Odd diffusive spheres and chiral active polymers

International Workshop “Chiral Particles and Odd Fluids”

TU Eindhoven

12.1-14.1.2026

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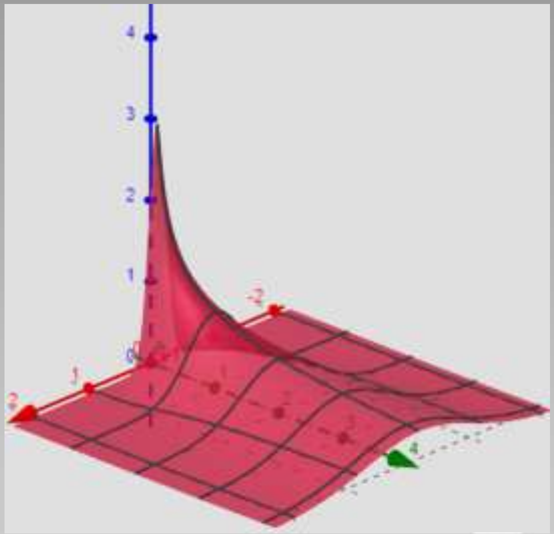
1) Odd diffusive spheres
2) Realization of odd and chiral particles
3) Chiral active polymers
4) Outlook
5) Conclusions

1) Odd diffusive spheres

Even versus Odd diffusion

start with the **ordinary diffusion** equation

$$\frac{\partial \mathrm{P}(\vec{r},\mathrm{t})}{\partial \mathrm{t}} = \mathrm{D}_0 \Delta \mathrm{P}(\vec{r},\mathrm{t})$$



this can be generalized to

$$\frac{\partial \mathrm{P}(\vec{r},\mathrm{t})}{\partial t} = \vec{\nabla} \cdot [\overline{\overline{D}} \vec{\nabla} P(\vec{r},t)] \quad (\bigstar)$$

the matrix $\overline{\overline{D}} = [D_{ij}] = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$ (2d) should be **symmetric** i.e. $D_{12} = D_{21}$

if brought to diagonalized form, changing space variables yields

$$\frac{\partial \mathrm{P}(\vec{r},\mathrm{t})}{\partial t} = \vec{\nabla} \cdot \begin{pmatrix} D_0^{(1)} & 0 \\ 0 & D_0^{(2)} \end{pmatrix} \vec{\nabla} P(\vec{r},t)$$

for $D_0^{(1)} > D_0^{(2)} > 0$: anisotropic diffusion

for $D_0^{(1)} = D_0^{(2)} = \mathrm{D}_0 > 0$: isotropic diffusion

Now we generalize $(\star)$ to a diffusion matrix $\overline{\overline{D}}$ which is not symmetric.

$$D_{ij} \neq D_{ji}$$

This is **odd** diffusion.

In the isotropic case we can write

$$\overline{\overline{D}} = D_0(\mathbf{1} + \kappa\boldsymbol{\epsilon})$$ with $$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ and $$\boldsymbol{\epsilon} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

oddness

κ measures the relative strength of oddness

odd diffusion is both, diffusion and rotation (vorticity)


E. Kalz


H. D. Vuijk


I. Abdoli


J.-U. Sommer


A. Sharma

E. Kalz, H. D. Vuijk, I. Abdoli, J.-U. Sommer, HL, A. Sharma,
Collisions Enhance Self-Diffusion in Odd-Diffusive Systems, Physical Review Letters **129**, 090601(2022).

the white arrows are the currents $\overline{\overline{D}}\vec{\nabla}P(\vec{r},t)$

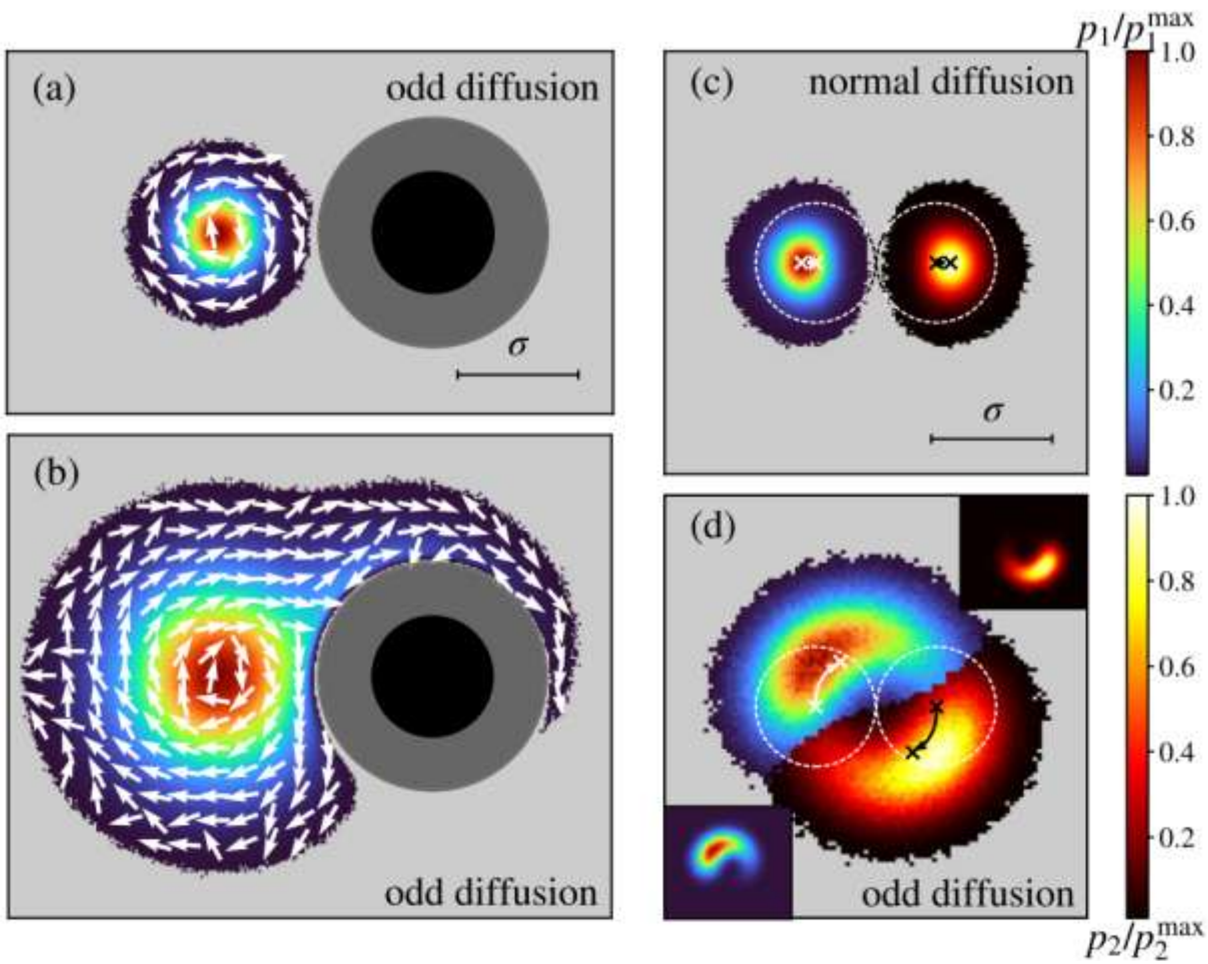
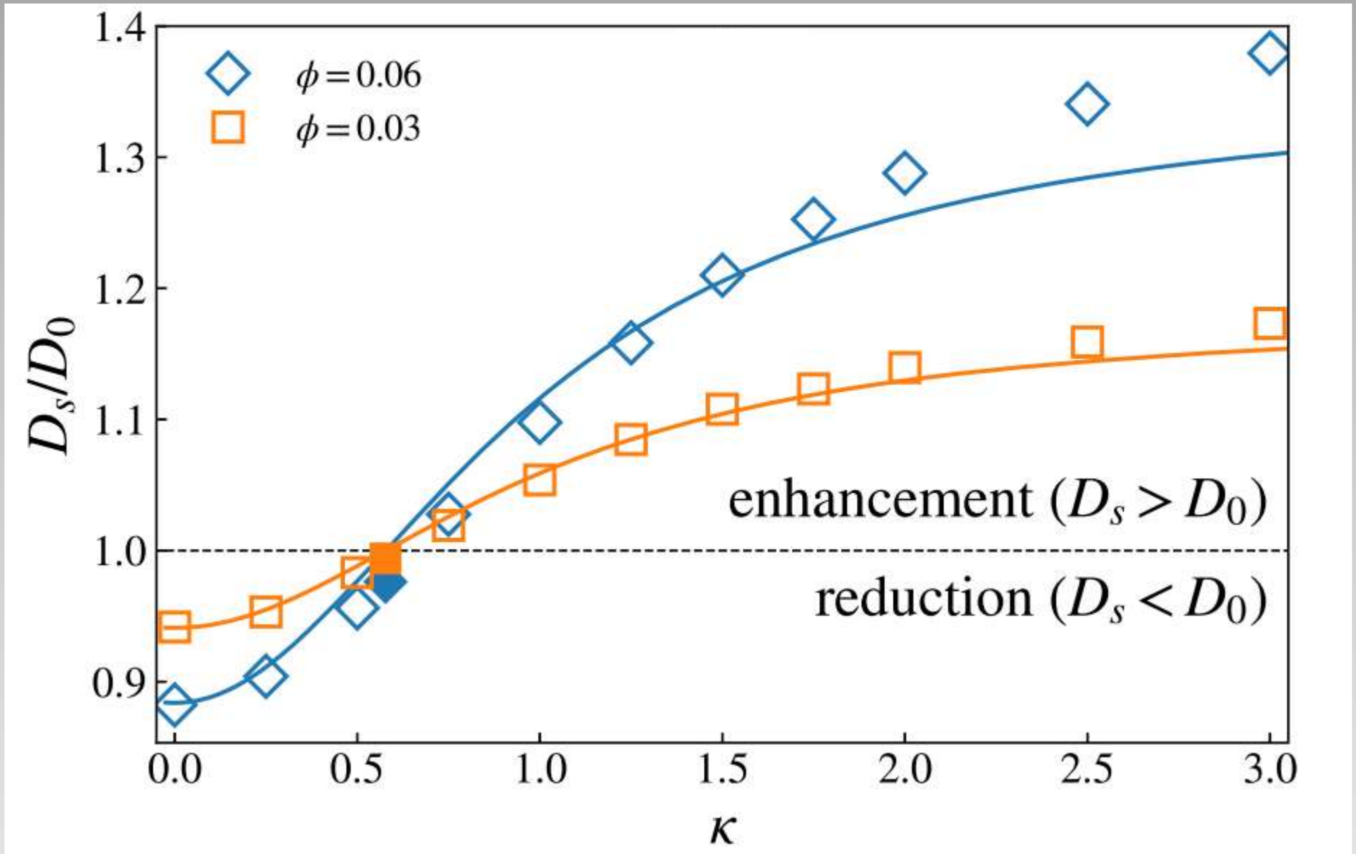


FIG. 1. Scaled density distribution (p_1/p_1^{max}) of an odd-diffusive particle near a fixed particle of same diameter σ at short (a) and long times (b). The curved probability fluxes (arrows) flow around the fixed particle in a preferred direction which can be flipped by reversing the odd-diffusivity parameter κ. Density distribution after a collision between two identical diffusing particles: whereas a collision results in a probability flow away from each other for normal particles (c) for odd-diffusive particles the probability flows around each other (d). This "mutual rolling" of the particles around each other facilitates the motion resulting in an enhanced self-diffusion. The dashed circles represent the initial configuration, and the crosses indicate the displacements of particles' centers. Insets in (d) show individual probability distributions of the two particles. The results are obtained from Brownian dynamics simulations of charged Brownian particles under Lorentz force with $\kappa = 5$.

