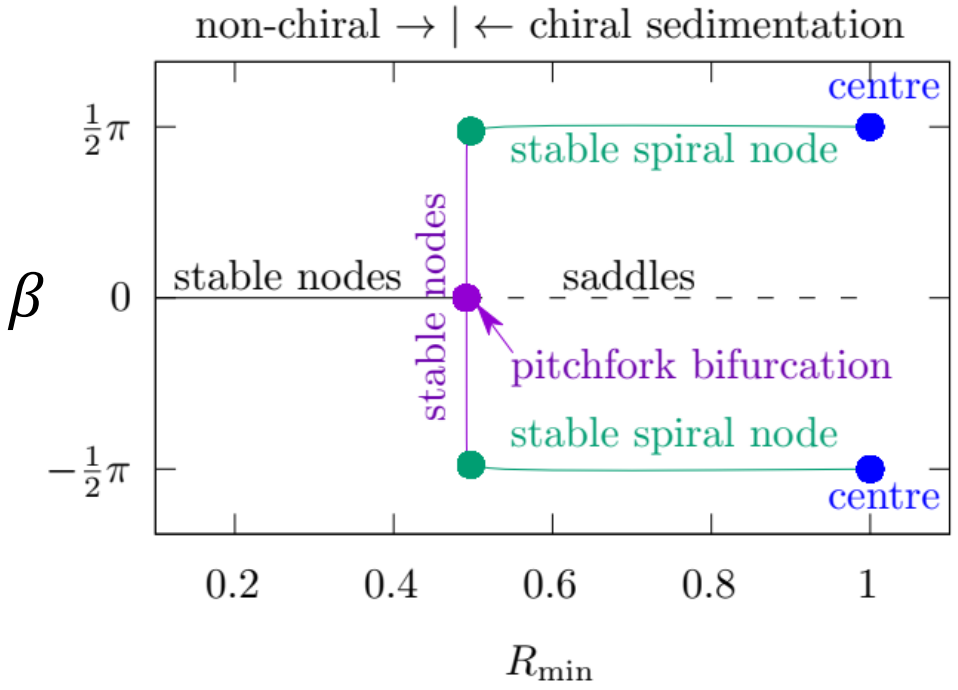
Conical disk:
one plane of symmetry; achiral particle






(a) $R_{\min} = 1\,R_{max}$
(b) $R_{\min} = 0.8\,R_{max}$
(c) $R_{\min} = 0.495\,R_{max}$
(d) $R_{\min} = 0.4 R_{max}$
$\times 10^3$
$x_3/\mathcal{R}$
$x_2/\mathcal{R}$
$x_1/\mathcal{R}$
Making disk more “pinched”

