Acoustic propulsion of nanoparticles and near-hydrodynamic electron flows

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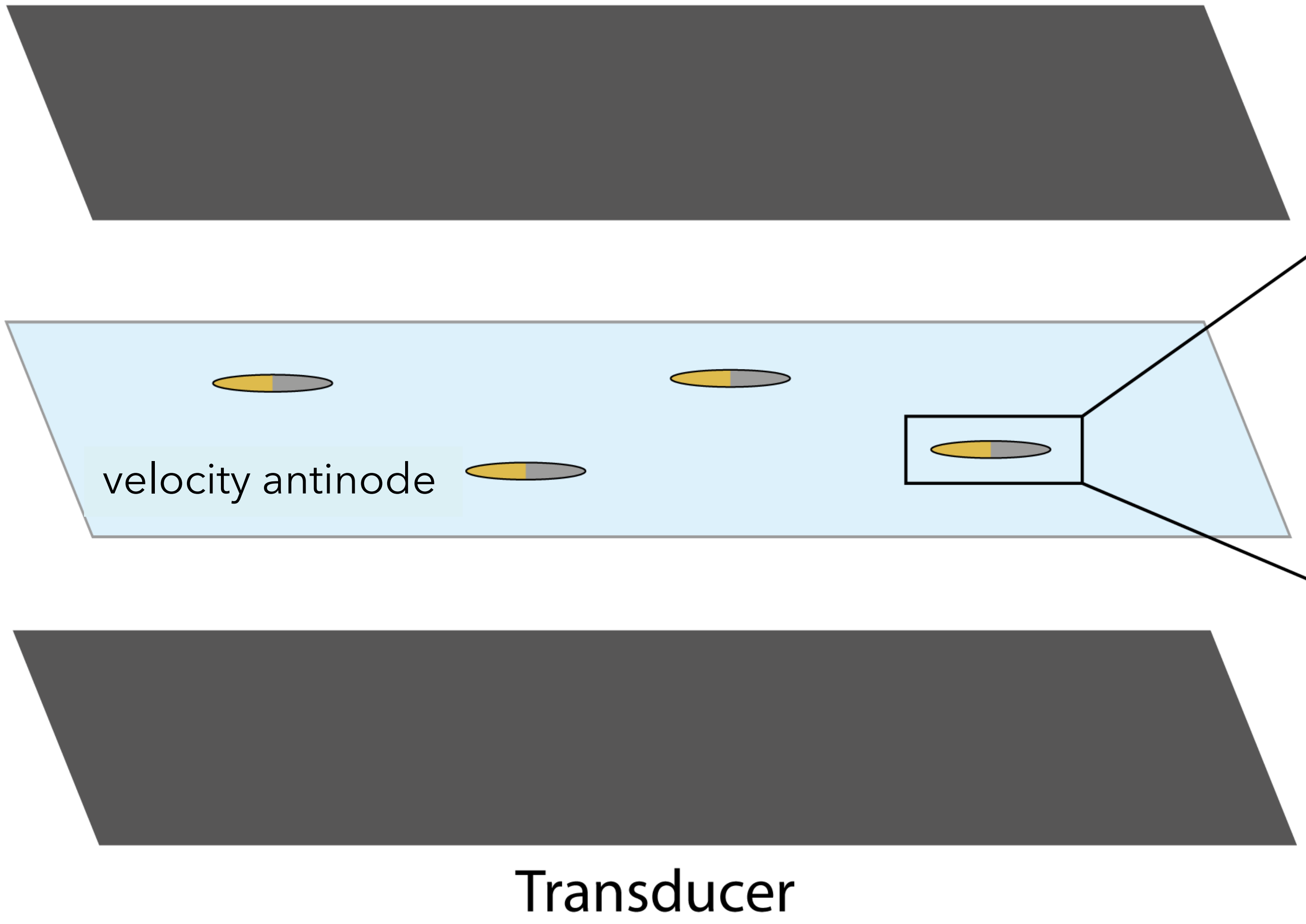
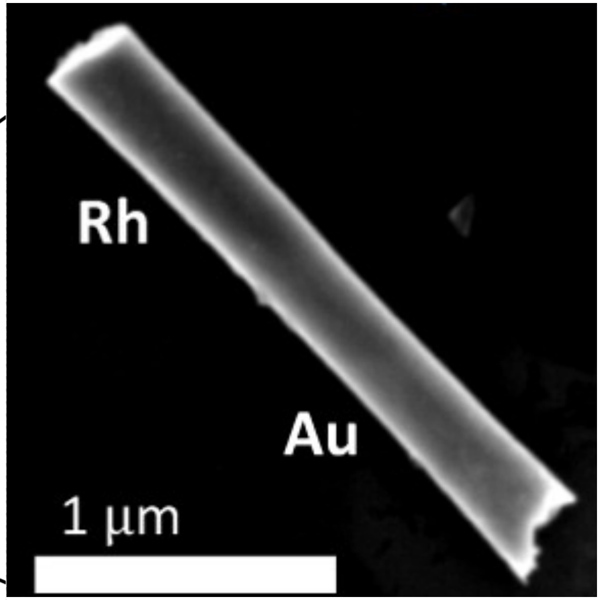
Autonomous nanoparticle propulsion

JF Collis, D Chakraborty & JE Sader, *Journal of Fluid Mechanics,* 825, 29 (2017); 935, E1 (2022)
P Li, AR Nunn, DR Brumley, JE Sader & JF Collis, *Journal of Fluid Mechanics,* 984, R1 (2024)

Acoustic field in a liquid cell (3.77 MHz)

Wang *et al*. ACS Nano 2012, ACS Nano 2016



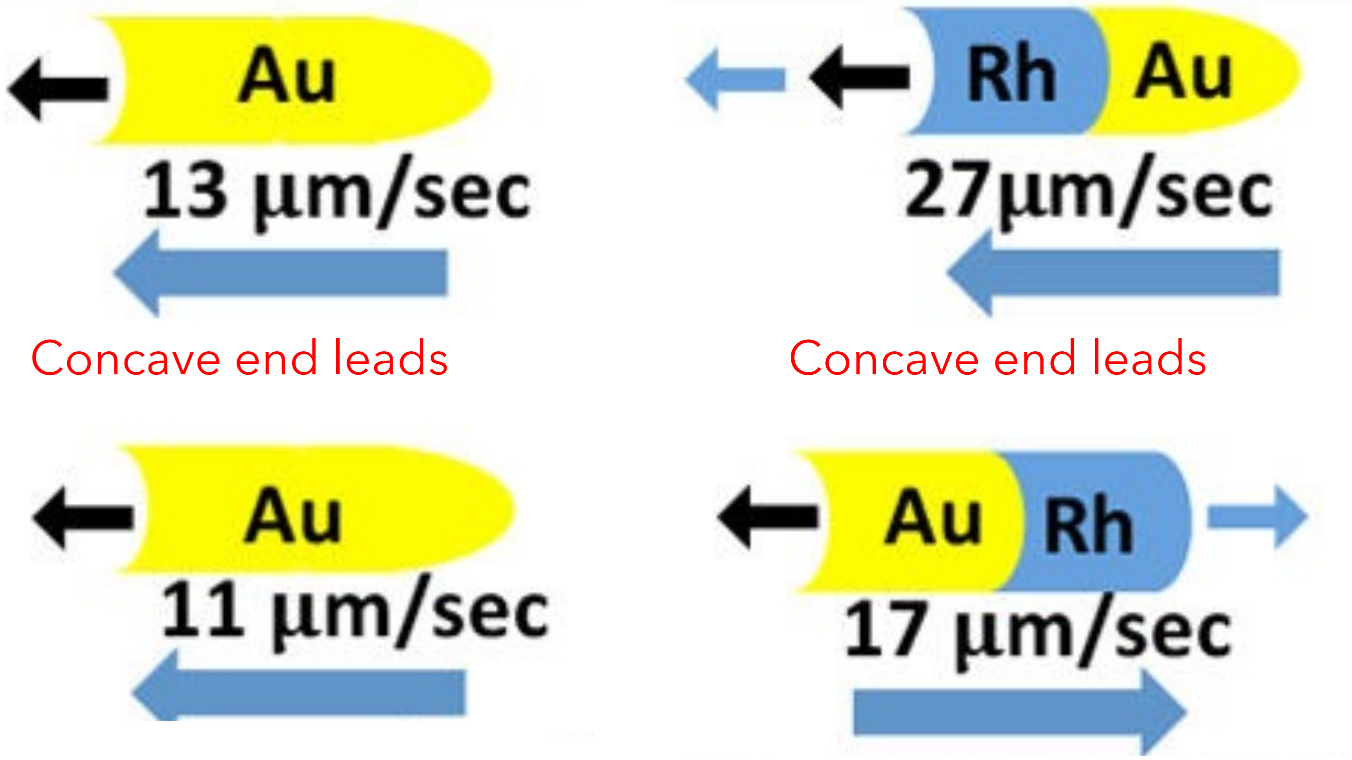


Ruthenium-gold nanorods
(autonomous propulsion)