Collisions Enhance Self-Diffusion in Odd-Diffusive Systems

rolling mechanism

$$D_s = D_0\left(1 - 2\phi\,\frac{1-3\kappa^2}{1+\kappa^2}\right)$$



E. Kalz, H. D. Vuijk, I. Abdoli, J.-U. Sommer, HL, A. Sharma,
Collisions Enhance Self-Diffusion in Odd-Diffusive Systems, Physical Review Letters **129**, 090601(2022).

Dynamical Density Functional Theory (DDFT) of odd diffusion

for interacting Brownian particles at high density

classical DDFT is a generalized diffusion equation:

$$\frac{\partial \rho(\vec{r},t)}{\partial t} = \beta \vec{\nabla}_{\vec{r}} \cdot D_0 \left(\rho(\vec{r},t) \vec{\nabla}_{\vec{r}} \frac{\delta \mathcal{F}(T,[\rho(\vec{r},t)])}{\delta \rho(\vec{r},t)} \right)$$

equilibrium free energy density functional

I. Abdoli

idea for odd diffusion: replace D_0 by an odd tensor $\overline{\overline{D}}$

review on DDFT

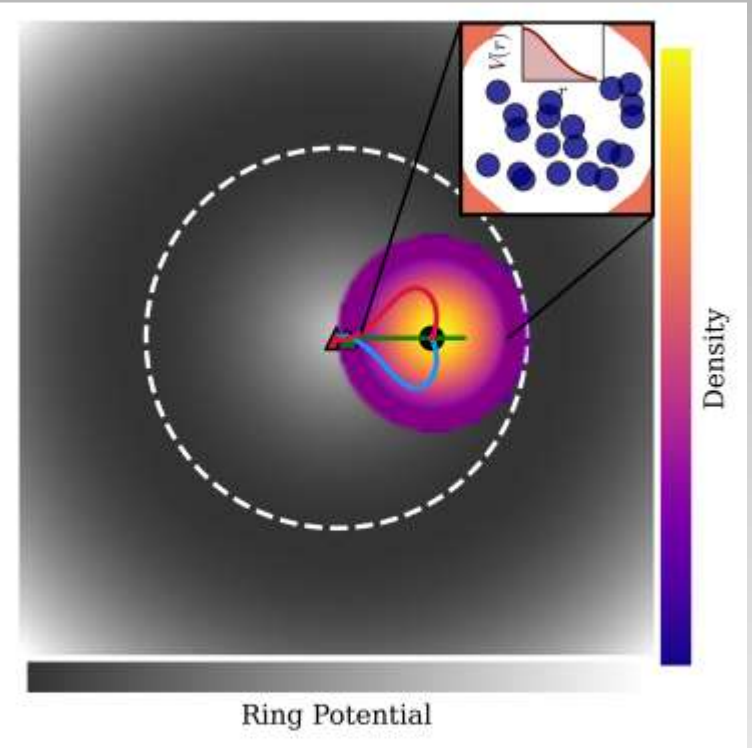
M. te Vrugt, HL, R. Wittkowski, *Classical dynamical density functional theory: from fundamentals to applications*, Advances in Physics **69**, 121 (2020)

Michael te Vrugt

Raphael Wittkowski

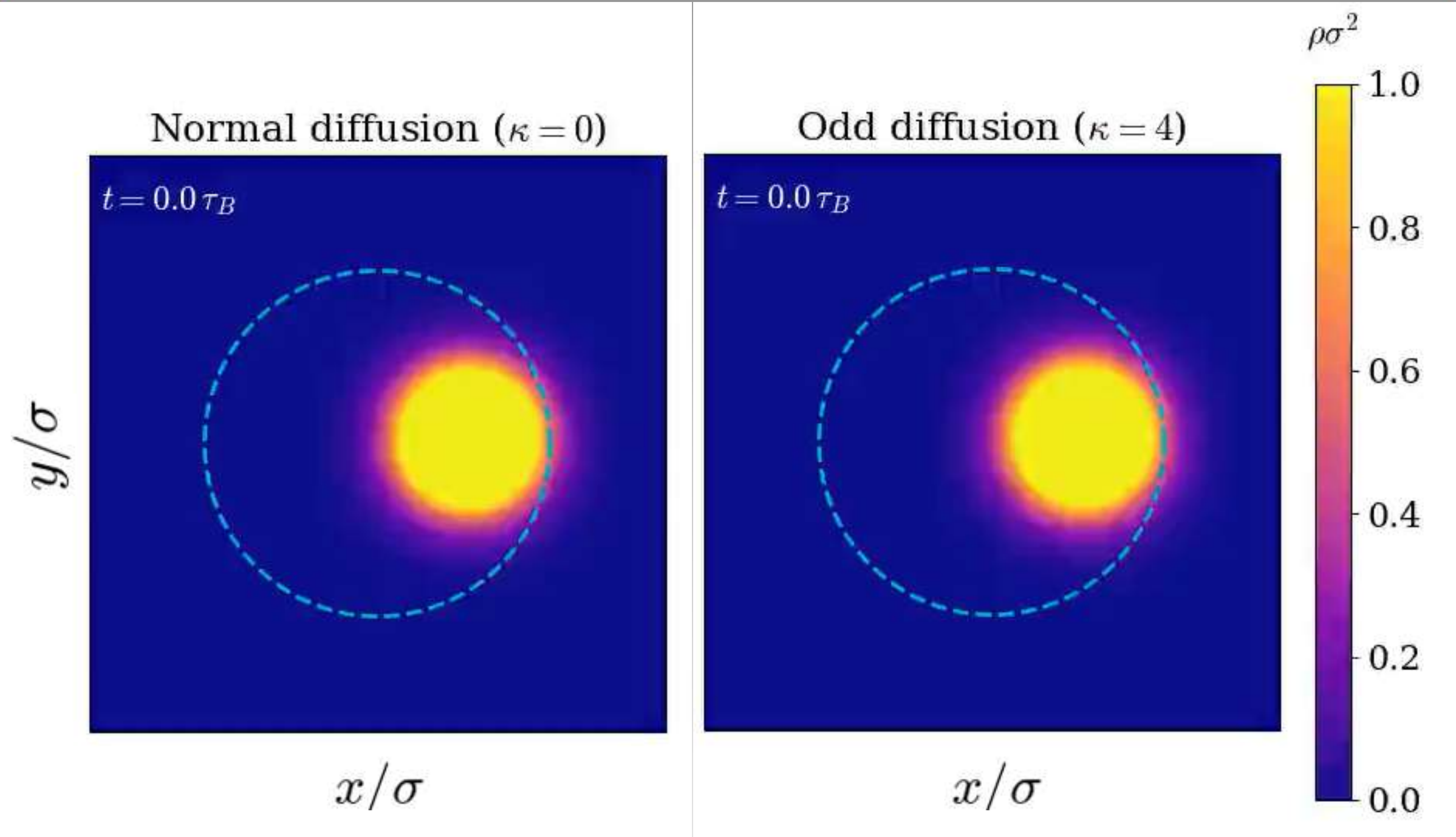
dense fluid with a **Gaussian interaction potential** $V(r) = \varepsilon \exp\left[-\left(\frac{r}{\sigma}\right)^2\right]$

in an **external confining ring potential** $V_{\text{ext}}(\mathbf{r}) = \frac{1}{2}k(r - R_0)^2$



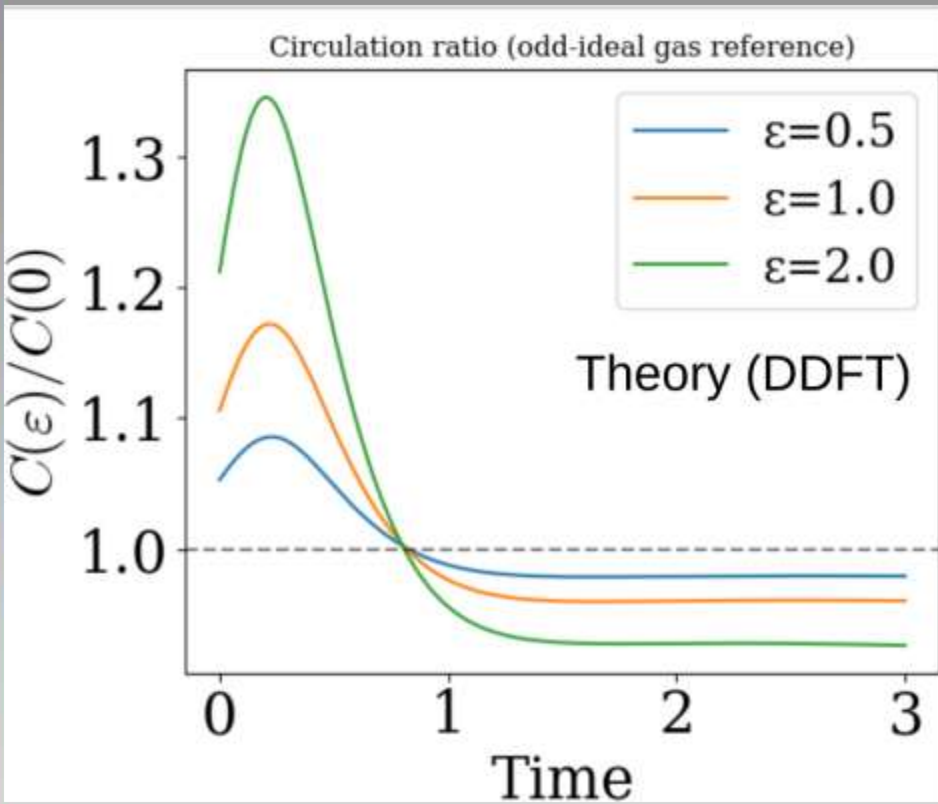
mean field approximation for the equilibrium free energy density functional $\mathcal{F}(T, [\rho(\vec{r}, t)])$