Transition from chiral to no-chiral sedimentation



$R_{min} = 0.4R_{max}$

$R_{min} = 0.52R_{max}$

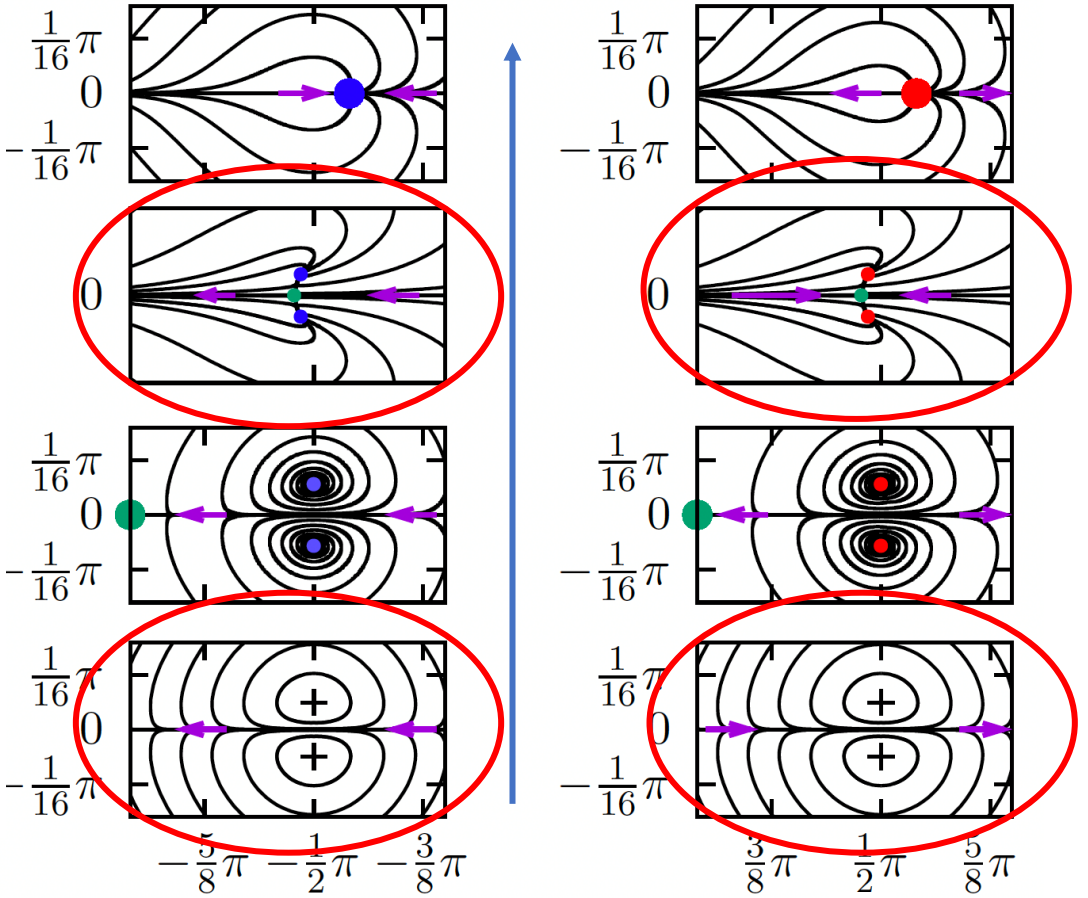
$R_{min} = 0.8R_{max}$

$R_{min} = R_{max}$

Nose down (stable)

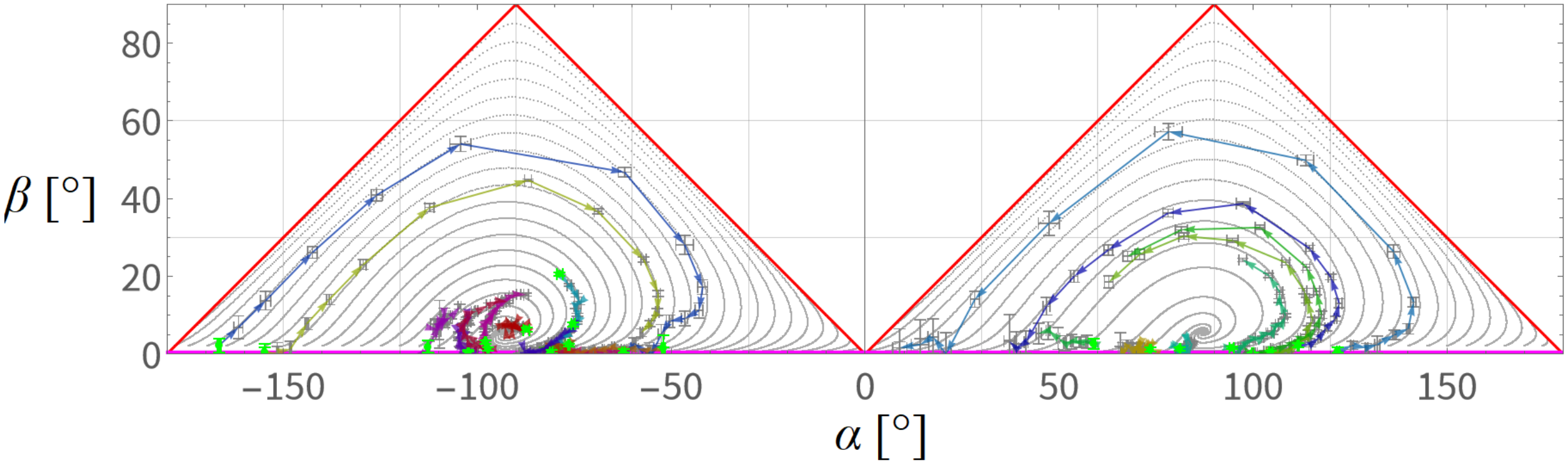
Making disk more "pinched"

Nose up (unstable)



- Transition from flutterers to drifters.

Experimental results



What about elastic sheets?

- Can we extend the framework established for rigid disks to elastic sheets?
- Do sheets just settle steadily as U-shapes?