Ahmed *et al*. ACS Nano 2016

Experimental observations

Ahmed *et al*. ACS Nano 2016



Concave end leads

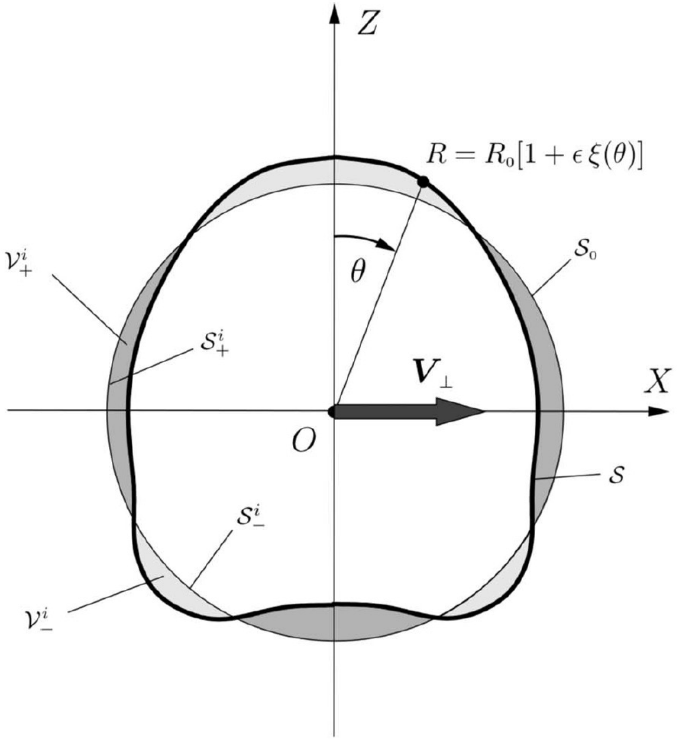
Concave end leads

Concave end leads

Convex end leads

Light and concave end leads motion!

Acoustic streaming
(near-spherical particle – no density asymmetry & low frequency)



Nadal & Lauga, Phys Fluids 2014

Van Dyke, 1982

Streaming structure:

P Li *et al.*, JFM, 974, A37 (2023)

$\beta_{\text{critical}} = 16.817$

Steady propulsion due to shape asymmetry!

Lorentz reciprocal theorem (Stokes flow)

Navier-Stokes equation:

$$\beta \frac{\partial \mathbf{u}}{\partial t} + \epsilon \beta \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u}$$

normalized oscillation amplitude

Lorentz reciprocal theorem (leading-order propulsion velocity):

$$U_{\text{prop}} = \frac{\beta}{4F_p} \iiint_V \mathbf{u}' \cdot \left(\bar{\mathbf{u}}^{(0)} \cdot \nabla \bar{\mathbf{u}}^{(0)*} + \bar{\mathbf{u}}^{(0)*} \cdot \nabla \bar{\mathbf{u}}^{(0)} \right) \mathrm{d}V$$

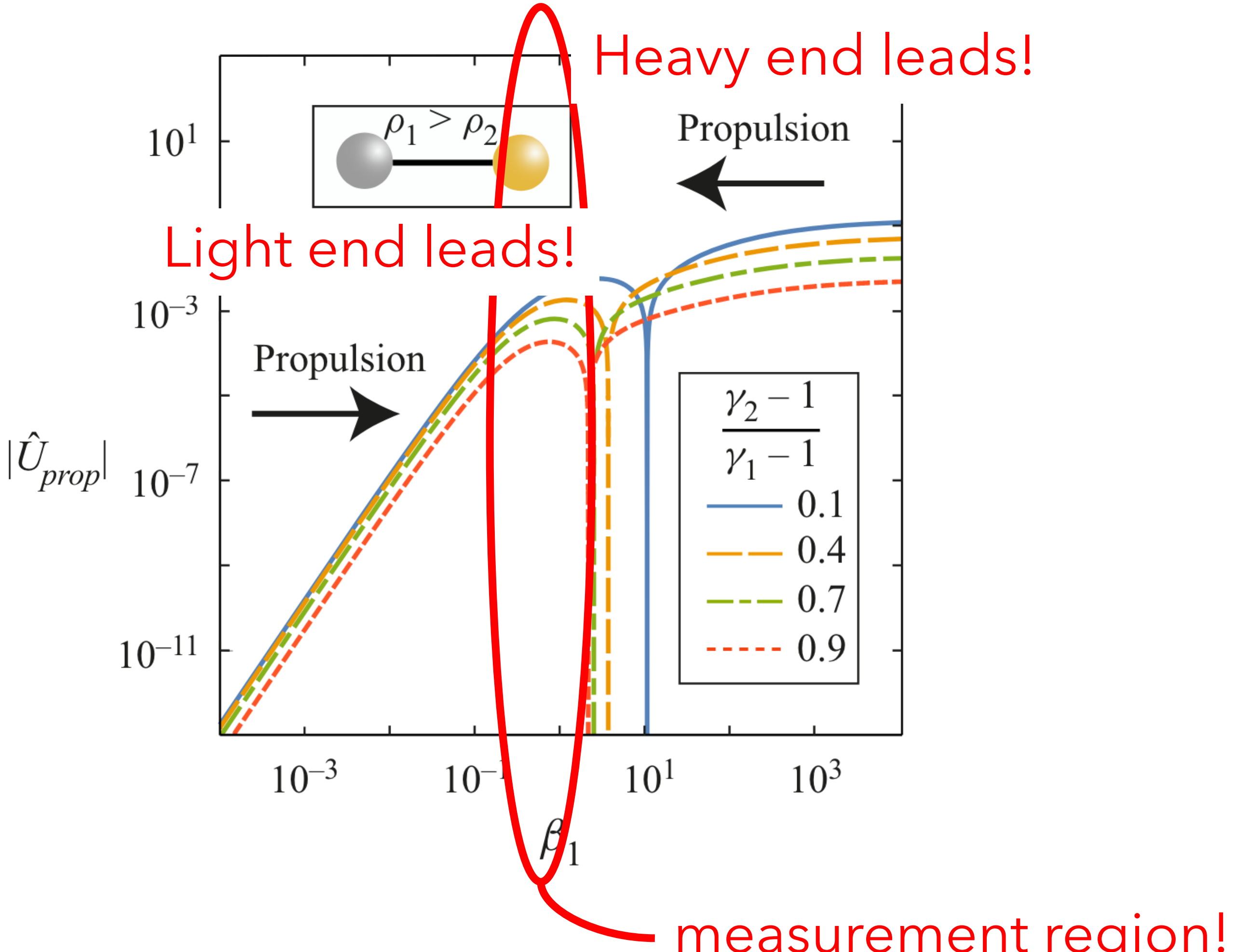
+ surface terms

unsteady (linear) Stokes flow

auxiliary (steady) Stokes flow

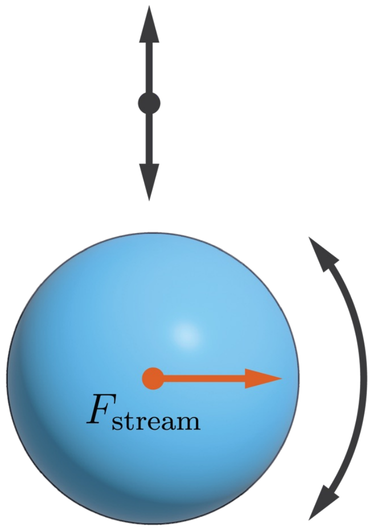
Dumbbell model

(DENSITY asymmetries only)