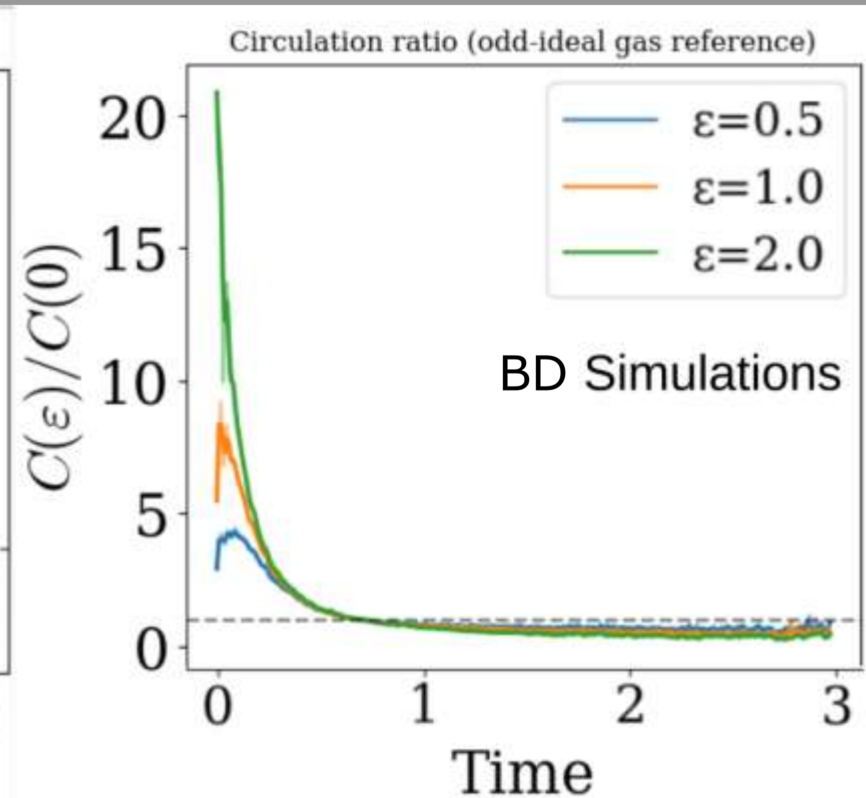
Brownian dynamics



Normal diffusion ($\kappa = 0$)
Odd diffusion ($\kappa = 4$)
$t = 0.0\,\tau_B$
$t = 0.0\,\tau_B$
y/σ
x/σ
x/σ
$\rho\sigma^2$
1.0
0.8
0.6
0.4
0.2
0.0

C: angular circulation along the full ring





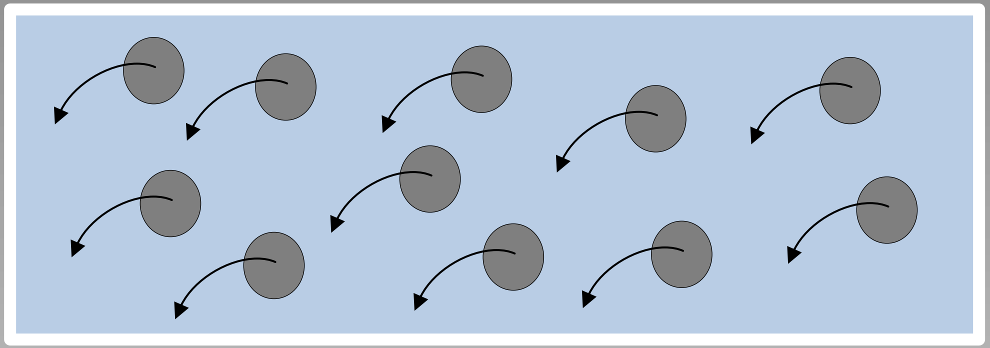
normal diffusion ($\kappa = 0$): C vanishes

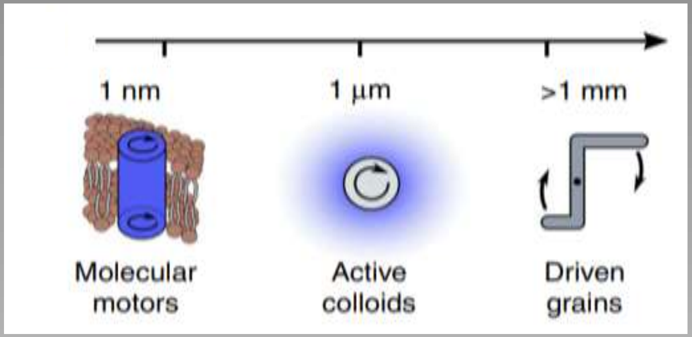
2) Realization of odd and chiral particles

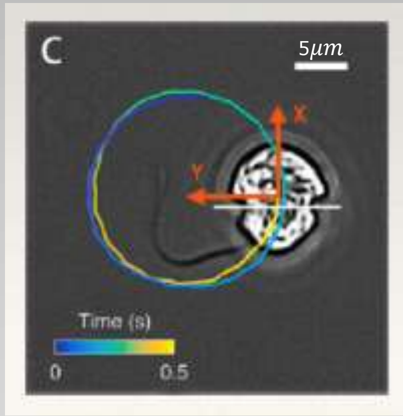
Is **odd diffusion** just a **theoretical** phantasm (even violating basic symmetry principles) or is it real??

It is **real** !

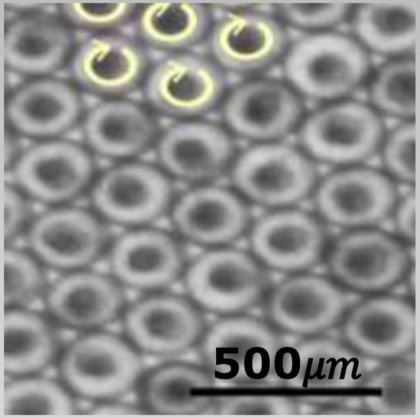
System of spinners/circlers



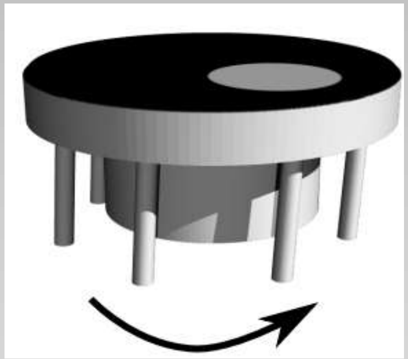




marine algae



starfish embryos



granular hopper

D. Banerjee, A. Souslov, A.G. Abanov, V. Vitelli, Nature Communications **8**, 1573 (2017).
M. Huang et al., PNAS **118**, e2100493118 (2021).
T.H. Tan et al., Nature **607,** 287 (2022).
L. Caprini, U.M.B. Marconi, B. Liebchen, HL, arXiv:2509.05053.

Circle swimming of Phalarope birds

https://www.youtube.com/watch?v=heEUPbxmYgQ

WHY?

Consider basic equations of an active chiral Brownian particle

underdamped (inertial) or overdamped circle swimmer

$$m\ddot{\vec{r}}(t)/\gamma + \dot{\vec{r}}(t) = v_0\hat{u}(t) + \vec{f}(t)$$

$$\dot{\phi}(t) = g(t) + \omega$$

$$\hat{u}(t) = \begin{pmatrix} \cos\phi(t) \\ \sin\phi(t) \end{pmatrix}$$

$$\overline{\vec{f}(t)} = 0 \quad \overline{f_i(t)f_j(t')} = 2D_t\delta(t-t')\delta_{ij}$$

$$\overline{g(t)} = 0 \quad \overline{g(t)g(t')} = 2D_R\delta(t-t')$$

circling radius

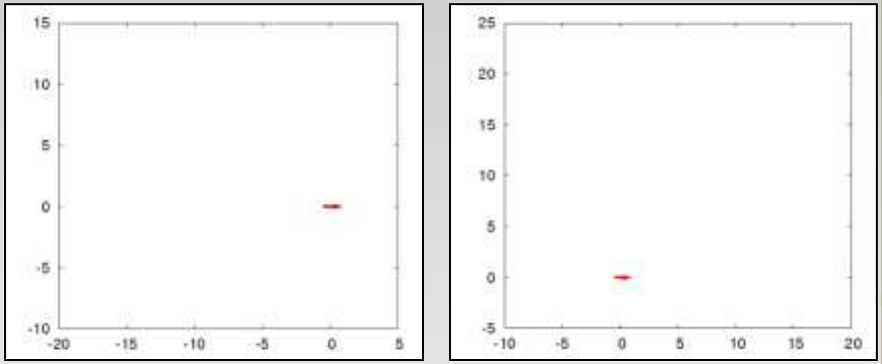
$$R = \frac{v_0}{\omega}$$

v_0 self propulsion velocity

ω internal spinning frequency

D_t translational diffusion coefficient

$\tau = \frac{1}{D_R}$ persistence time



van Teffelen, HL, PRE **78**, 020101 (2008)

Compute the diffusivity matrix for a Brownian circle swimmer

$$\overline{\overline{D}} = \int_0^\infty dt \, \langle \vec{v}(t) \otimes \vec{v}(0) \rangle$$

Green-Kubo

$\langle \ldots \rangle$ noise average

limit of vanishing inertia: $\mathrm{m} \to 0$

long time limit

$$\overline{\overline{D}} = \begin{pmatrix} D^{diag} & D^{odd} \\ -D^{odd} & D^{diag} \end{pmatrix}$$

$$D^{odd} = v_0^2 \tau \frac{\omega\tau}{1+\omega^2\tau^2}$$

$$D^{diag} = D_t + v_0^2 \tau \frac{1}{1+\omega^2\tau^2}$$

$\kappa \approx \omega\tau$

C. Hargus, J. M. Epstein, K. K. Mandadapu, Physical Review Letters **127**, 178001 (2021).

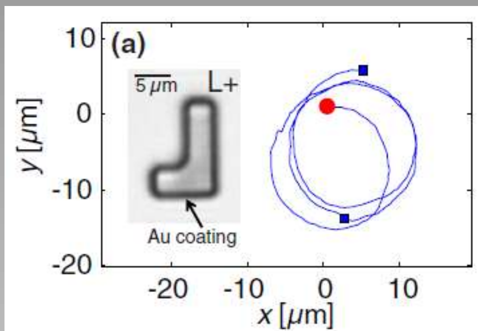
And are there realizations of chiral active Brownian particles?

