Gyroviscosity modifies the laminar-turbulent transition in pipe flow

CHIRAL PARTICLES AND ODD FLUIDS: 12-JANUARY, 2026

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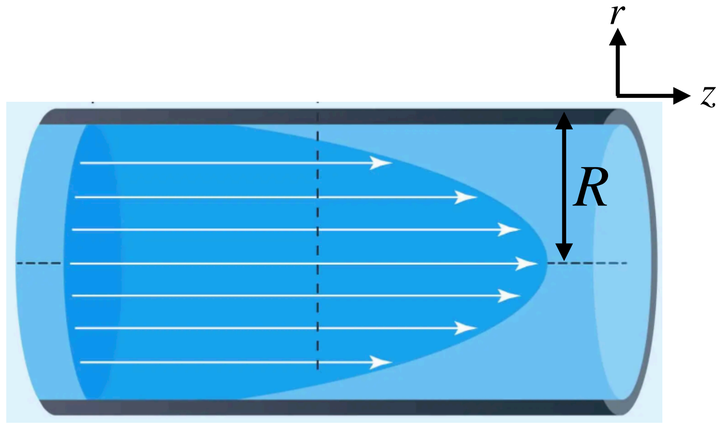
Laminar flow profile: transition to turbulence

- Poiseuille flow

$$\bar{u}_z = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$



- Classical result: Poiseuille flow is linearly stable for all Reynolds number

- But we observe turbulence in pipe at a critical Reynolds number.

Odd (Gyro) viscosity: transition to turbulence

How odd viscosity changes the nature of transition?

- Relevance to
  1. Active fluids
  2. Chiral fluids
  3. Magnetised plasma (Tokamaks)

Odd (Gyro) viscosity

- Odd viscosity: Antisymmetric part of viscous stress tensor

$$\partial_i u_j = \text{symmetric} + \text{antisymmetric}$$

- Appears when:
  1. Time-reversal symmetry is broken
  2. Parity is broken

- Does not dissipate energy

Governing equations

- The Navier-Stokes with odd viscosity

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \frac{\nu_o}{\nu} \hat{\mathbf{z}} \times \nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = 0; \;\; r = 1$$

- Where
  - Re : Reynolds number
  - $\frac{\nu_o}{\nu}$: Ratio of odd to normal viscosity

Base flow and perturbation

- Net velocity field $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$

- With base flow and perturbation

  1. $\bar{\mathbf{u}} = (0,0,1 - r^2)$
  2. $\mathbf{u}' = \tilde{\mathbf{u}}(r)e^{i(kz+n\theta)+\omega t}$