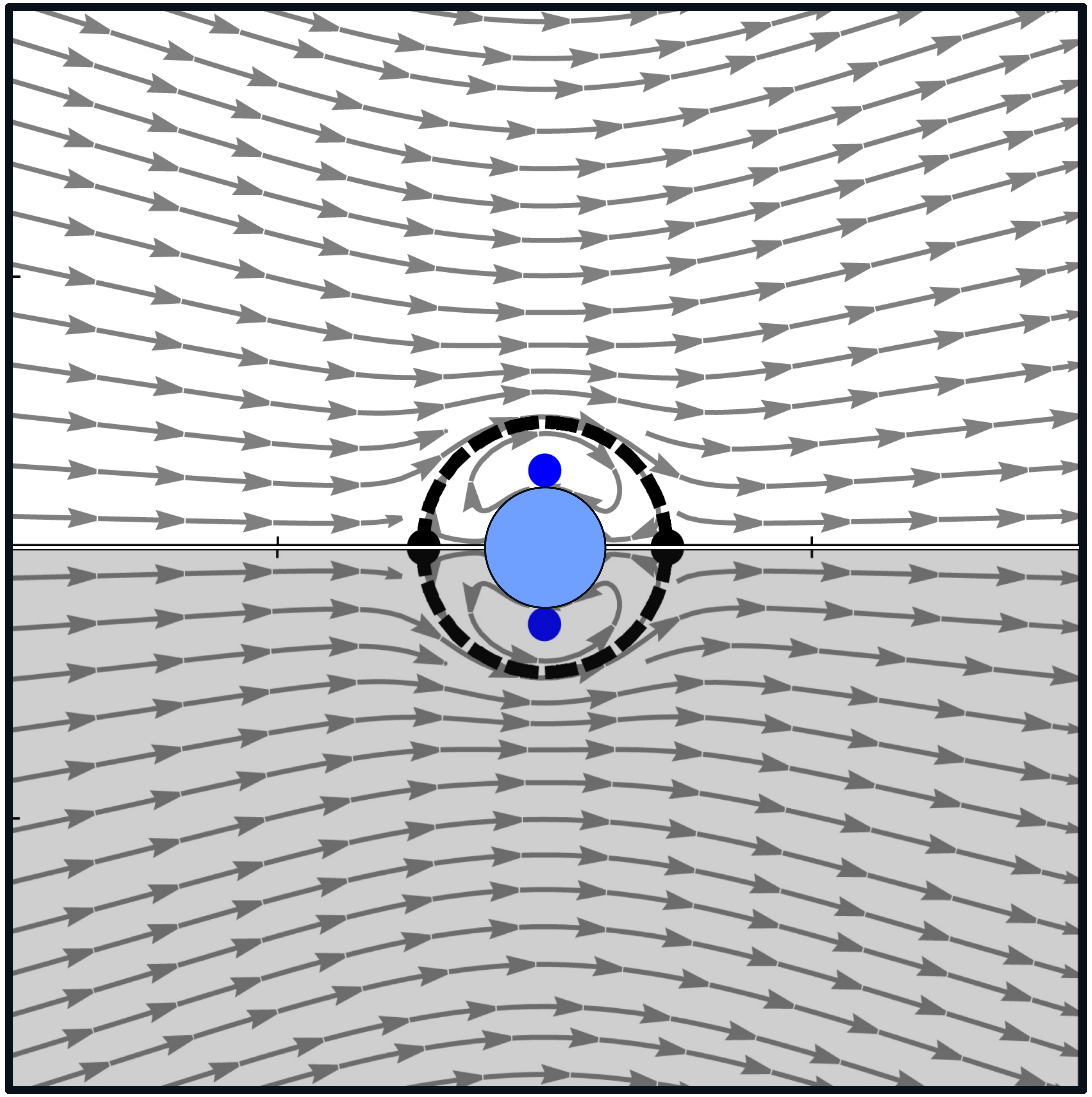
Operation at LOW frequency yields experimental observation

Sphere engine



Stokeslet strength is NET FORCE

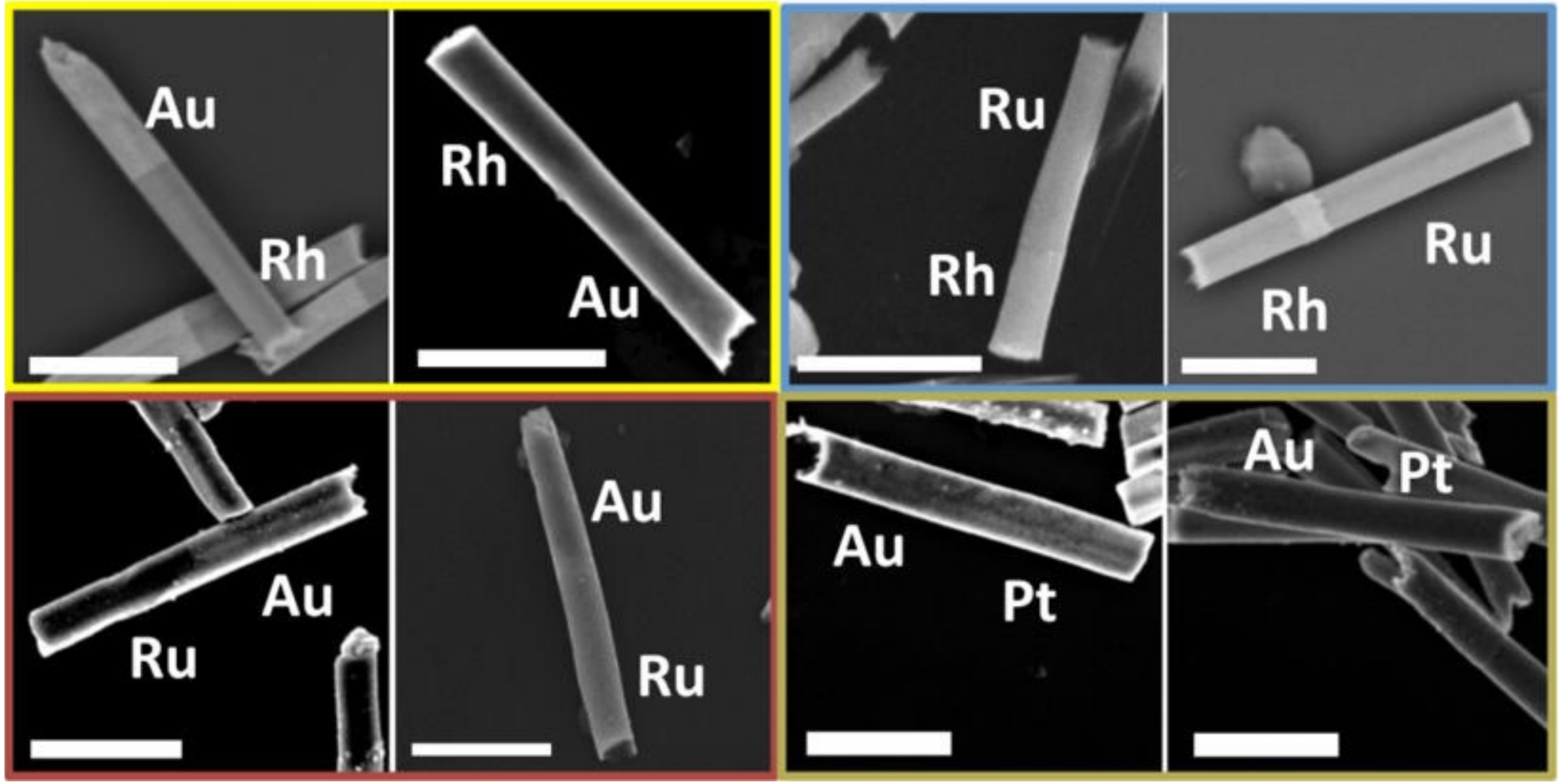
$\beta = 50$



P Li *et al.*, JFM, 974, A37 (2023)

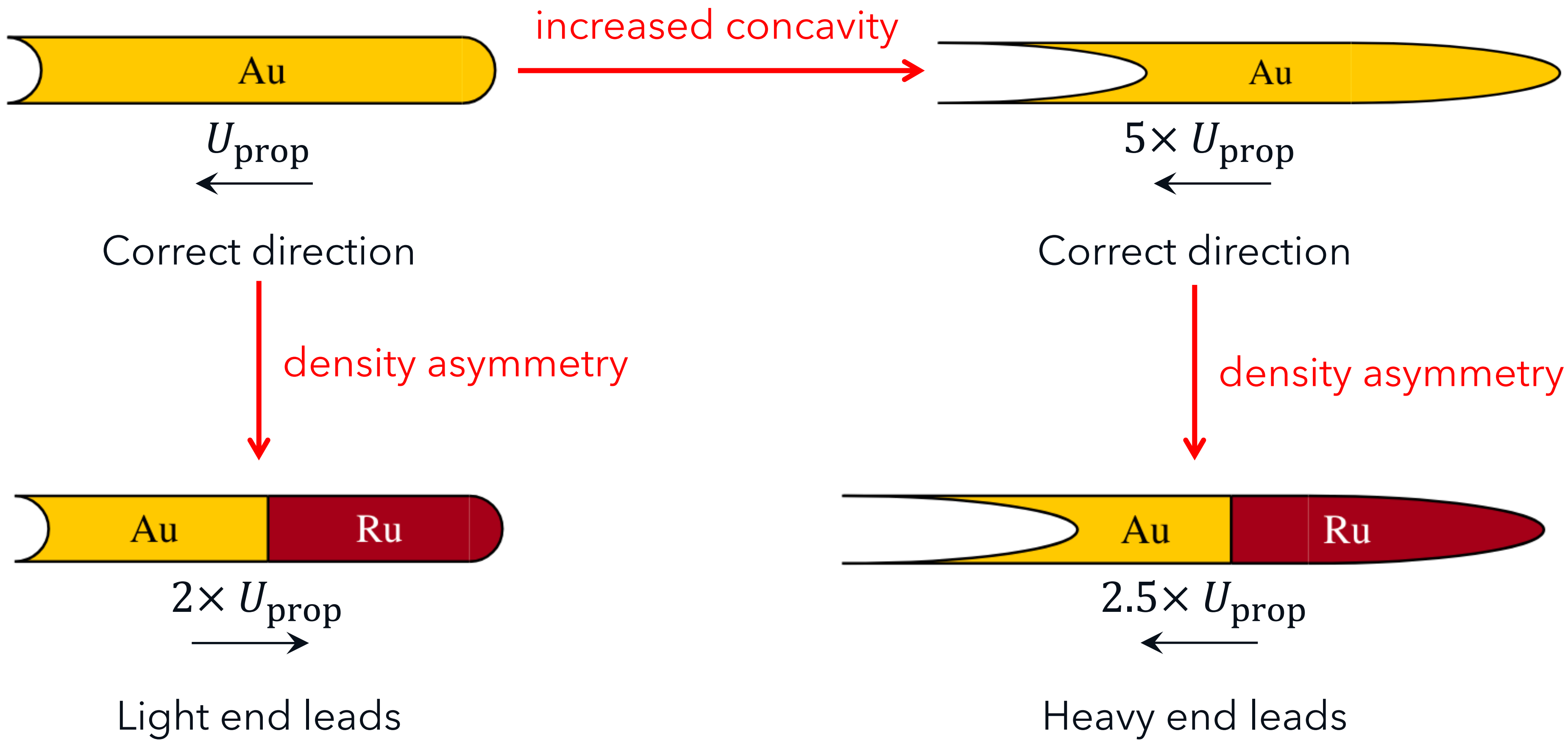
Experimental nanorods

(dimensionless frequency: $\beta = 0.53 \pm 0.16$)