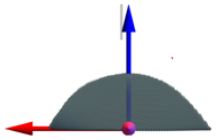
Flexible fibre

Marchetti et al., *Phys. Rev. Fluids* **3**, (2018) 104102

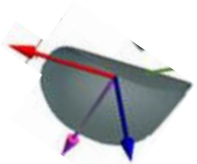
Elastic sheet

Yu & Graham, *Phys. Rev. Fluids* **9**, (2024) 054104

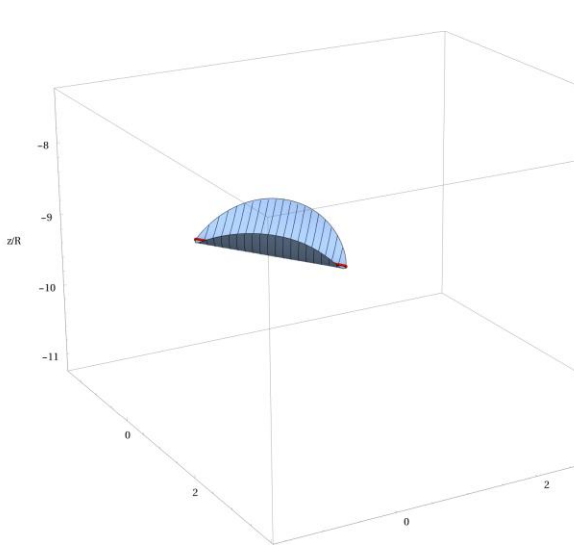
Pitching & rolling of elastic disk



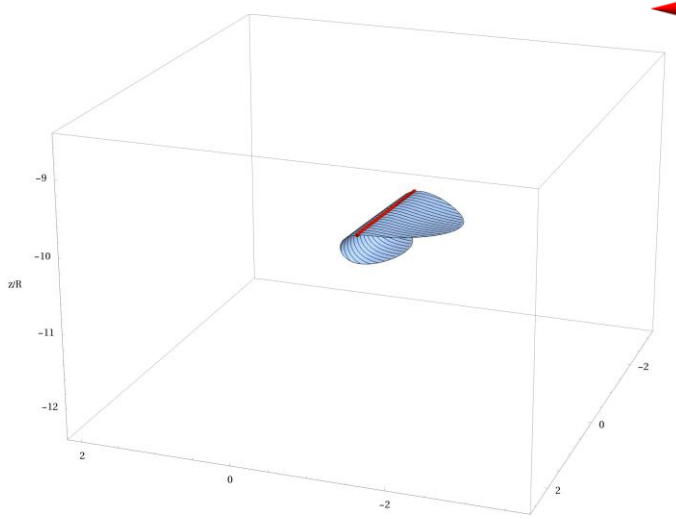
Unstable to Pitching.



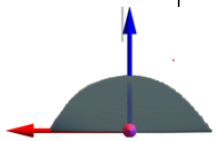
Unstable to rolling.



Pitching

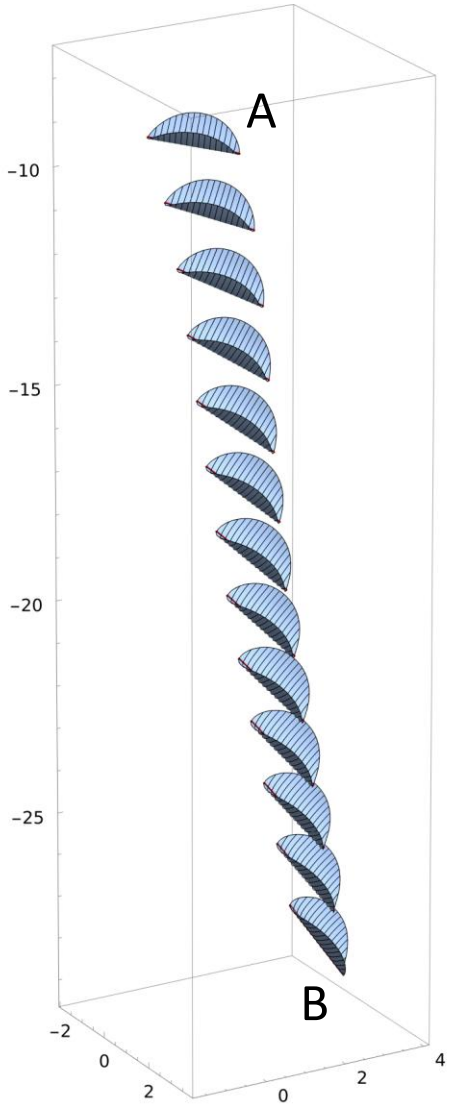


Rolling

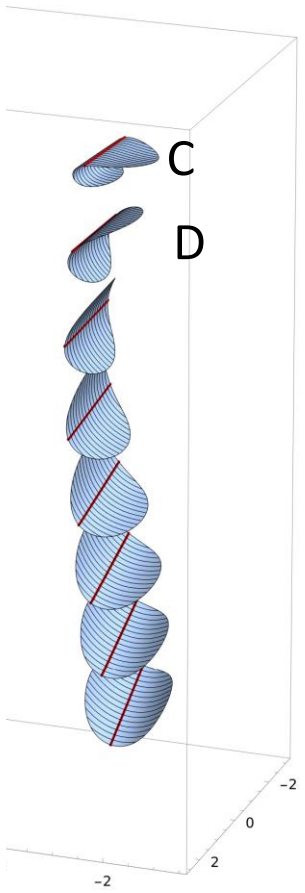


- Relaxation of spine curvature as sheet reorients near vertical ($\alpha = 90°$).