  - Where

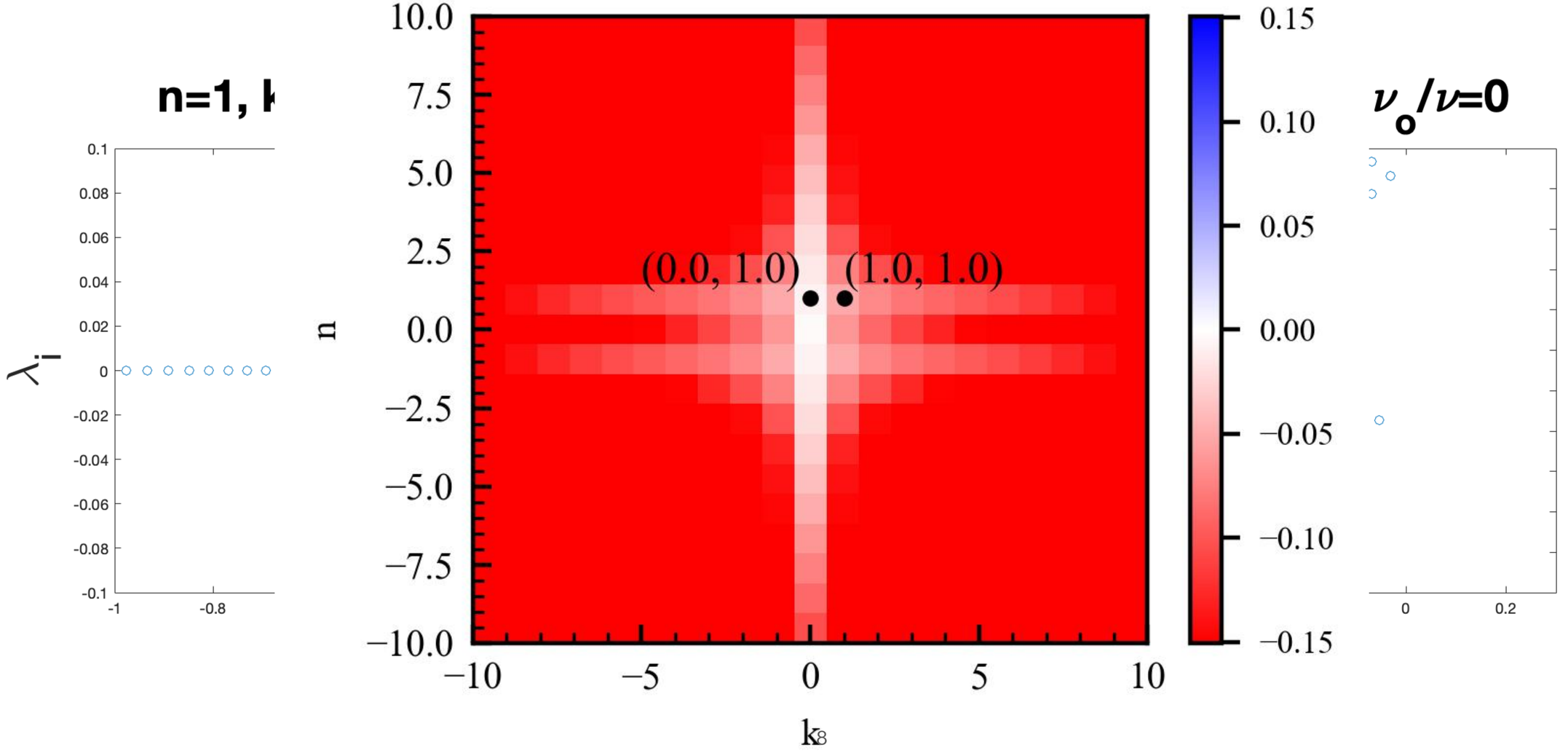
    1. k : axial wavenumber
    2. n : azimuthal wavenumber

Linear Stability Analysis (LSA)

- Linearise Navier-Stokes around the base flow
- Obtain coupled ODEs in r for $(\tilde{u}_r, \tilde{u}_\theta, \tilde{u}_z)$
- Spectral expansion using Chebyshev polynomial
- Solve eigenvalue problem:

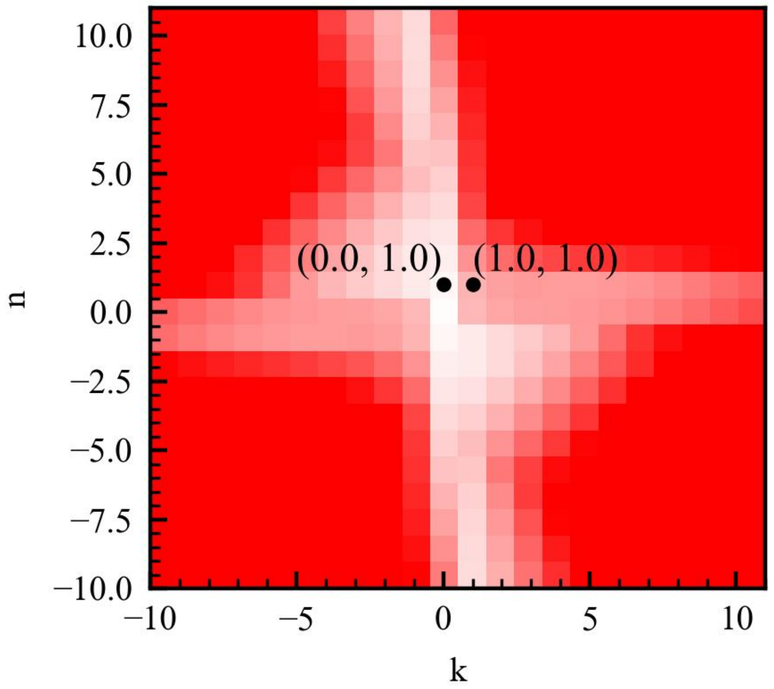
$$\omega(k, n; \nu_o/\nu, Re)$$

LSA: Classical result $\nu_o/\nu = 0$

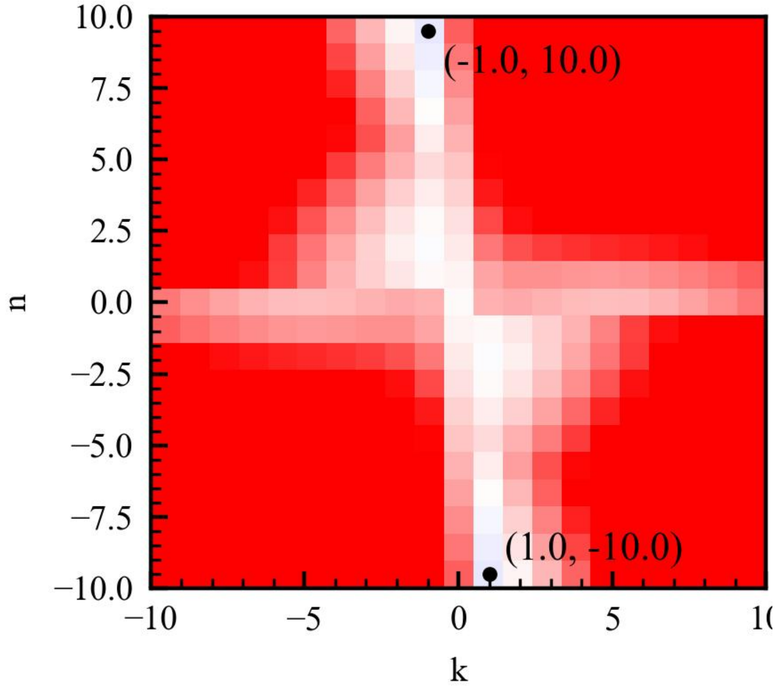


LSA: Increasing odd viscosity $Re = 2000$

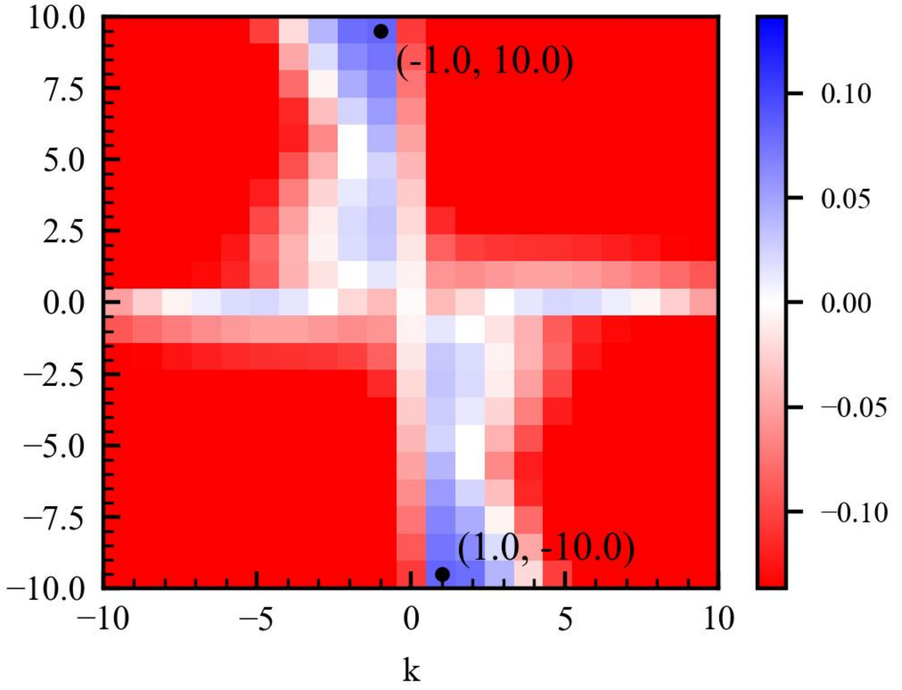
$\frac{\nu_o}{\nu} = 4$



$\frac{\nu_o}{\nu} = 6$