Ahmed *et al*. ACS Nano 2016

Numerical simulations
($\beta = 0.53$)



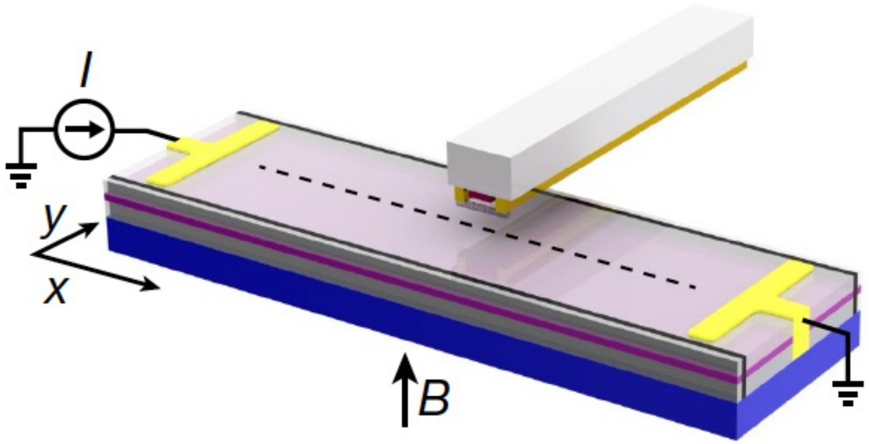
Agrees with experiment!

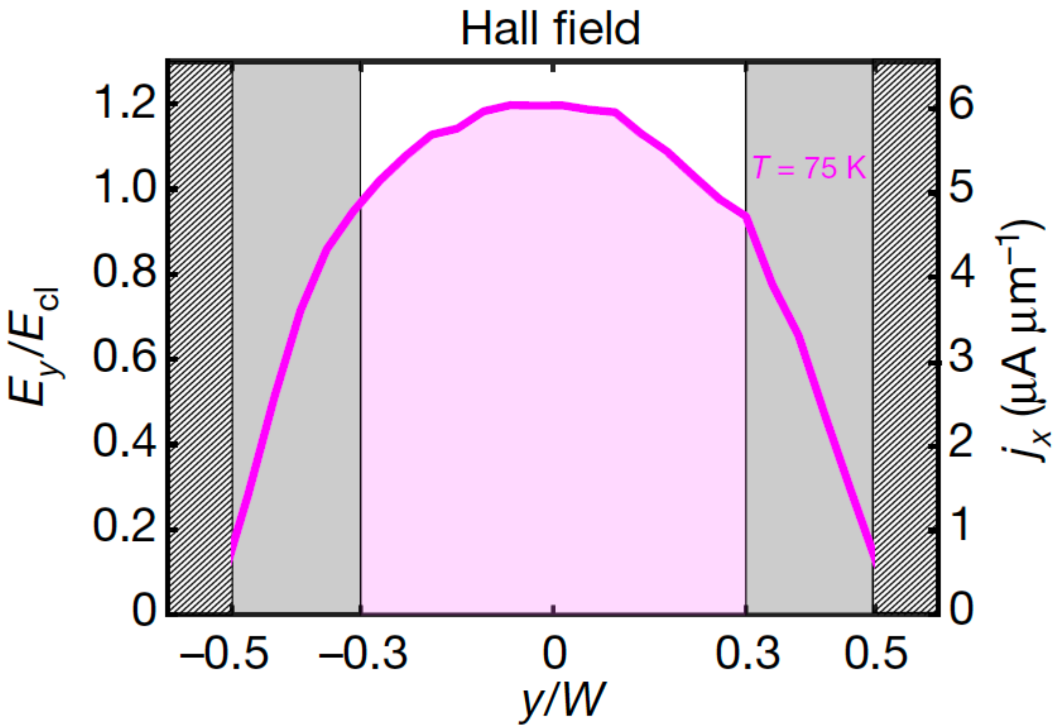
Near-hydrodynamic electron flows

N Ben-Shachar, JT Johnson, M Madadi, DR Brumley, J Nassios & JE Sader

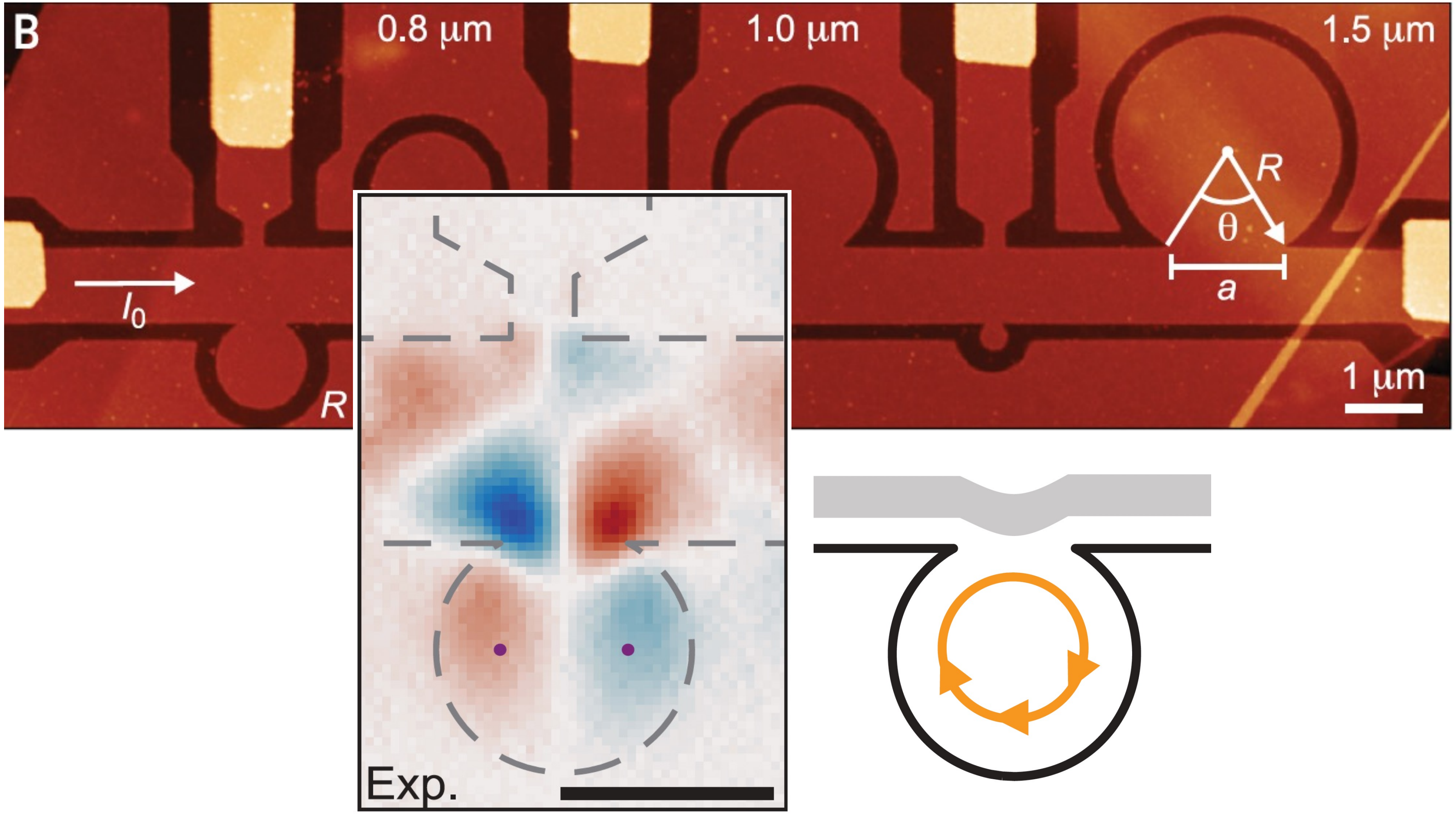
Physical Review B, 111, L121107 (2025); 111, 125145 (2025)