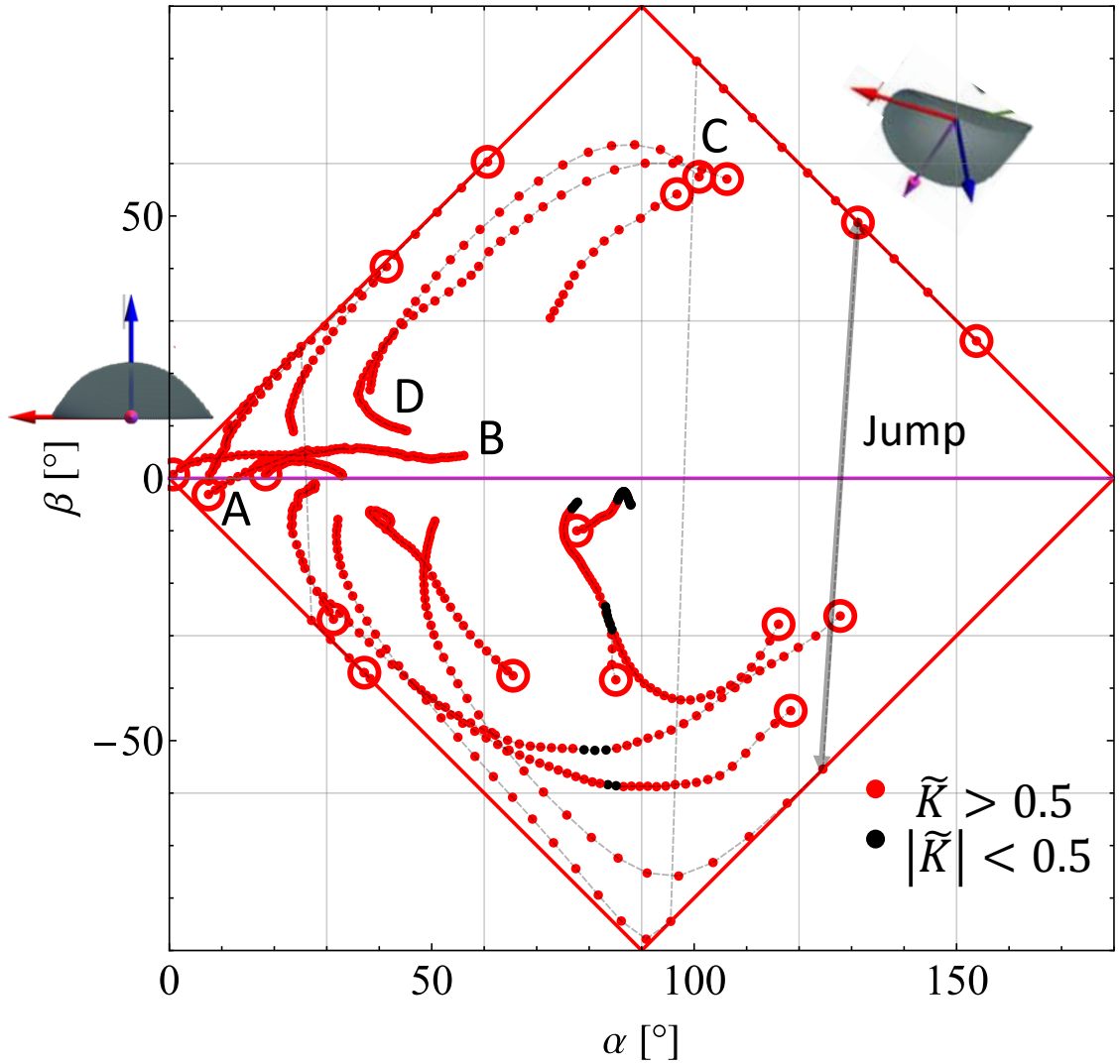
Pitching & rolling



Pitching



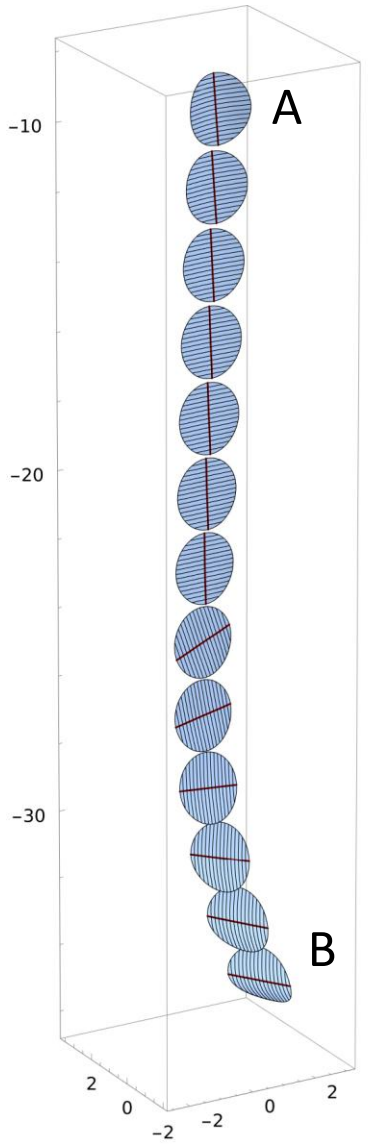
Rolling

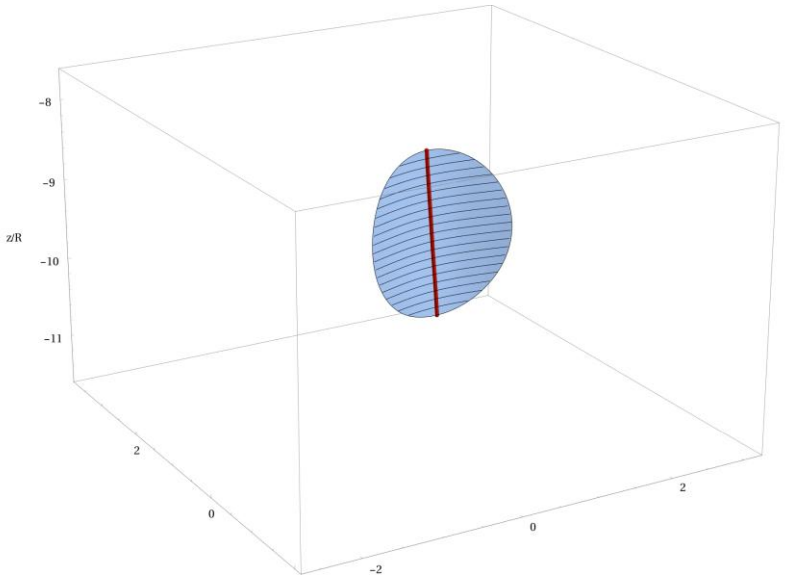