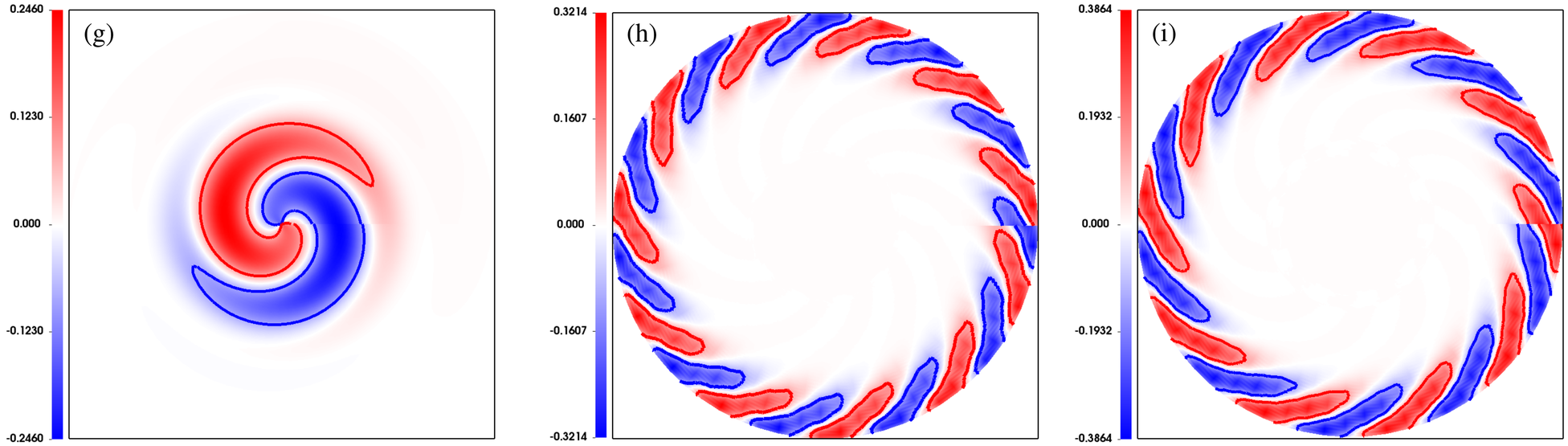


$\frac{\nu_o}{\nu} = 10$



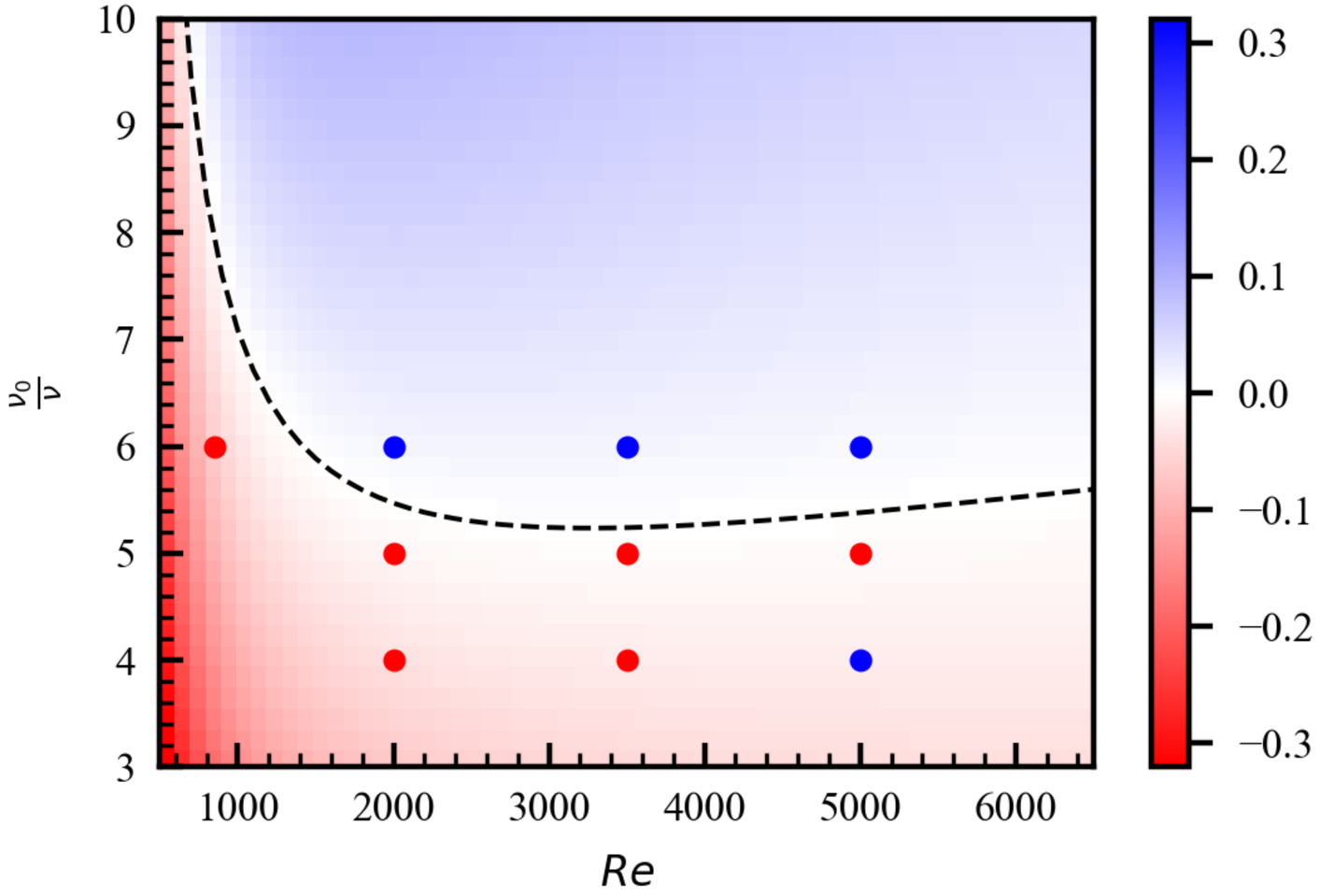
LSA: Eigenmodes $Re = 2000$

$\frac{\nu_o}{\nu} = 4$ $\frac{\nu_o}{\nu} = 6$ $\frac{\nu_o}{\nu} = 10$



- Unstable modes:
  1. Localised near pipe wall
  2. Strong azimuthal velocity component