Experiments: Sulpizio *et al.* Nature (2019)





Experiments: Palm *et al.* Science (2024)



Standard hydrodynamic model

$$-\frac{1}{m}\nabla\mu + \nu\nabla^2\boldsymbol{u} + \nu_H(\nabla^2\boldsymbol{u})\times\widehat{\boldsymbol{B}} - \frac{1}{\tau_{\mathrm{imp}}}\boldsymbol{u} = -\frac{e}{m}(\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B})$$

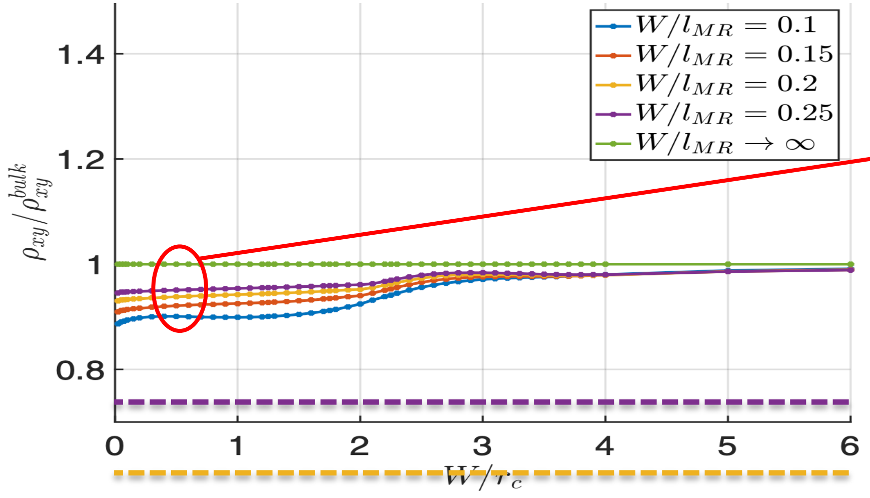
pressure viscosity Hall (odd) viscosity Ohmic drag body forces

- Applies in the continuum limit only!

$$\frac{l_{MC}}{L} \equiv k \ll 1$$

- BCs: no-slip or Navier slip

Discrepancy with kinetic theory (Hall resistivity)



Hall (odd) viscosity effect!

hydrodynamic theory (k = 0.2)

hydrodynamic theory (k = 0.25)

Scaffidi *et al.* PRL (2017)

hydrodynamic theory (k = 0.33)

hydrodynamic theory (k

"The exact value [of the Hall resistance]… can be calculated **only** by a numerical solution of the kinetic [Boltzmann] equation"

Afanasiev *et al.* PRB (2022)

Matched asymptotic expansion of Boltzmann equation

Linearized non-dimensional Boltzmann equation:

$$\mathbf{v} \cdot \nabla_{\mathbf{x}} h + 2k\boldsymbol{E} \cdot \mathbf{v} - \frac{L}{r_c}(\mathbf{v} \times \boldsymbol{B}) \cdot \nabla_{\mathbf{v}} h = \frac{k}{G^2}(\mu - h) + \frac{1}{k}(\mu + 2\mathbf{v} \cdot \boldsymbol{u} - h)$$

Voltage and bulk velocity:

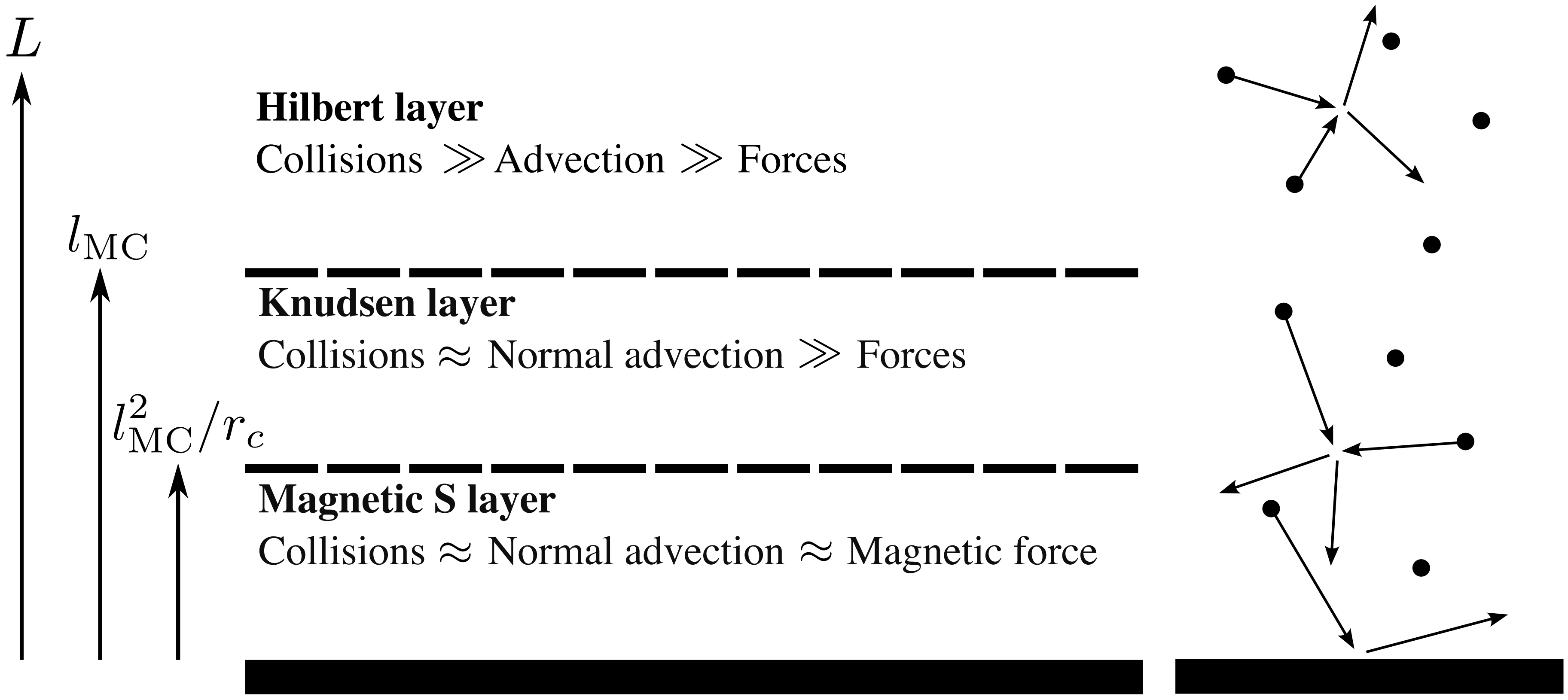
$$\mu = \frac{1}{2\pi}\int h\, d\Omega \qquad \boldsymbol{u} = \frac{1}{2\pi}\int \mathbf{v}\, h\, d\Omega$$

Small-*k* expansion:

$$h = h^{(0)} + kh^{(1)} + k^2 h^{(2)} + \ \ldots$$

Gas dynamics: J Nassios & JE Sader, *Journal of Fluid Mechanics*, 708, 197-249 (2012)

Boundary layer structure



Diffuse boundary

Gas dynamics: J Nassios & JE Sader, *Journal of Fluid Mechanics*, 708, 197-249 (2012)

Hilbert layer - momentum equations

$$O(1): \quad \frac{1}{2}\nabla\mu^{(0)} - \frac{L}{r_c}\mathbf{u}^{(0)} \times \mathbf{B} = \mathbf{0}$$

$$O(k): \quad \frac{1}{2}\nabla\mu^{(1)} - \frac{L}{r_c}\mathbf{u}^{(1)} \times \mathbf{B} = \frac{1}{4}\nabla^2\mathbf{u}^{(0)} - \frac{\mathbf{u}^{(0)}}{G^2} - \mathbf{E}$$

$$O(k^2): \quad \frac{1}{2}\nabla\mu^{(2)} - \frac{L}{r_c}\mathbf{u}^{(2)} \times \mathbf{B} = \frac{1}{4}\nabla^2\mathbf{u}^{(1)} - \frac{L}{2r_c}\nabla^2\left(\mathbf{u}^{(0)} \times \mathbf{B}\right) - \frac{\mathbf{u}^{(1)}}{G^2}$$

Hall viscous force

Equations identical to literature!