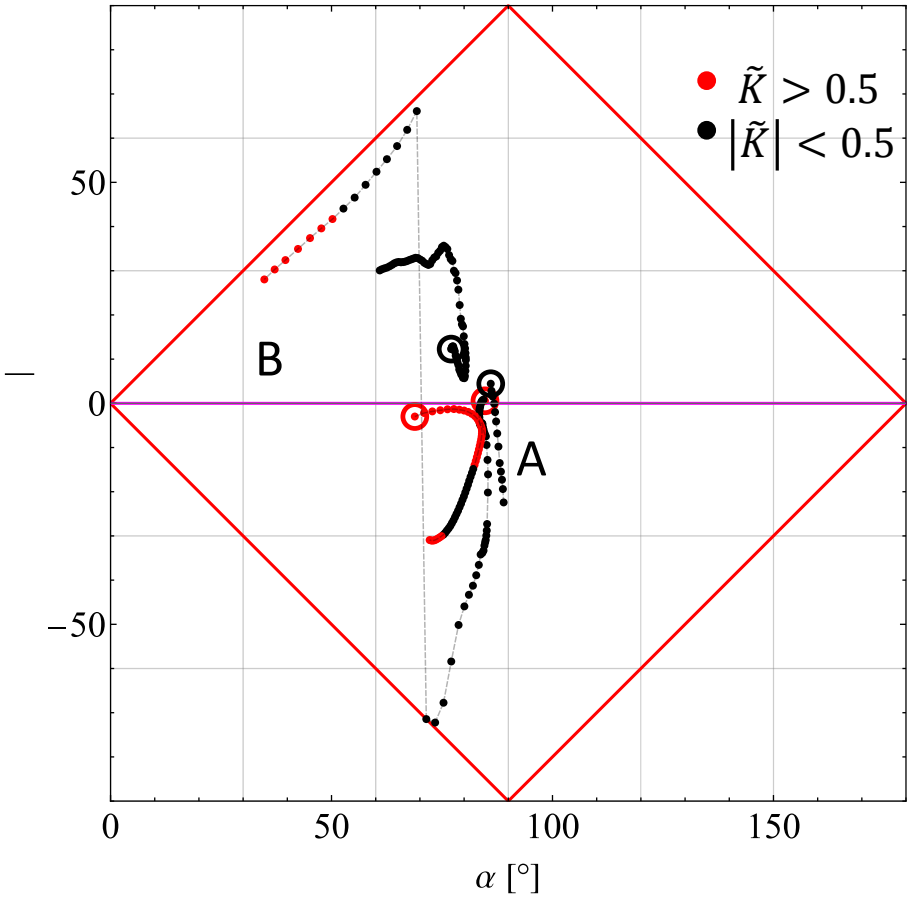
- Relaxation of spine curvature as sheet reorients near vertical ($\alpha = 90°$).

Spine reconfiguration

- Elastic sheets does not pitch beyond $\alpha = 90°$.