- colloids (without inertia, $m=0$)
- granules/hoppers (with inertia)

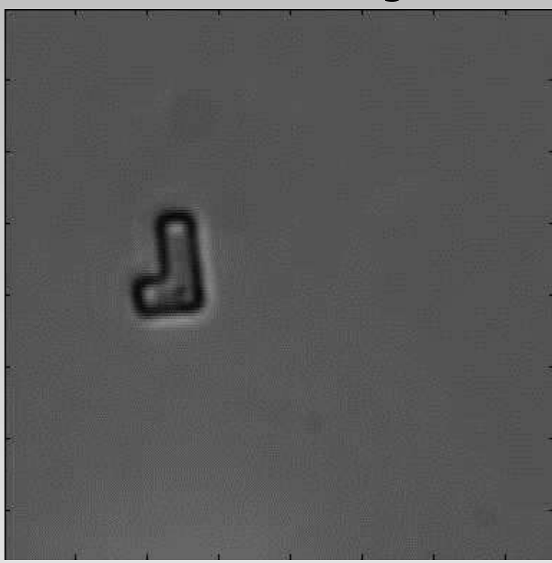
review on chiral active matter: B. Liebchen, D. Lewis, EPL **139**, 67001 (2022)

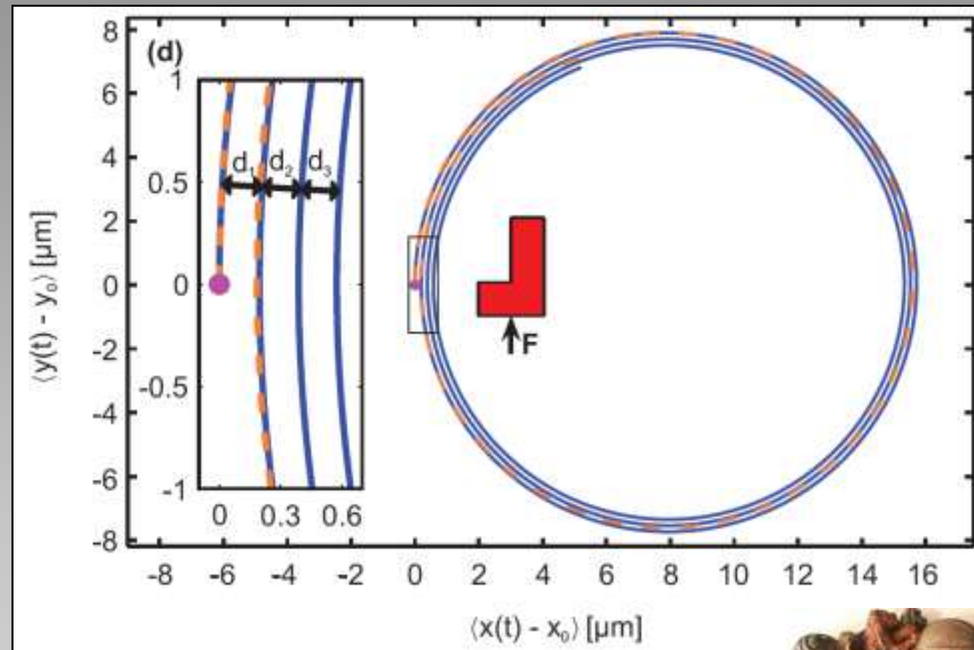
Chiral colloids

chiral L-shaped particles



circle swimming





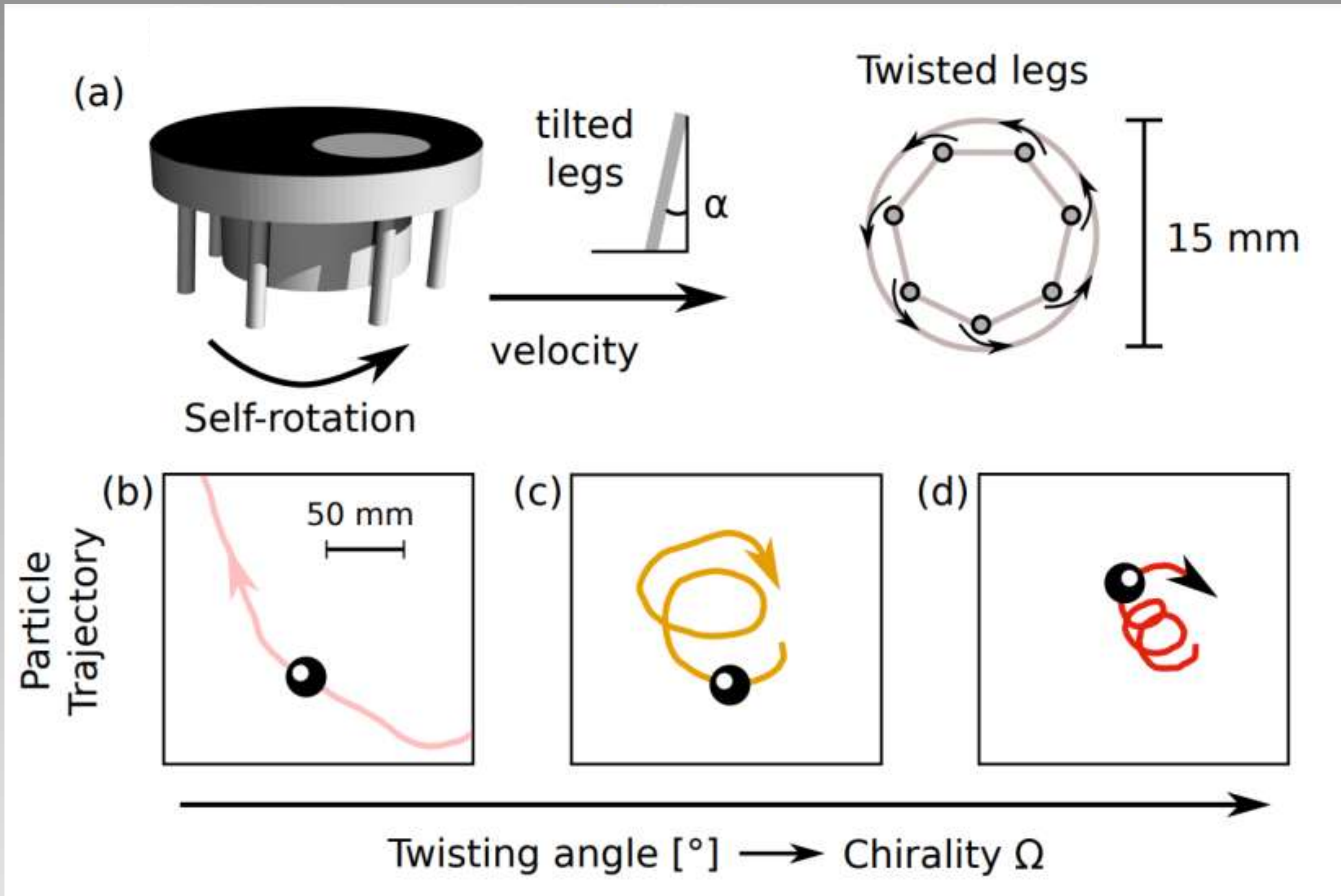
spira mirabilis for the mean displacement for a prescribed initial position (x_0, y_0) and orientation $\hat{u}_0 = (0{,}1)$

S. van Teeffelen, HL, Phys. Rev. E. **78**, 020101 (2008)

F. Kümmel, B. ten Hagen, R. Wittkowski, I. Buttinoni, G. Volpe, H. Löwen, C. Bechinger, PRL **110**, 198302 (2013)

https://de.wikipedia.org/wiki/Datei:Basler_Muenster_Bernoulli.jpg

Chiral active granular particles

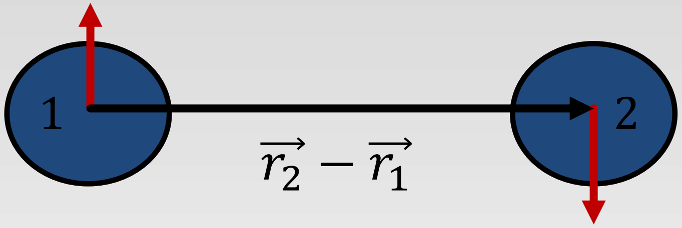


L. Caprini, U.M.B. Marconi, B. Liebchen, HL, arXiv:2509.05053

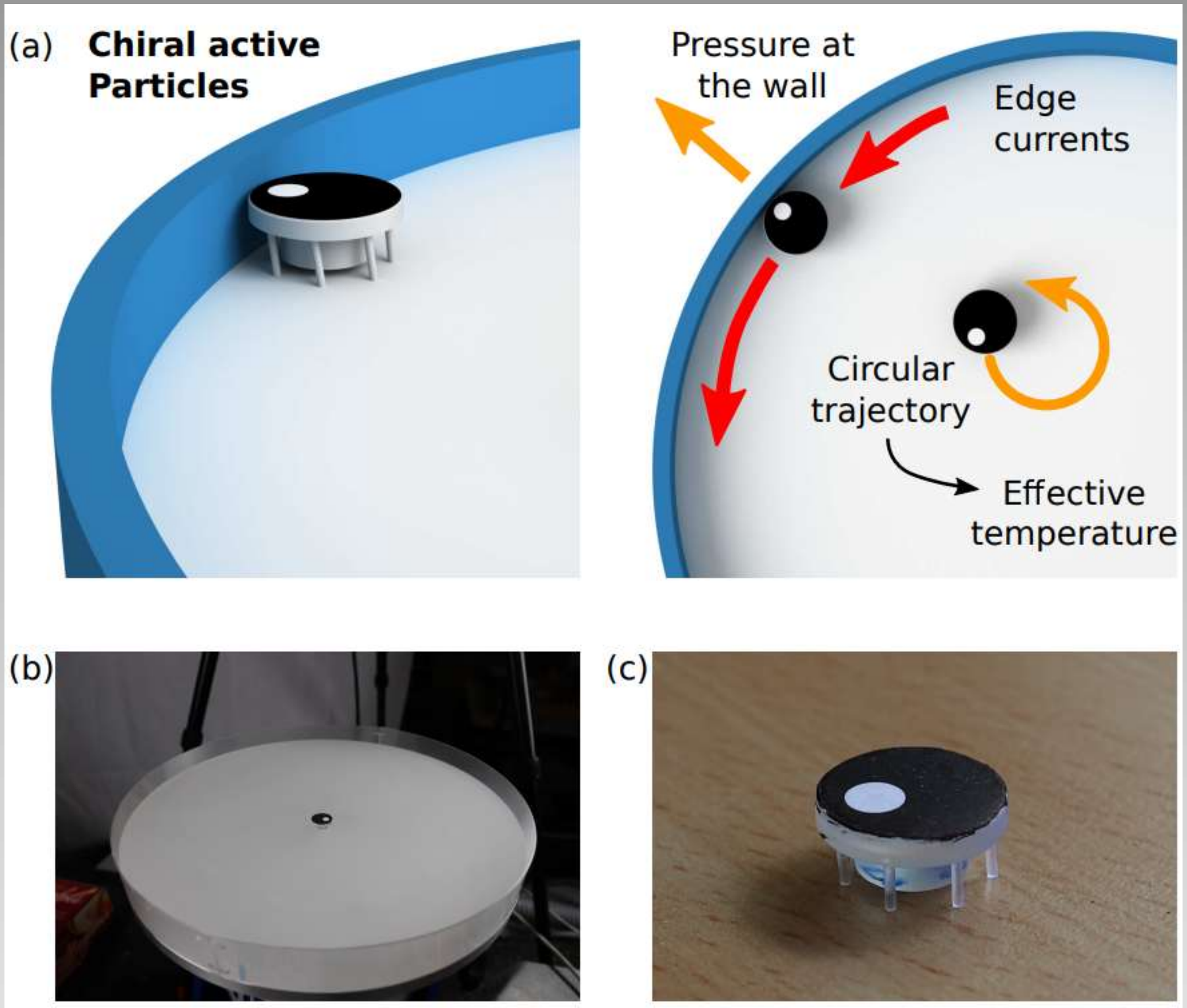
Other rotating systems

- **forces ⊥ velocity**
- Lorentz-force (charged particles)
  - huge magnetic fields are required
- Magnus-force
  - macroscopic particles needed
- Coriolis-force (in non-inertial frames)
- internal torques…