Hilbert layer – new boundary conditions

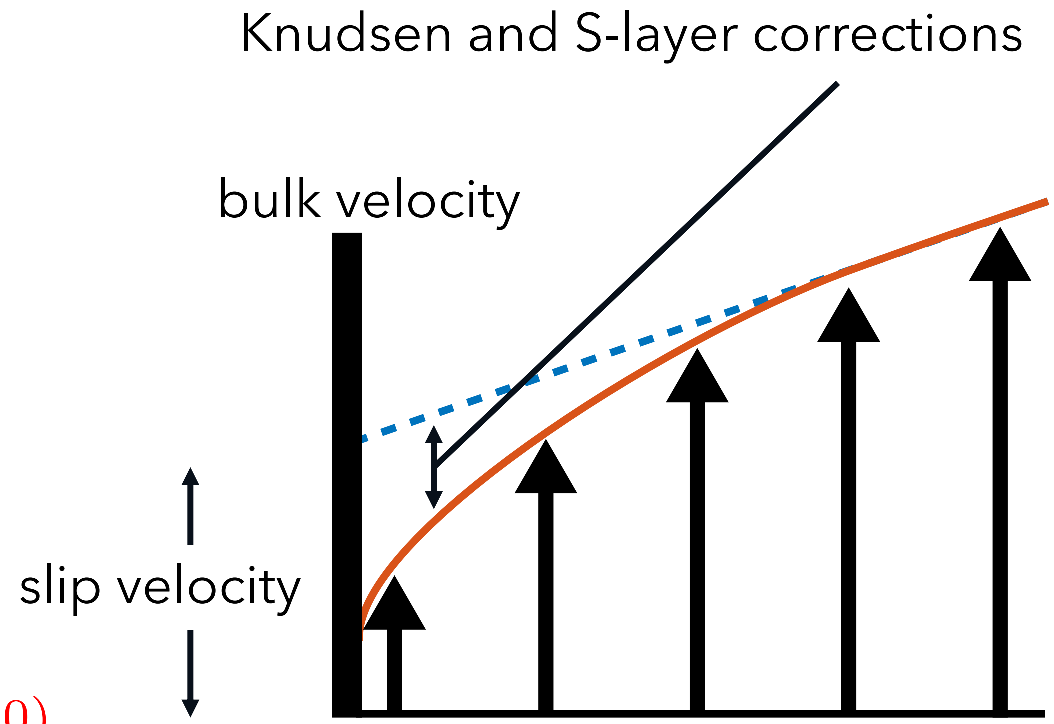
$$O(1): \quad \mathbf{u}^{(0)} \cdot \hat{t} = 0$$

$$O(k): \quad \mathbf{u}^{(1)} \cdot \hat{t} = k_0 \, \hat{n}\hat{t} : S^{(0)}$$

$$O(k^2): \quad \mathbf{u}^{(2)} \cdot \hat{t} = k_0 \, \hat{n}\hat{t} : S^{(1)} + a_1 \, \hat{t}\hat{n}\hat{n} : \nabla S^{(0)} + a_2 \kappa \, \hat{n}\hat{t} : S^{(0)}$$

$$\mathbf{u}^{(2)} \cdot \hat{n} = \frac{1}{2} \int_0^\infty Y_0(\eta_0) d\eta_0 \, \hat{n}\hat{n}\hat{n} : \nabla S^{(0)}$$

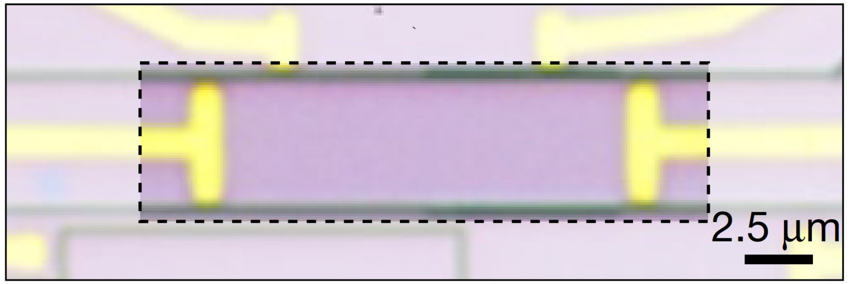
… and Knudsen and S-layer corrections



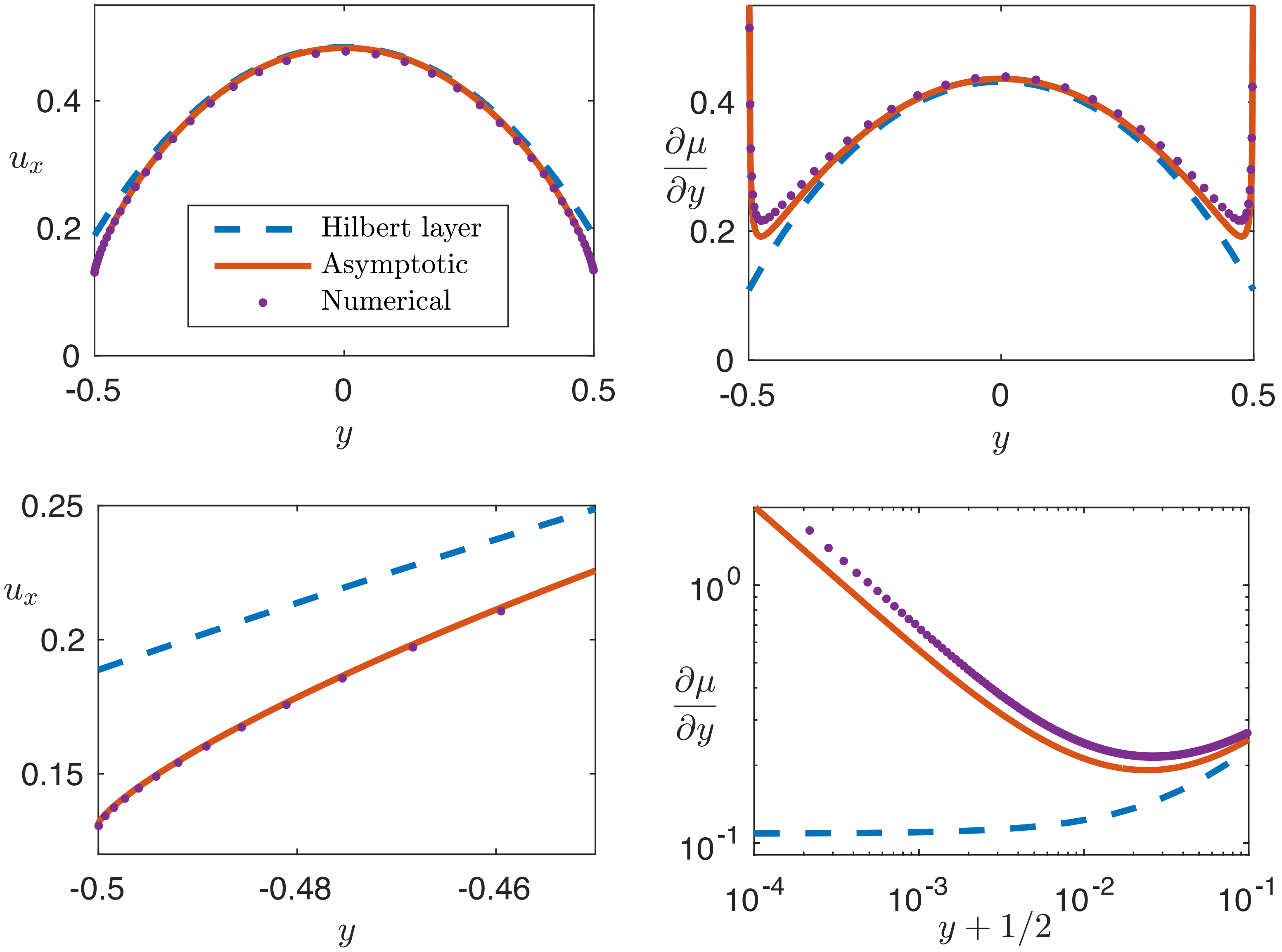
Poiseuille flow

$k = 0.2$

Sulpizio *et al.* Nature (2019)



$\otimes B$



Poiseuille flow – Hall resistivity

Deviation from (continuum) $R_{xy}^{\text{bulk}} = B/(ne)$ is due to the Hall viscosity ONLY!