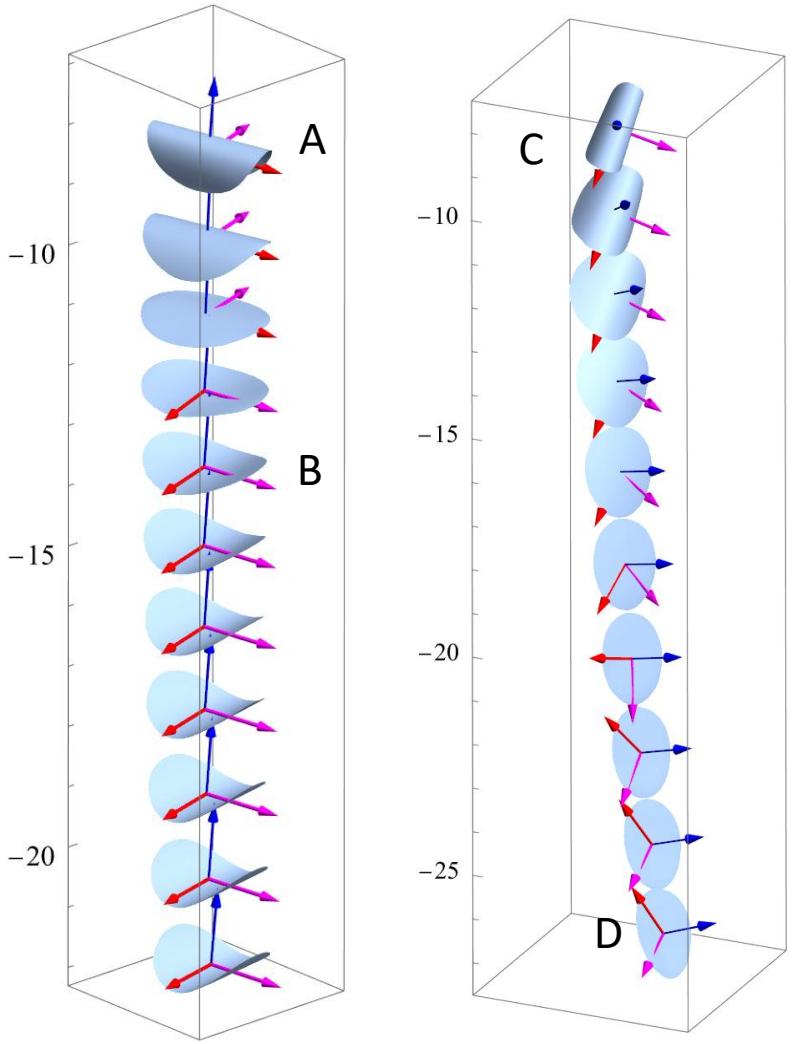


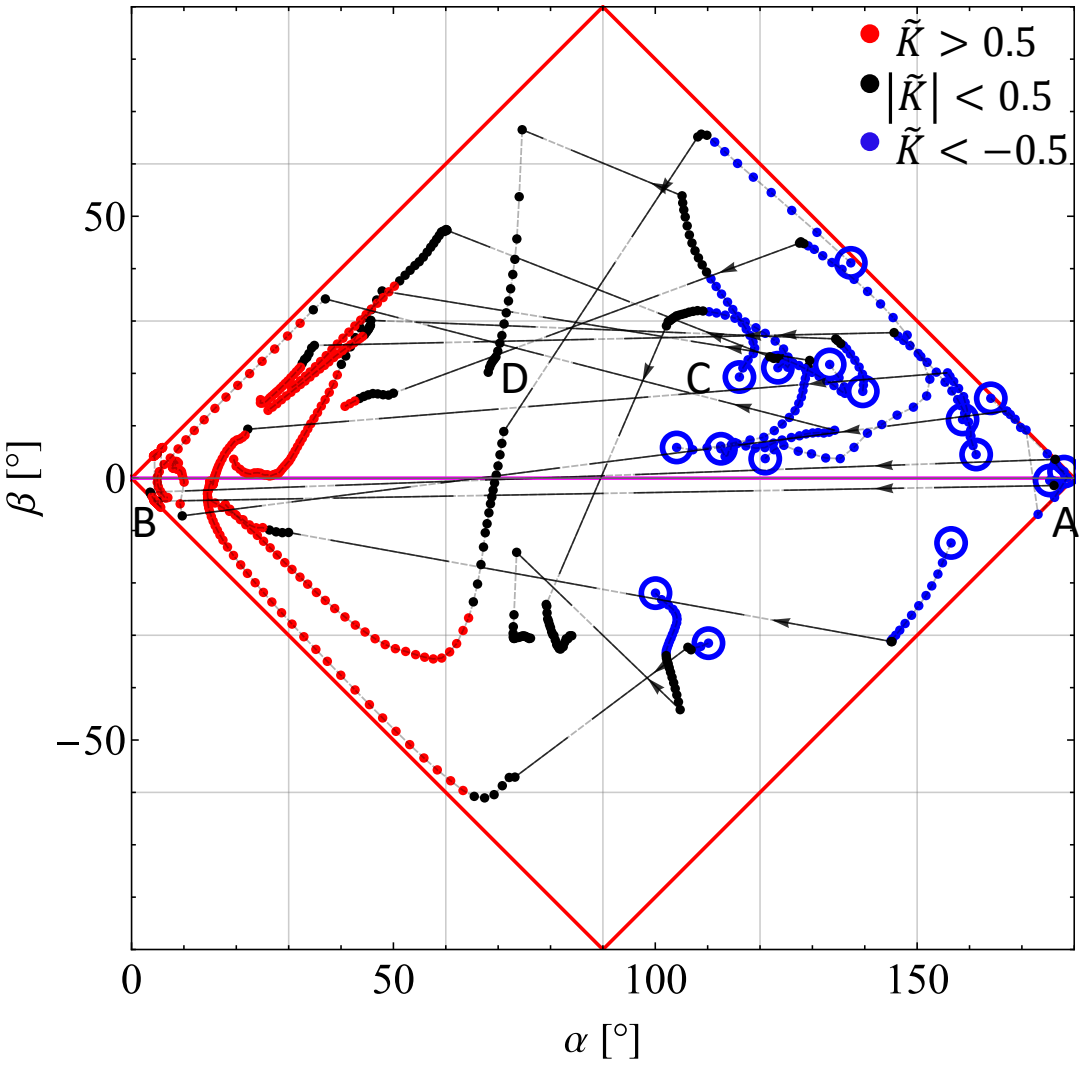
- Reconfiguration of the spine (without rotation of material elements)