LSA: Phase space



Direct Numerical Simulation (DNS)

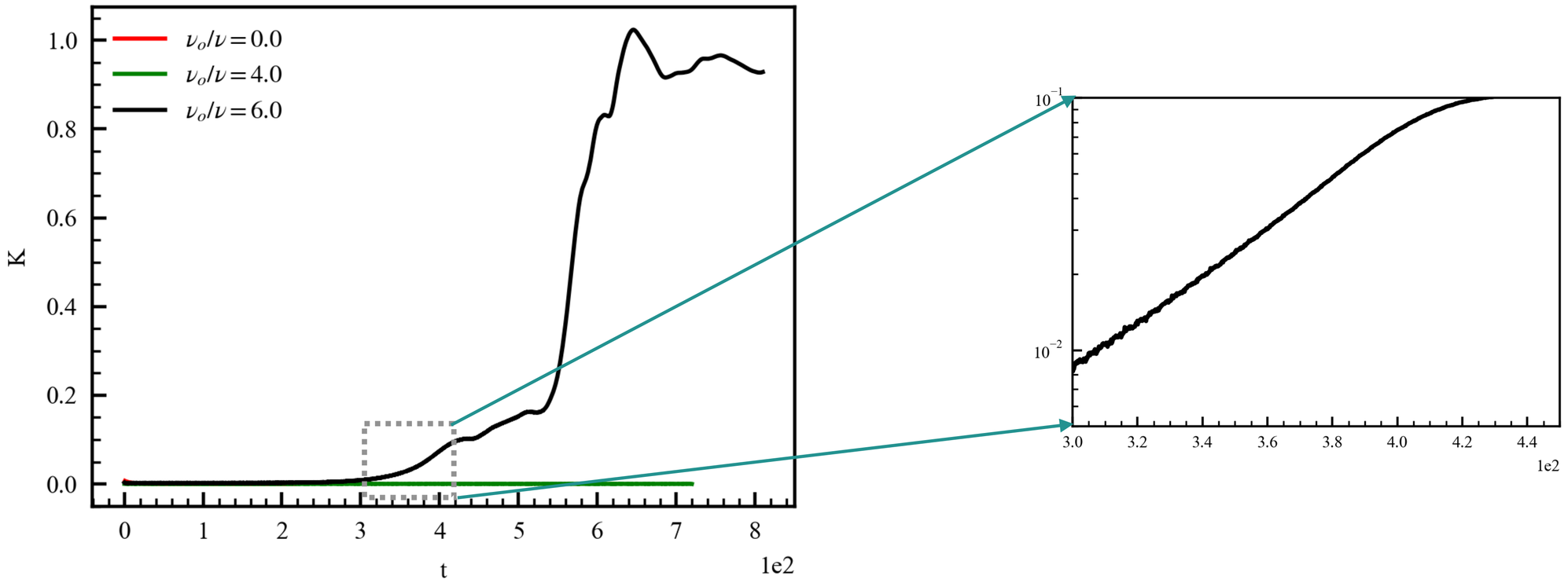
- Fourier decomposition in θ and z direction
- Finite difference scheme along radial r direction
- White noise as initial condition

nsCouette: https://github.com/dfeldmann/nsCouette

by Prof. Marc Avila et. al.