$$\Delta R_{xy} = -2\frac{\tanh(G^{-1})}{G - G^2\tanh(G^{-1})}$$

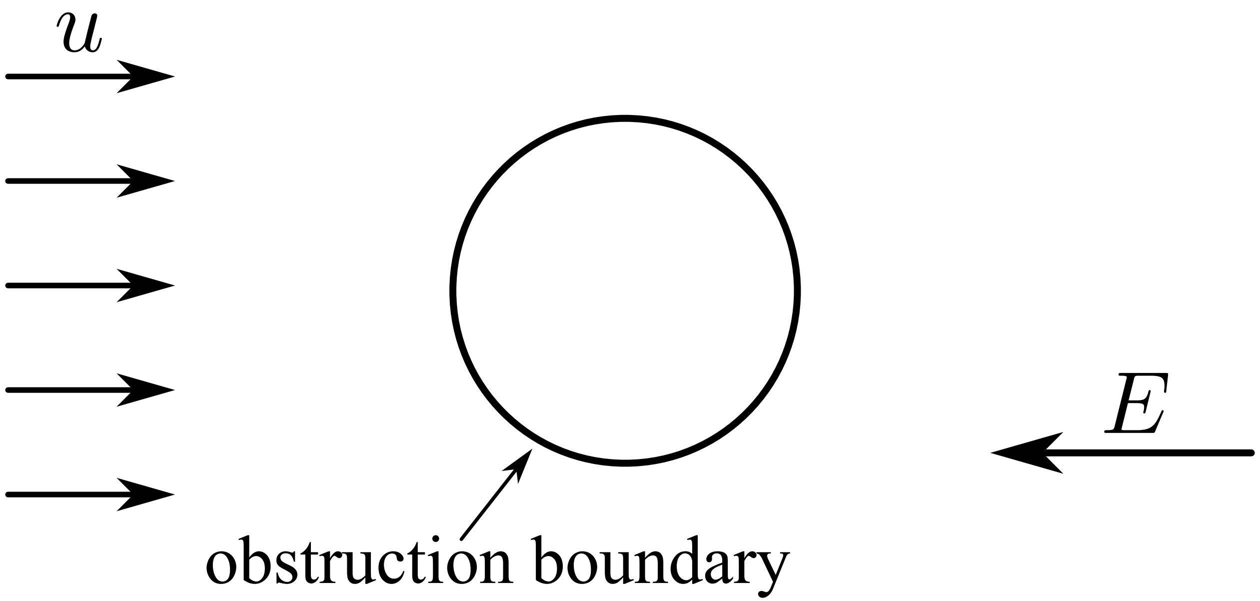
✗

[Scaffidi *et al*. PRL 2017; Holder *et al*. PRL 2019; Matthaiakakis *et al*. PRB 2020]

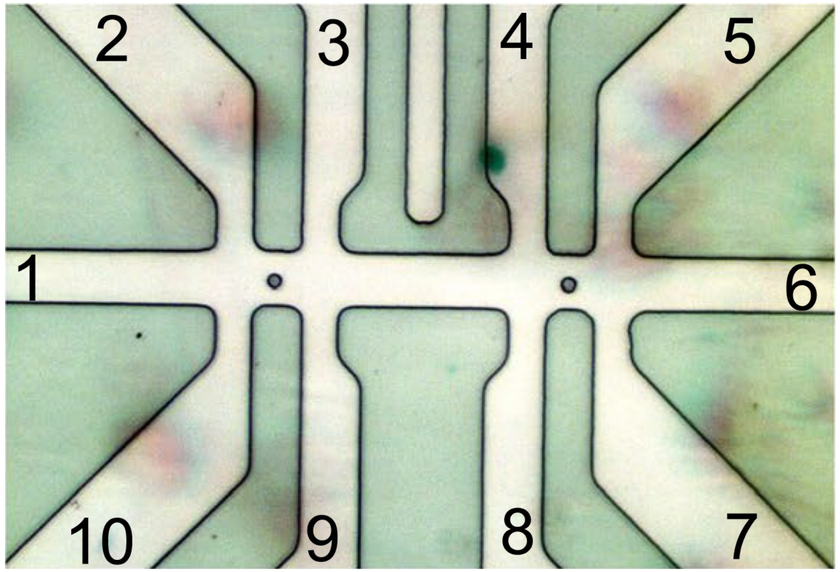
$$\boxed{\Delta R_{xy} = -1.207\frac{\tanh(G^{-1})}{G - G^2\tanh(G^{-1})} + O(k\log k)}$$

correct BCs + Knudsen layer + magnetic S-layer

Flow around a circle – specular reflection

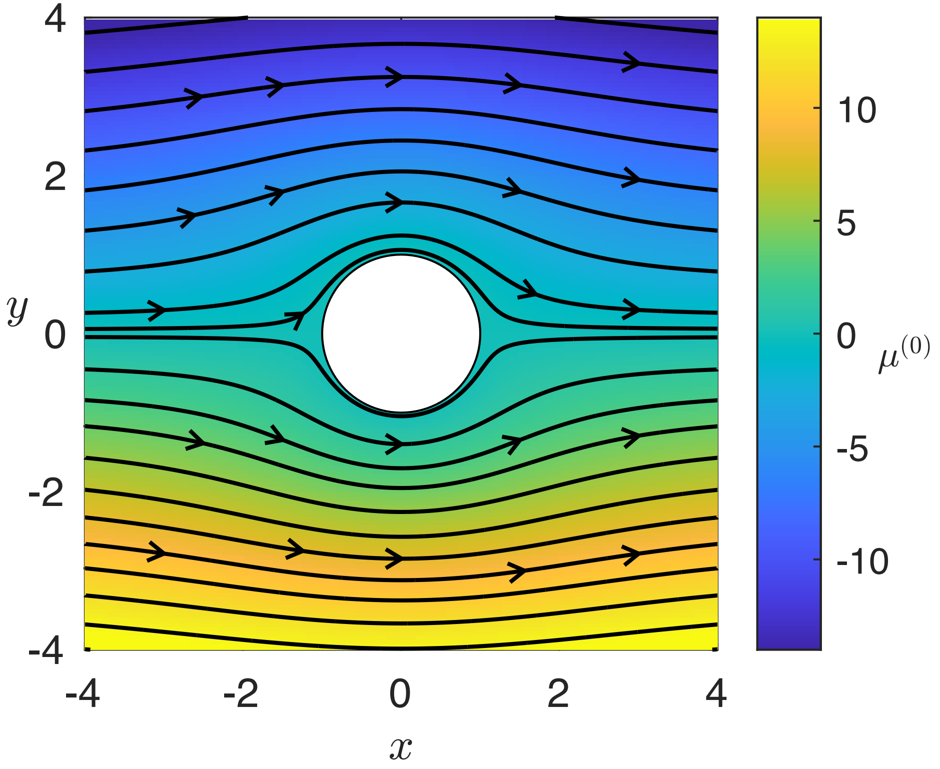


Gusev *et al.* Sci. Rep. (2020)



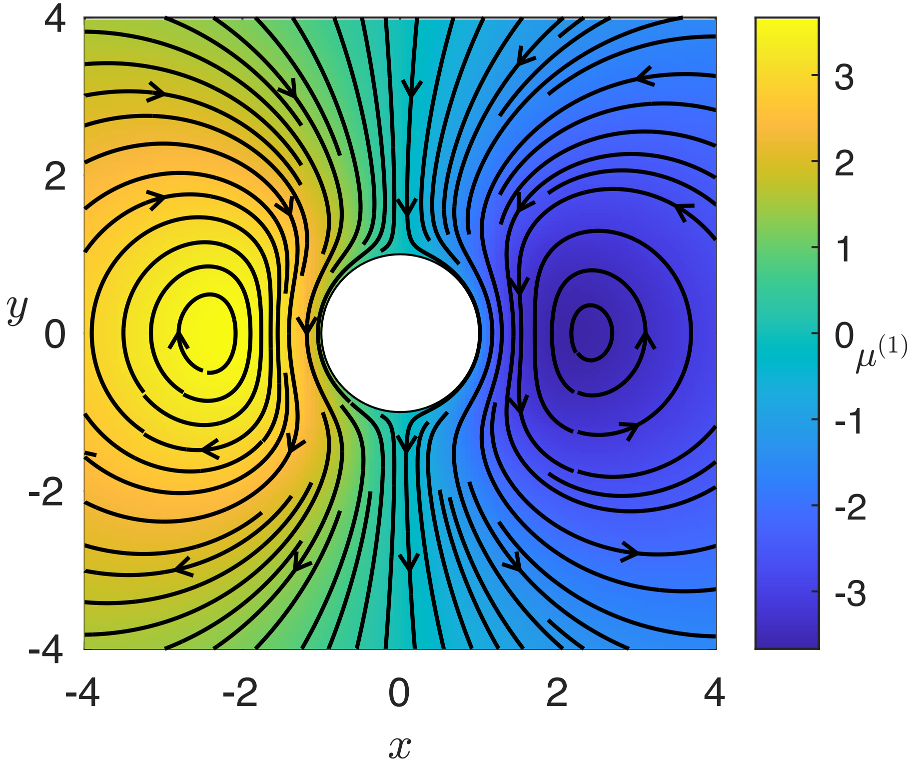
Flow around a circle – asymptotic solution

$O(1)$



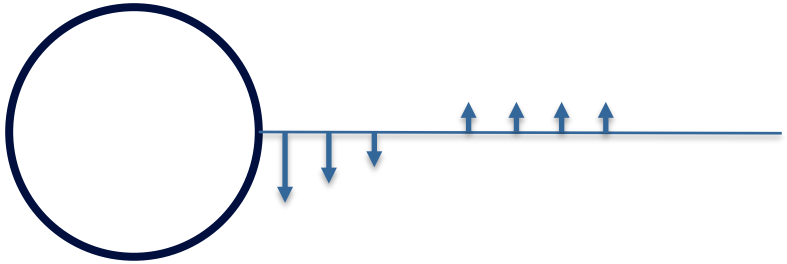
$$\hat{n} \cdot S^{(0)} \cdot \hat{t} = 0$$