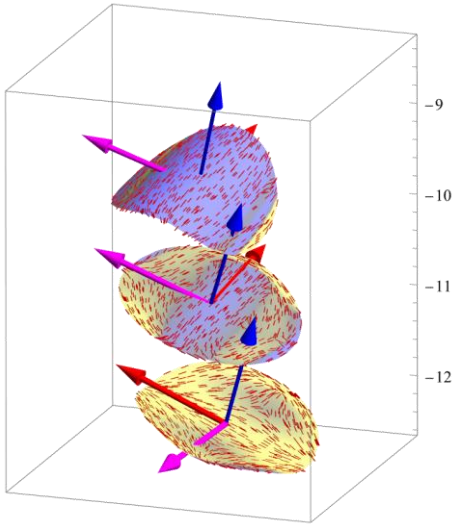
Periodic cycle of reorientation preserved!
Pitching, spine reconfiguration, rolling.

Elastic reconfiguration for $\alpha > 90°$

- Curvature inversion with 90° spine rotation.

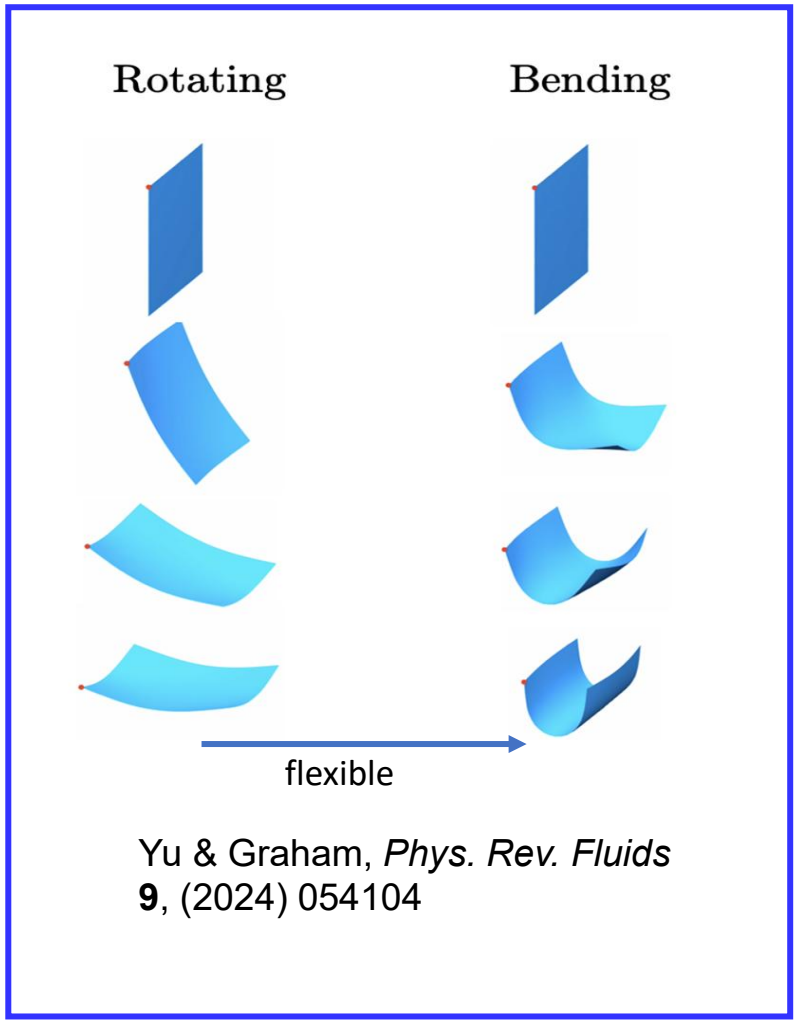






- Subtle changes in configuration due to fluid-structure interaction when disk almost flat.