pairwise transversal forces



Active granular spinners/circlers



L. Caprini, U.M.B. Marconi, B. Liebchen, HL, arXiv:2509.05053

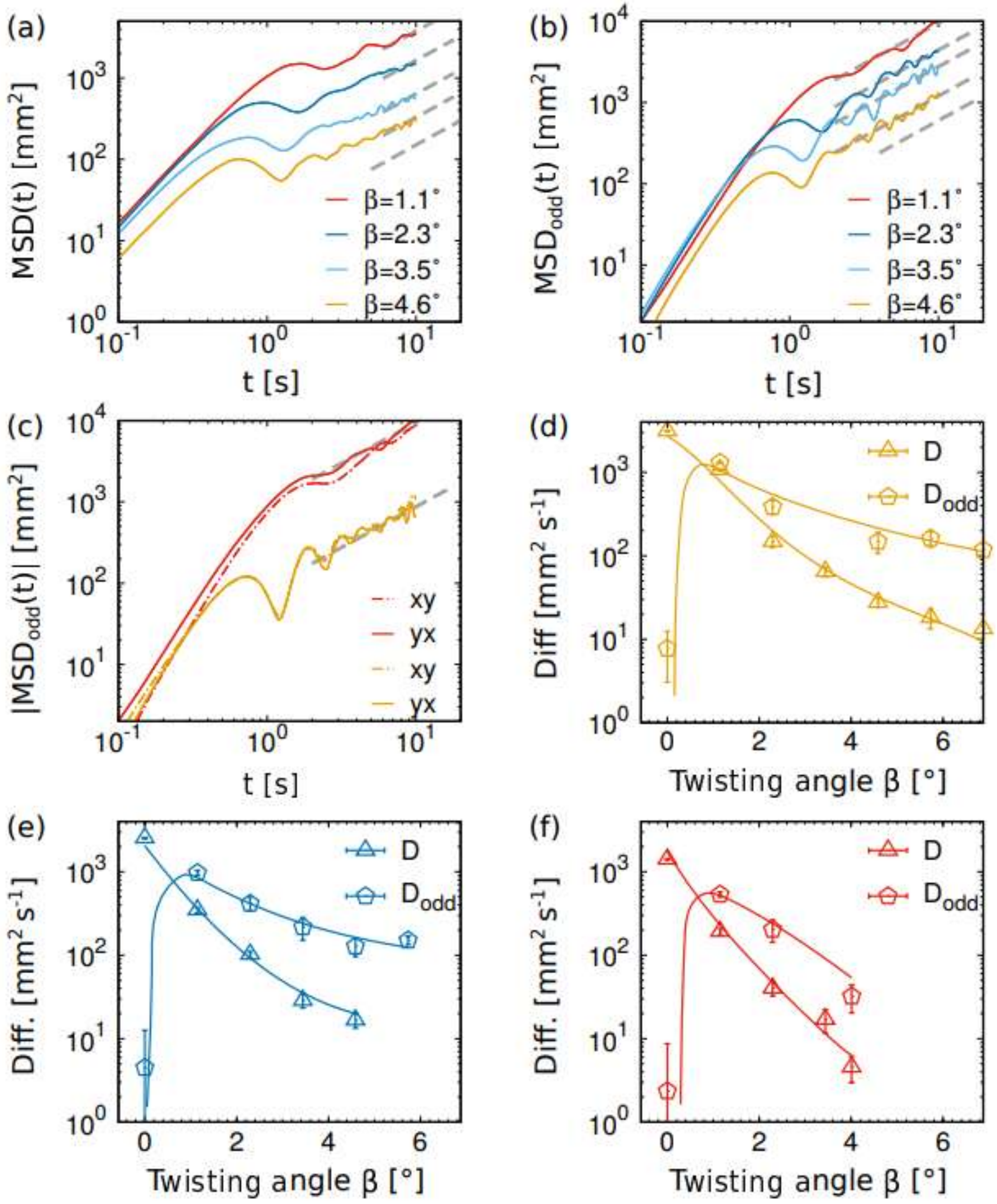
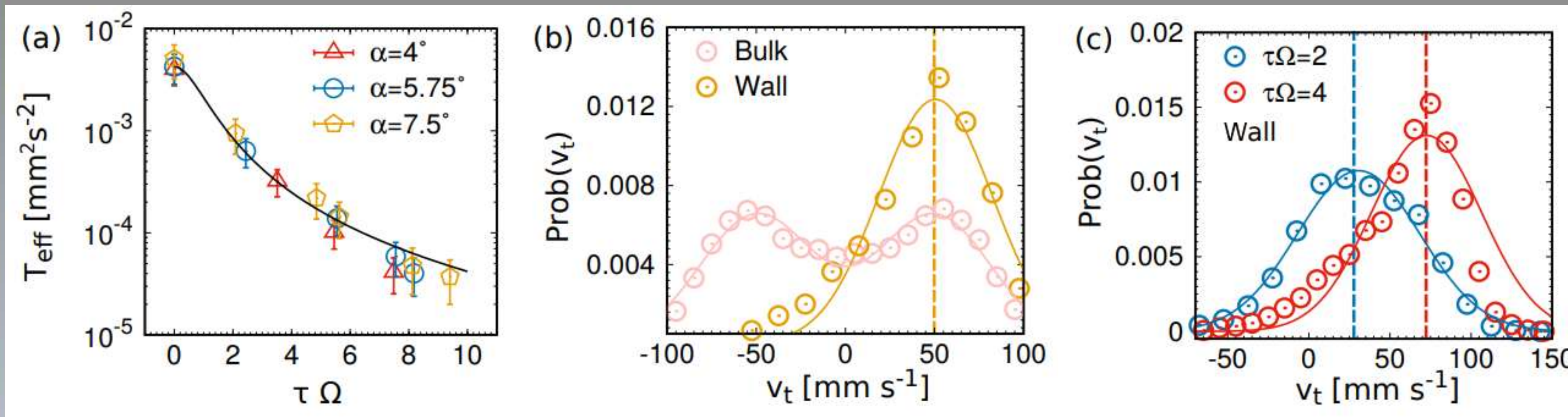


Figure 3. Chirality-induced odd diffusion in chiral active granular particles. (a)-(b) Mean-square displacements for several values of the twisting angle for tilting angle $\alpha = 5.75$ degrees. Specifically, (a) plots the diagonal mean-square displacement, MSD(t), and (b) plots the odd mean-square displacement MSD$_{\text{odd}}(t)$. (c) compares MSD$_{\text{odd}}(t)$ for the xy and yx elements, showing that the two observables are one the opposite of the other. Colored dashed lines in (a)-(c) mark the linear scaling while the black dashed line is an eye guide marking the linear scaling. (d), (e), (f) Long-time diffusion coefficient, D, and longtime odd diffusion coefficient, D_{odd}, as a function of the twisting angle for tilting angle $\alpha = 7.5$ degrees (d), $\alpha = 5.75$ degrees (e) and $\alpha = 4$ degrees (f). Here, points are obtained by experiments and solid colored lines by numerical predictions, Eq.(2) and Eq.(4). Errors in (d)-(f), calculated from the standard deviation, are smaller than the point size.



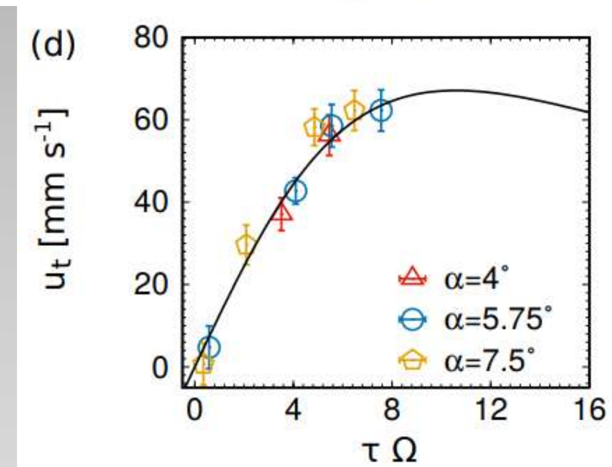
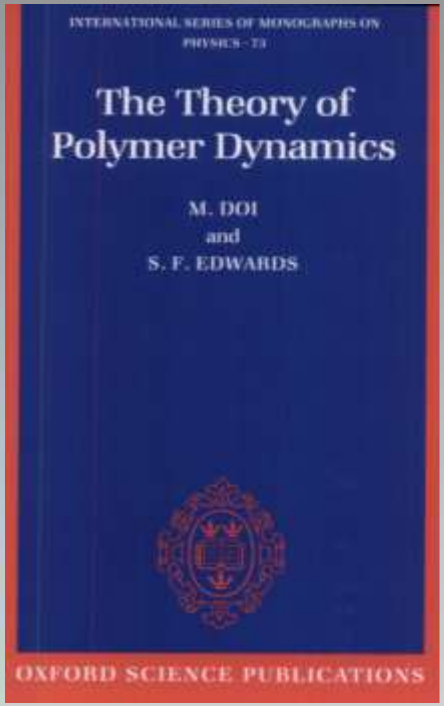


Figure 4. Effective temperature and edge currents. (a) Effective temperature $T_{eff} = \gamma D_t + \frac{v_0^2 \tau \gamma}{1+\Omega^2\tau^2}$ as a function of the rescaled chirality $\tau\Omega$ - generated by different twisting angles β and tilting angles α. (b) Probability distribution Prob(v_t) of the tangential velocity v_t from the middle of the plate, for particles moving close (wall) and far (bulk) from the wall for $\tau\Omega = 3$. (c) Prob(v_t) close to the wall for two values of $\tau\Omega$. (d) Average tangential velocity $u_t = \langle v_t(\mathbf{x} \approx R\rangle$ calculated close to the wall. Colored points are obtained from experimental data, black lines in (a) and (d) represent the scaling functions $\sim 1/(1+\tau^2\Omega^2)$ and $\sim \Omega\tau/(1+k\,\tau^2\Omega^2)$, while colored curves in (b) and (c) are guides for the eyes. Errors in (a) and (d) are calculated from the standard deviation.