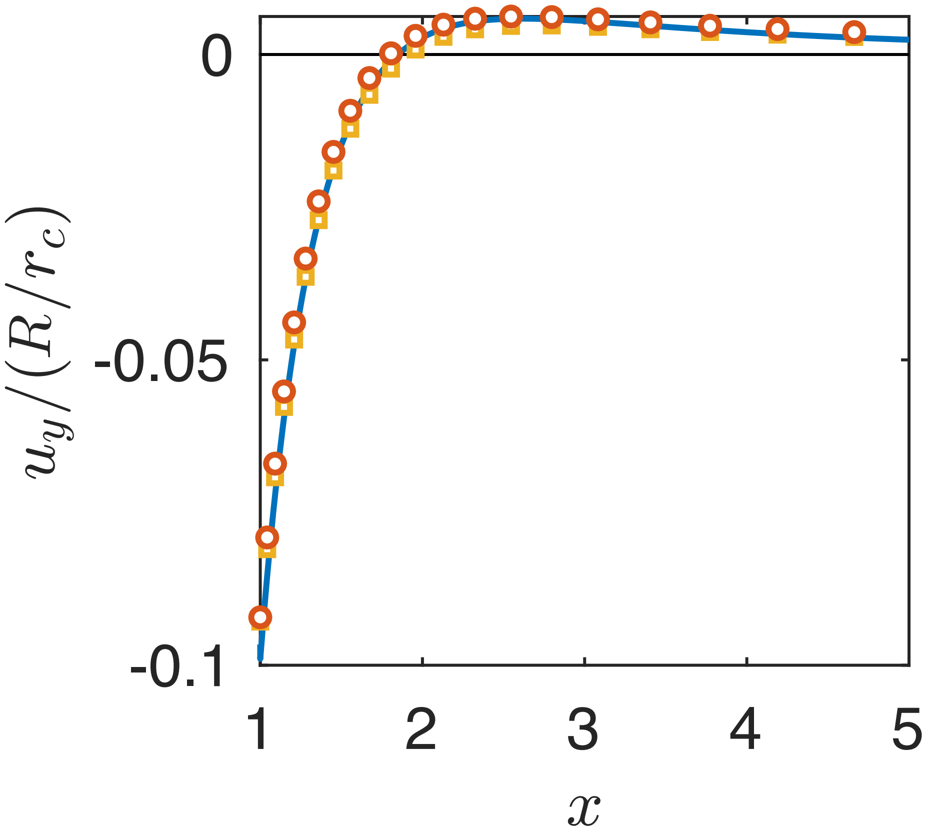
$O(k)$

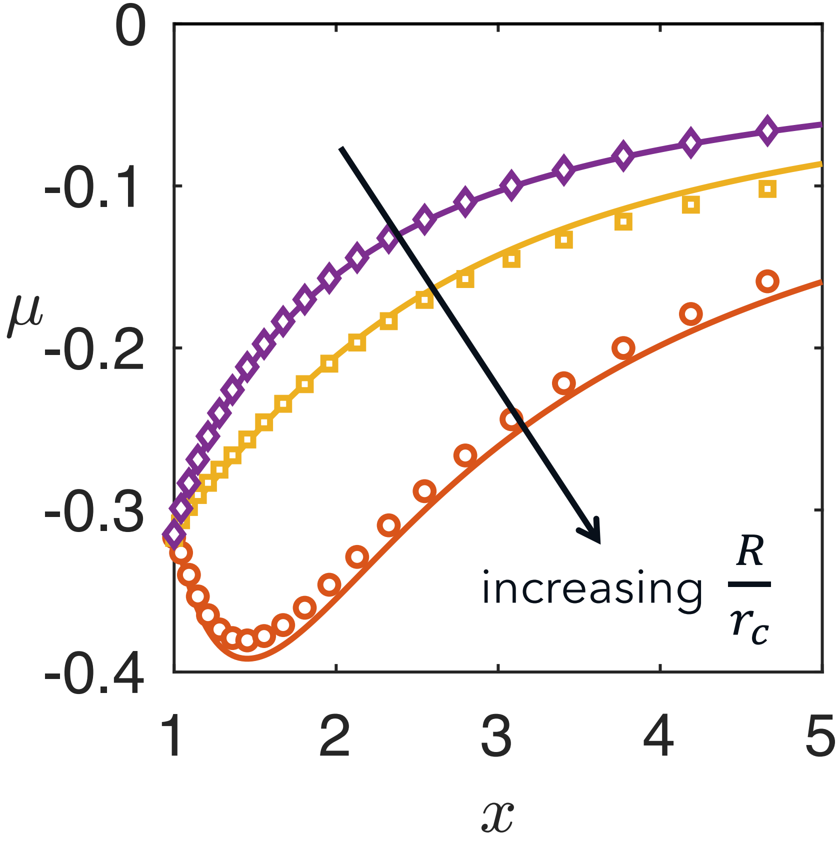


$$\hat{n} \cdot S^{(1)} \cdot \hat{t} = \frac{2R}{r_c}\, \hat{n} \cdot S^{(0)} \cdot \hat{n}$$

Numerical validation (k = 0.1)







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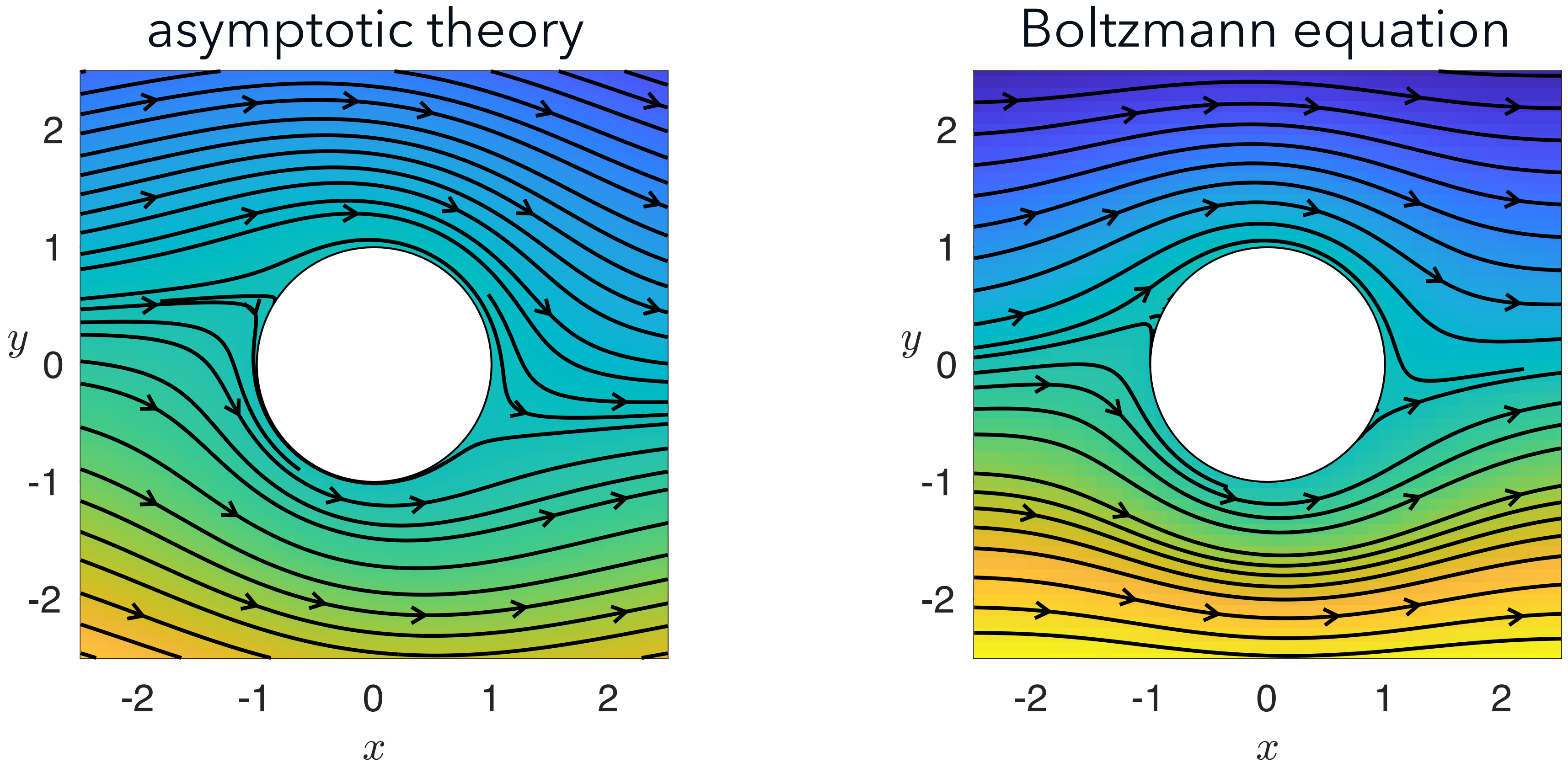

Flow rotation due to odd viscosity!

Collaborators & funding

Caltech

Nitay Ben-Shachar
Alex Nunn

University of Melbourne

Jesse Collis
Debadi Chakraborty
Douglas Brumley
Joseph Johnson
Mahyar Madadi
Jason Nassios (VU)

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References

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Electron hydrodynamics

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