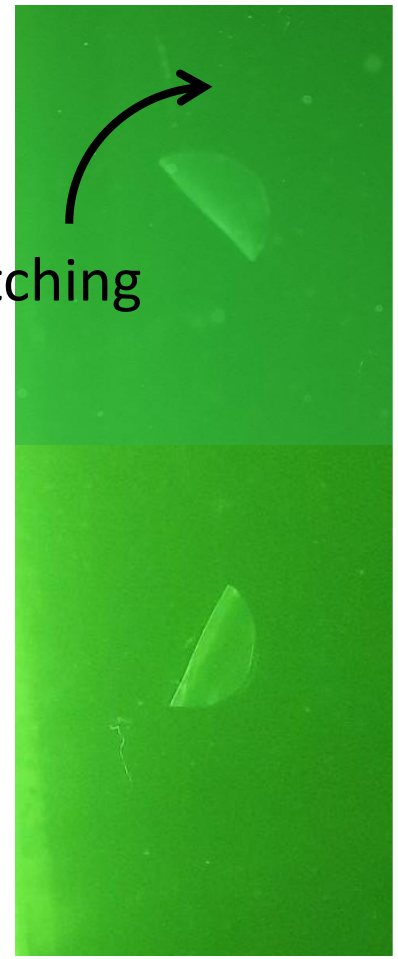
Rectangle versus disk



Yu & Graham, *Phys. Rev. Fluids* **9**, (2024) 054104

Rigid particles: disk versus rectangle

U-bent disk



time
or
vertical
position

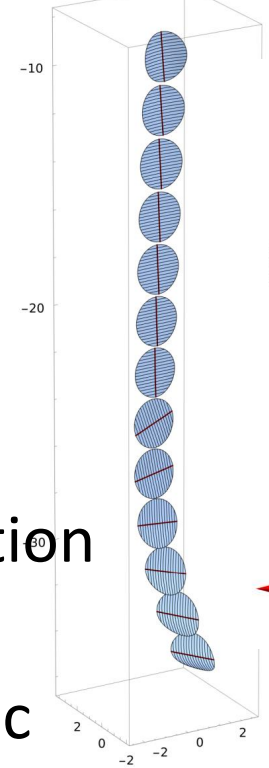
U-bent rectangle

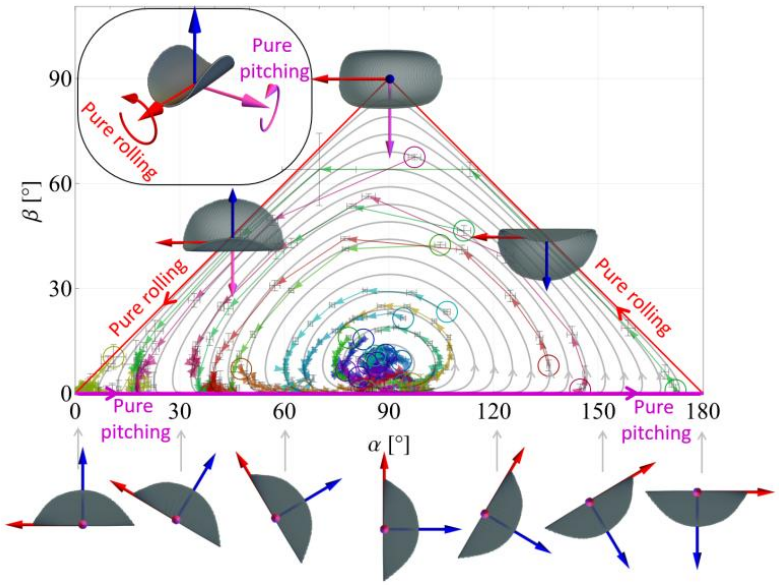
- Both roll.
- But they pitch in opposite directions.

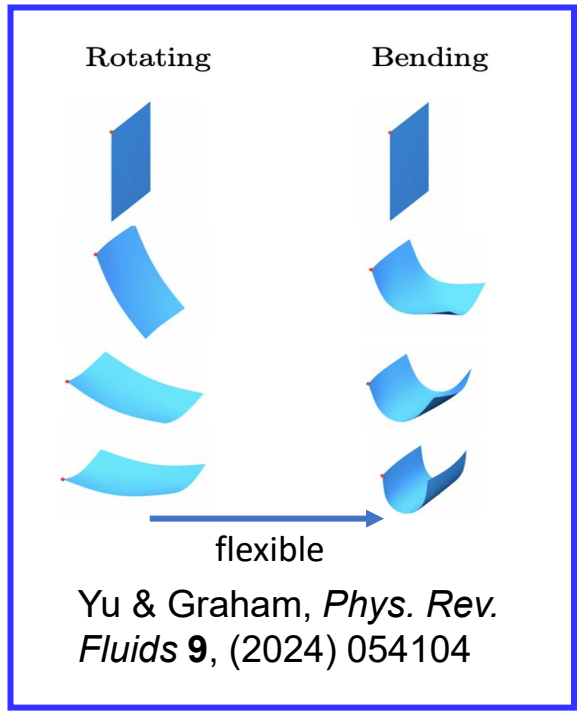
- Both particles have the same symmetries!

Conclusions

- Intuition fails despite linear flow problem!
- Non-chiral rigid objects can flutter at Re →0.
- Symmetry-breaking of rigid U-shape yields transition from fluttering to drifting.
- U-shaped elastic disk maintains flutter via periodic reorientation.
- But elastic rectangles sediment vertically!
- In fact, U-shaped rigid rectangle also sediments vertically in contrast with U-shaped rigid disk.
  - But no change in symmetry!
- Still work to do to predict behaviour of arbitrary shape…







Yu & Graham, *Phys. Rev. Fluids* **9**, (2024) 054104