3) Chiral active polymers



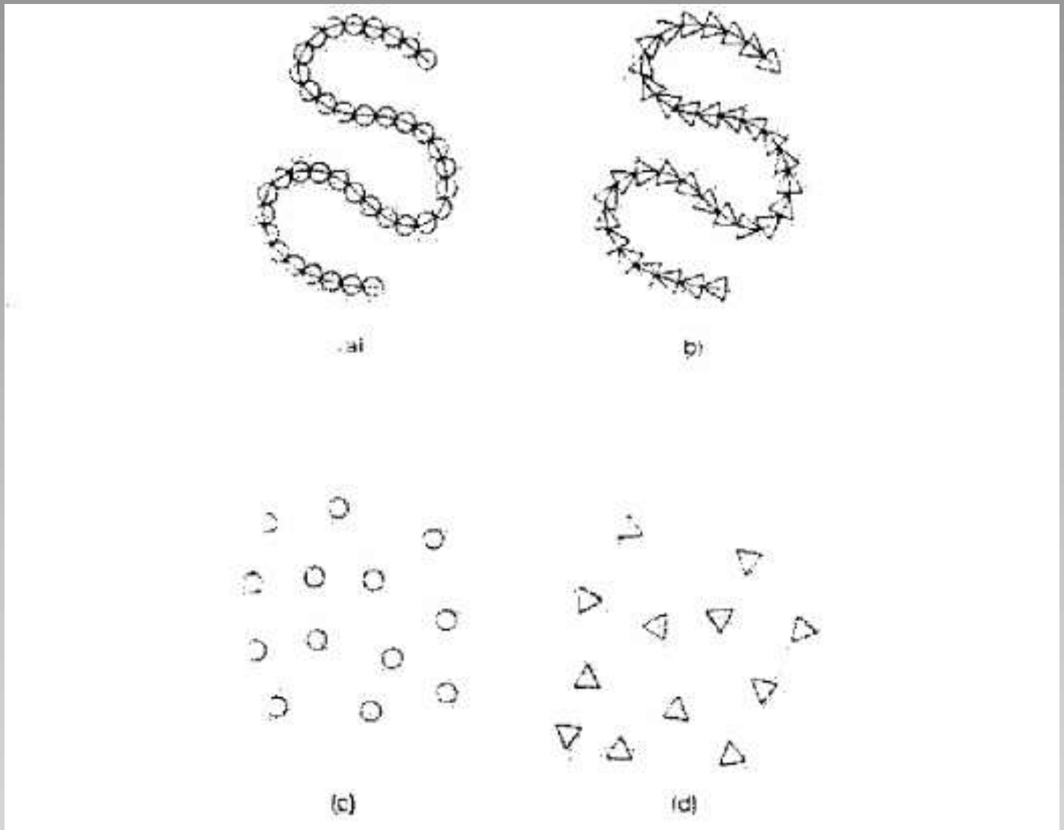
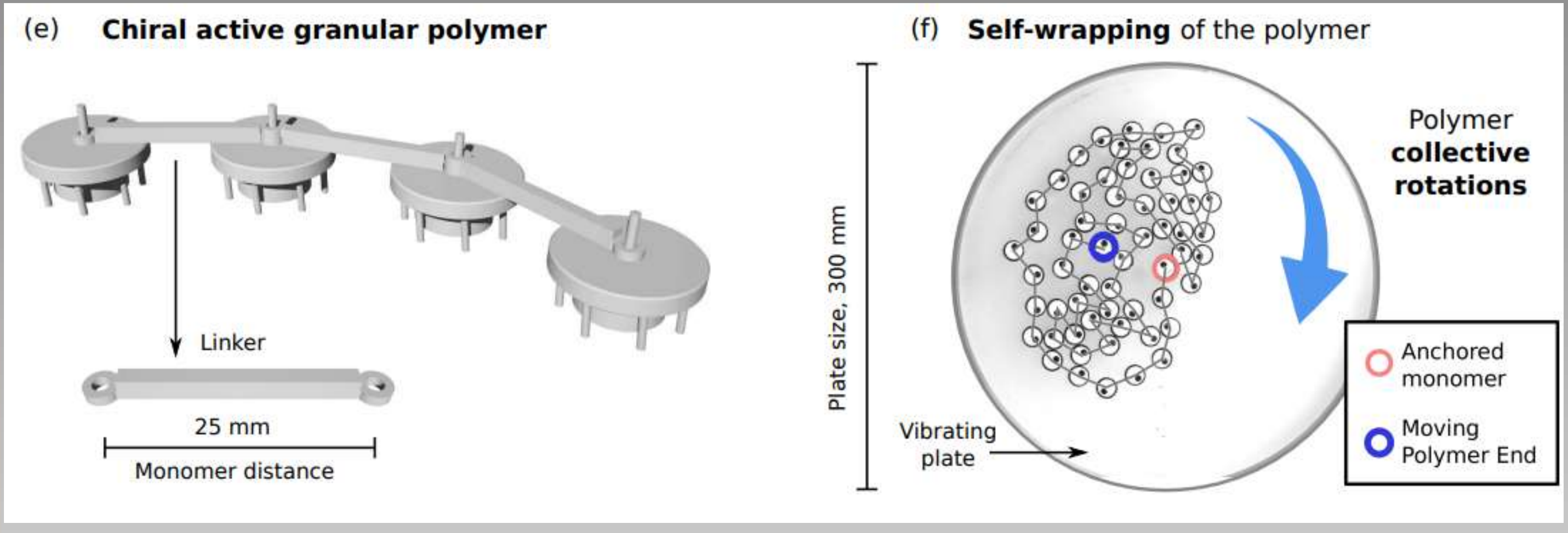


Fig. 1.2. Polymers composed of (*a*) spherical segments and (*b*) triangular segments with (*c*) and (*d*) their segments separated.

What happens if the monomers themselves exhibit chiral dynamics (circle swimming)?

chiral active polymers



L. Caprini

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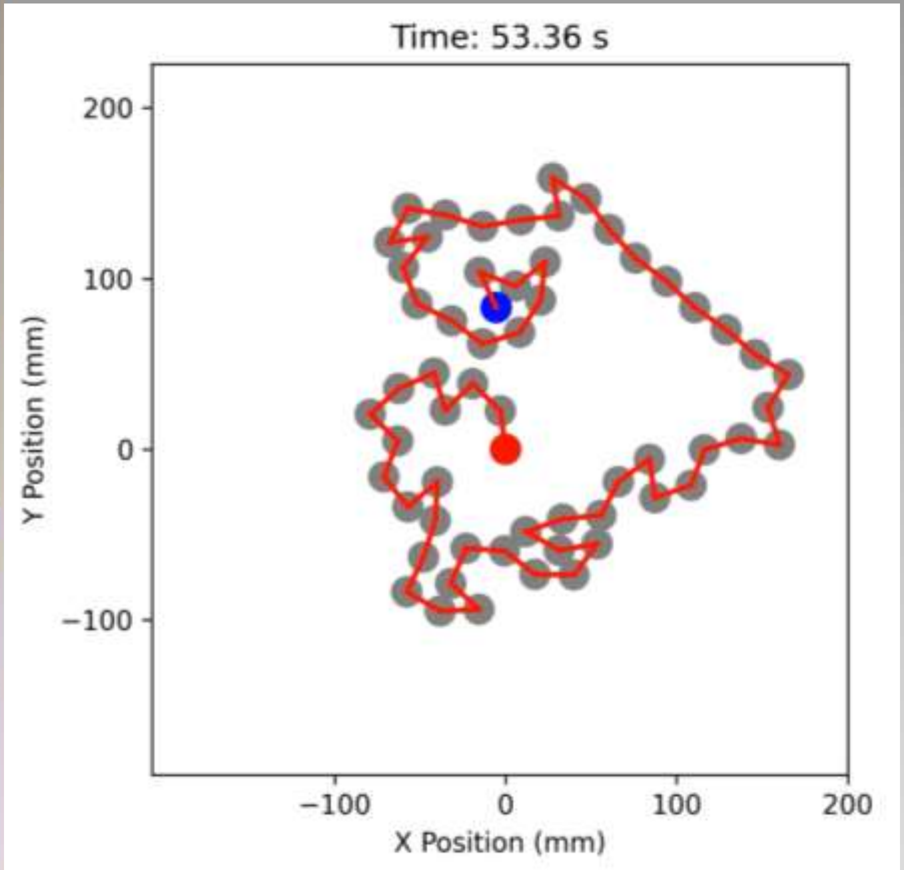
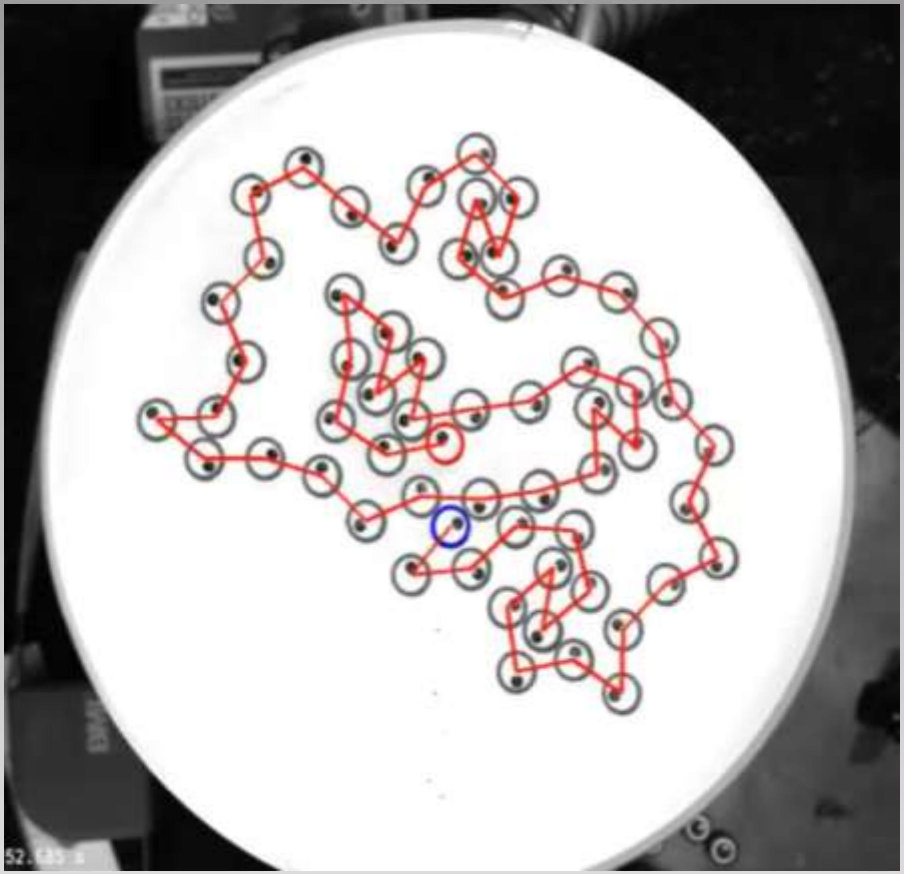
U. M. B. Marconi

L. Caprini, I. Abdoli, U. M. B. Marconi, HL, *Spontaneous self-wrapping in chiral active polymers*, Newton **1**, 100253 (2025)

chiral active polymer dynamics



computer simulation



experiment

self-wrapping

armadillo

woodlouse

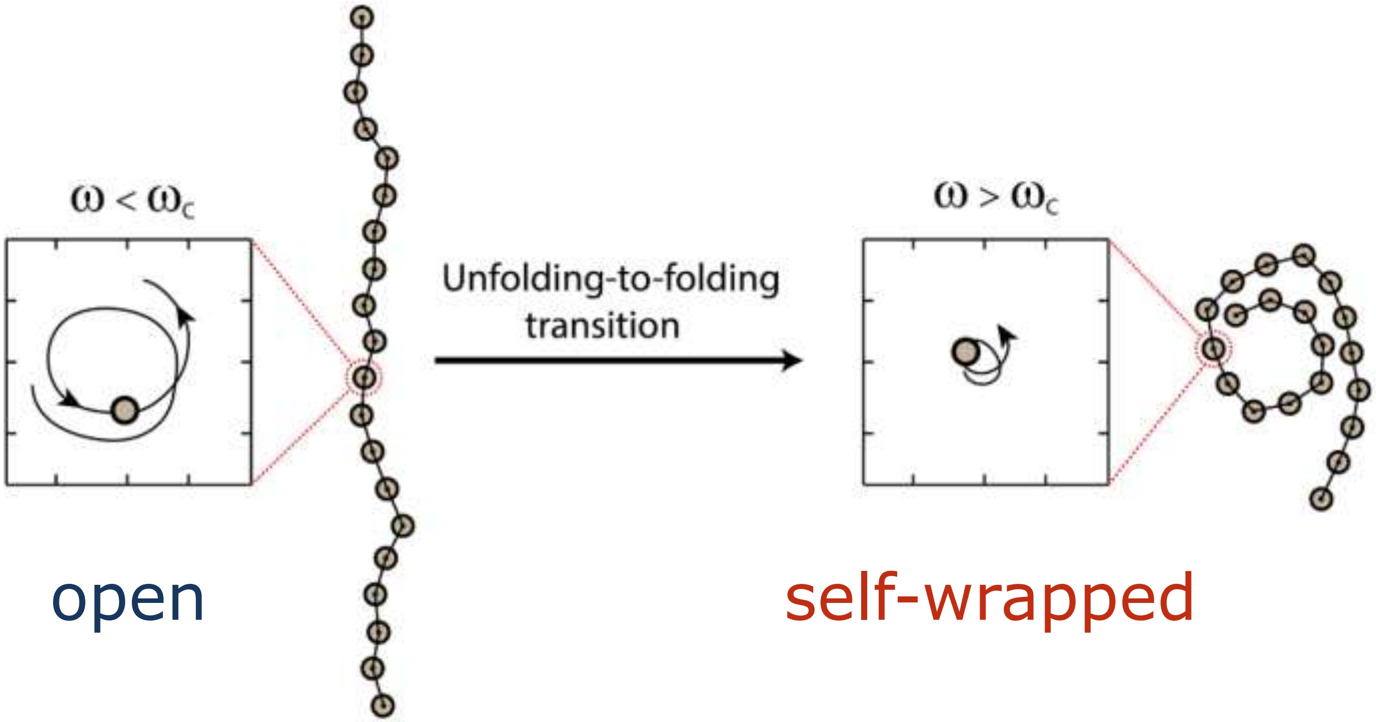


Figure 1. Chiral activity drives the unfolding-folding transition in a granular chiral active polymer

comment on our work

U. Tiwari, and R. Ganapathy, *A twist on active polymers,* Newton **1**, 100257 (2025)

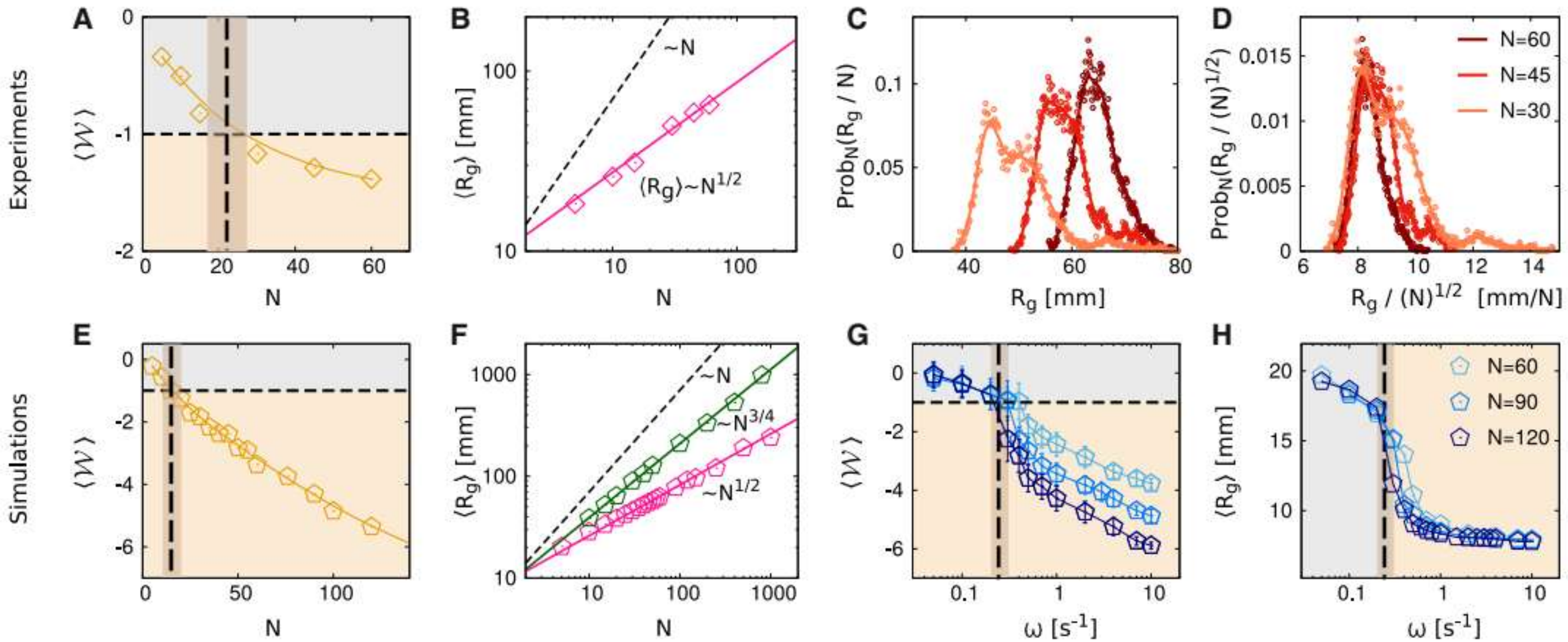


Figure 3. Chirality-induced unfolding-folding transition

(A and E) Average winding number $\langle \mathcal{W} \rangle$ as a function of the number of monomers N from experiments (A) and simulations (E).

(B and F) Average gyration radius $\langle R_g \rangle$ as a function of N, from experiments (B) and simulations (F). In (F), pink dots are obtained under experimental conditions with chirality $\omega = 4\ s^{-1}$ and green dots with $\omega = 0.1\ s^{-1}$. Pink and green lines fit the functions $\sim N^{1/2}$ and $\sim N^{3/4}$, while dashed black lines show the $\sim N$ scaling.

(C) Probability distribution, Prob(R_g), of the polymer gyration radius R_g.

(D) Prob(R_g) for the rescaled gyration radius R_g/N. Both (C) and (D) are obtained for $N = 60, 45, 30$.

(G) $\langle \mathcal{W} \rangle$ as a function of the chirality ω.

(H) $\langle R_g \rangle$ as a function of ω.

(G) and (H) are calculated from simulations. Vertical dashed lines in (G) and (H) mark the critical value of ω where the folded-unfolded transition occurs. In (A)–(H), self-wrapping occurs if the winding number is smaller than −1 (yellow region). Errors in (A)–(H) are calculated from the standard deviation. When error bars are not present, these errors are smaller than the point size. The parameters of the simulations are reported in Tables 1 and 2.

L. Caprini, I. Abdoli, U. M. B. Marconi, HL, *Spontaneous self-wrapping in chiral active polymers*, Newton **1**, 100253 (2025)

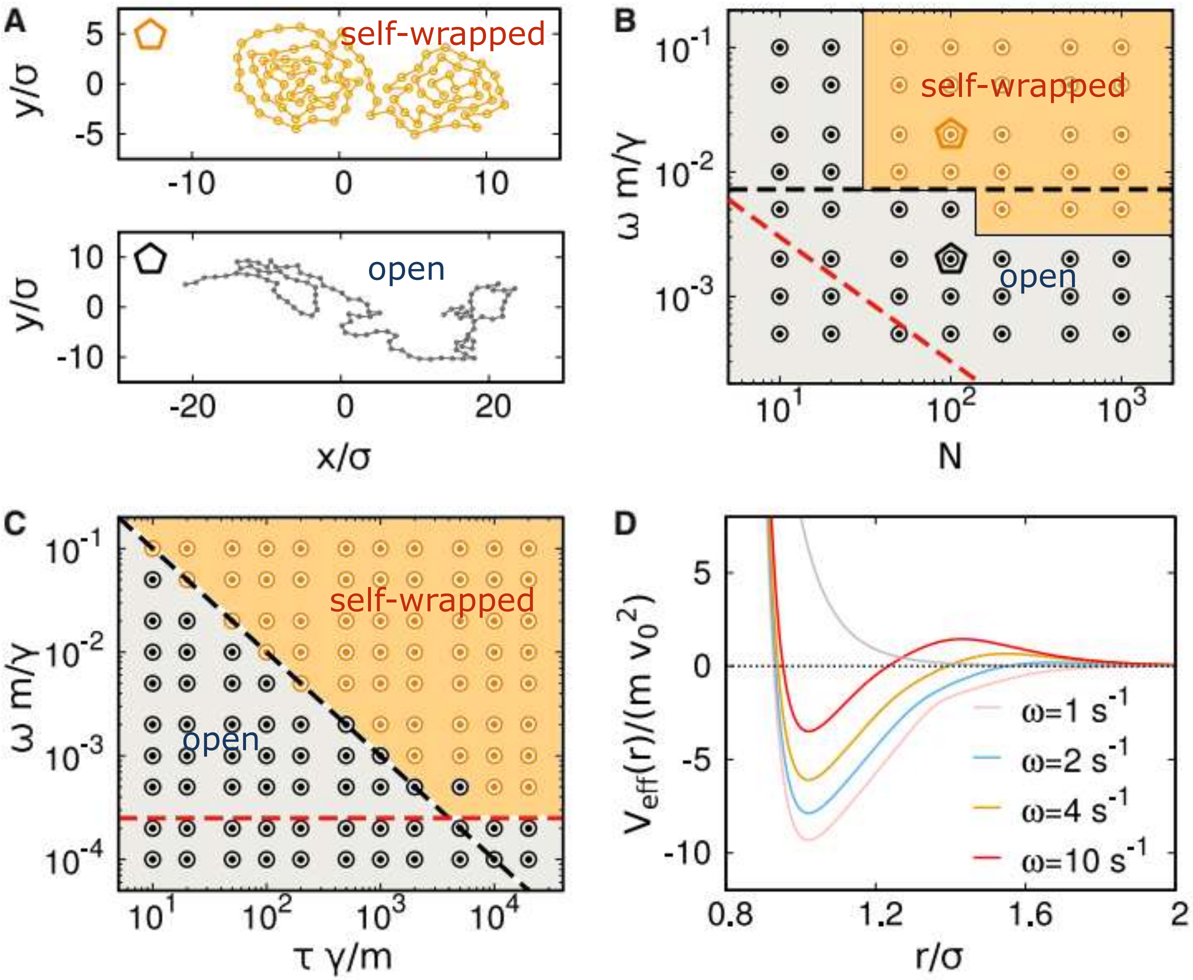


Figure 4. Phase diagrams for non-anchored polymers and effective attraction

(A) Self-wrapped and open configurations for a non-anchored polymer numerically explored with $N = 100$.

(B and C) State diagrams displaying self-wrapping (yellow) or open configurations (gray), where (B) shows the state diagram in the plane of chirality ω and number of monomers N (with fixed $\tau = 1/D_r = 1.4$), and (C) shows it in the plane of ω and persistence time $\tau = 1/D_r$ (with fixed $N = 100$). Time is normalized with the inertial time m/γ. In both diagrams, the dashed black line reports the curves $\omega = 1/\tau$, while the dashed red line reports the curves $\omega = v_0/(NL)$. Yellow and gray pentagons in (B) correspond to the configurations reported in (A).

(D) Effective potential $V_{eff}(r)$ governing the dynamics of chiral active particles for different values of the chirality ω. Effective potential is derived in Equation S35 and is normalized by the typical energy scale of inertial active particles $\sim mv_0^2$. The gray line denotes the repulsive potential $\sim 1/r^{12}$. The parameters to perform simulations and plot Equation S33 are reported under methods (Tables 1 and 2).

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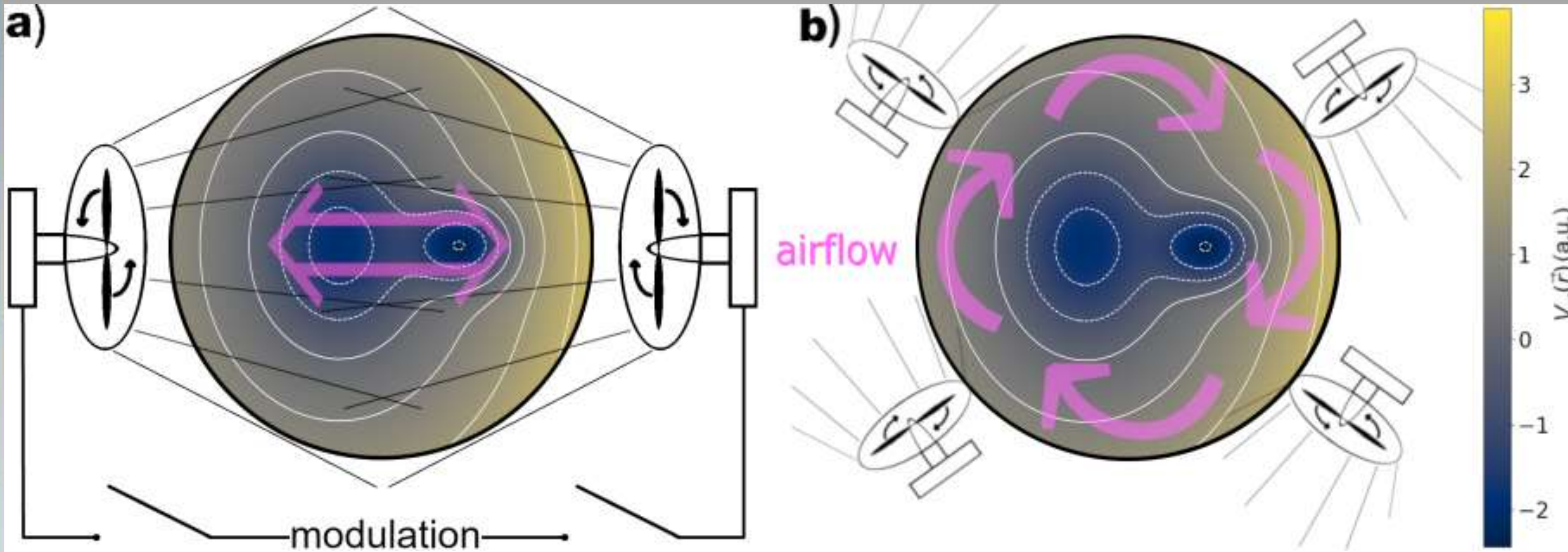
4) Outlook

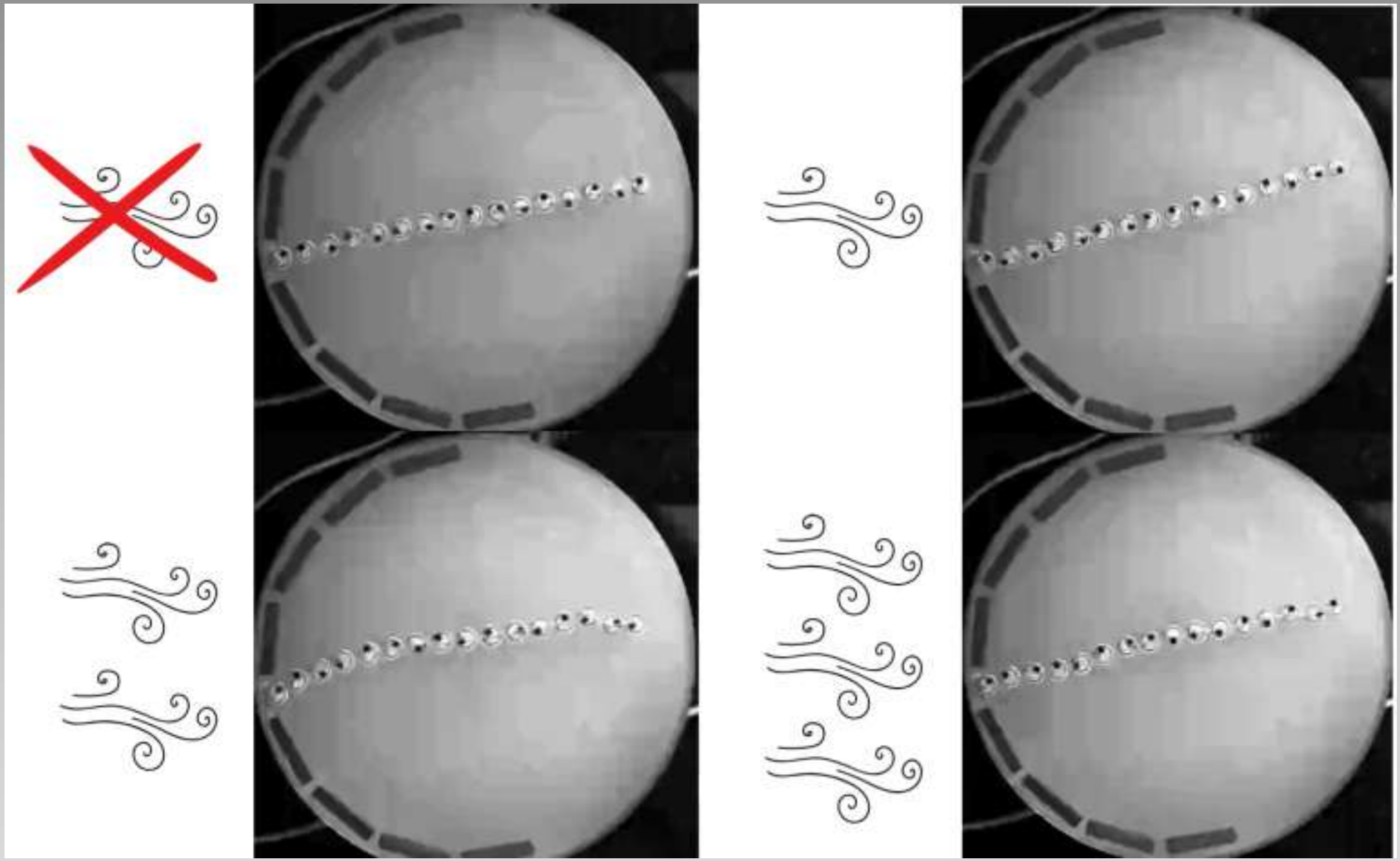
active particles in air flow



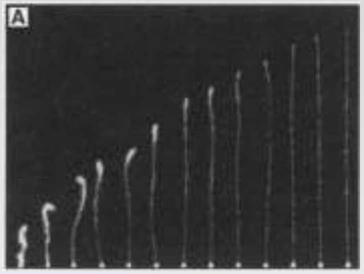
Schematic top view of the experimental set-up with air-blowing fans (“hair-dryers”) around the asymmetric double-well potential $V_{ext}(\vec{r})$ indicated by iso-energy lines and an energetic color scale. The fans create an air flow field indicated by the arrows which acts as an external body force on the granular particles. a) Two fans with mutual oscillatory electric power create a time-dependent oscillatory flow. b) Four fans create a time-independent vortex-like air flow field.

use air-blowing fans

active polymers in air flow (Marco Musacchio)



T. T. Perkins, D. E. Smith, R. G. Larson, S. Chu,
Stretching of a Single Tethered Polymer in a Uniform Flow,
Science **268**, 83 (1995)



5) Conclusions

odd-diffusive chiral active particles exhibit fascinating new physics

Acknowledgement:
L. Caprini, I. Abdoli, U.M.B. Marconi, C. Bechinger, I. Buttinoni, C. Scholz, E. Kalz, H. D. Vuijk, J.-U. Sommer, A. Sharma, B. Liebchen, M. te Vrugt, R. Wittkowski, M. Musacchio…