DNS: Kinetic energy growth

$Re = 2000$



- Exponential fit: $\lambda \sim 0.019$

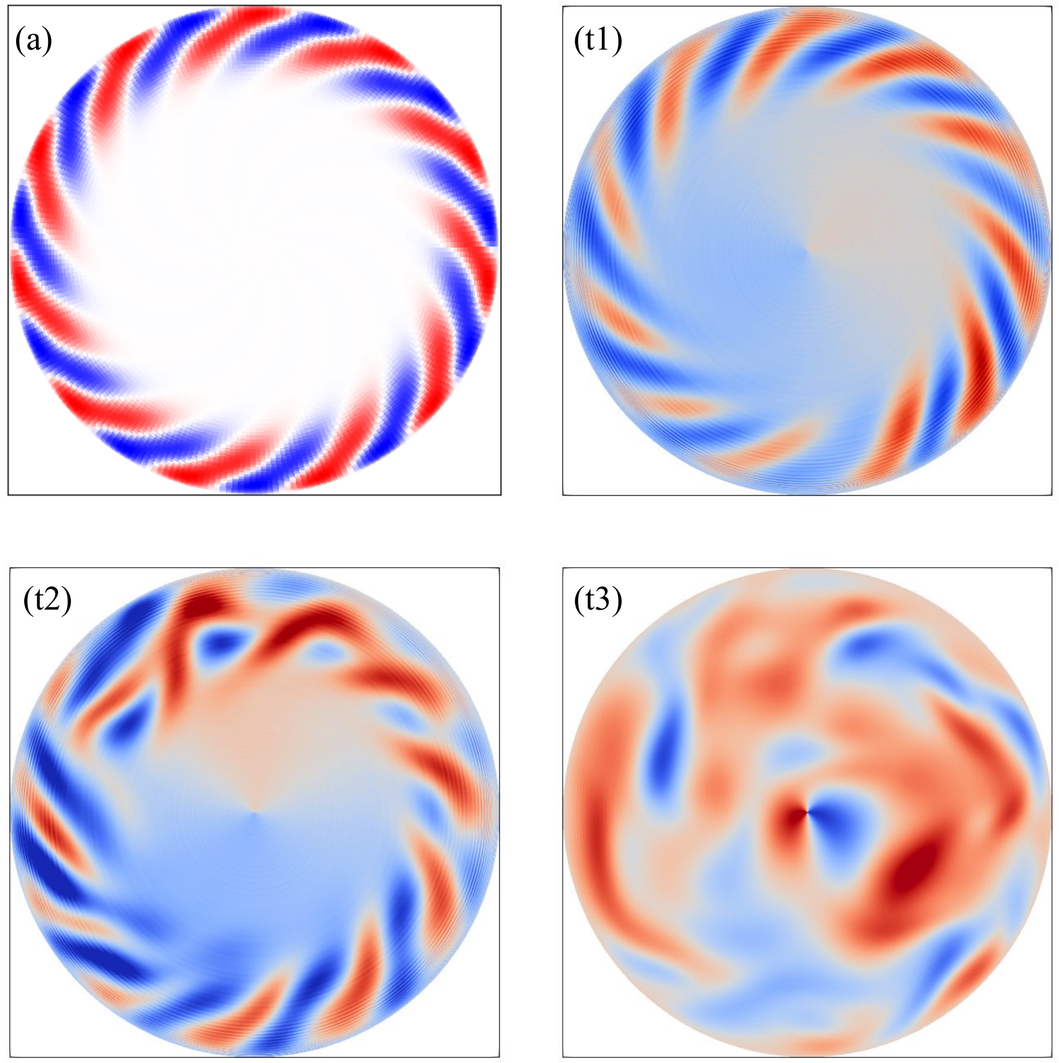
DNS: Flow structures in u_θ

$Re = 2000$

(a): least stable mode from LSA

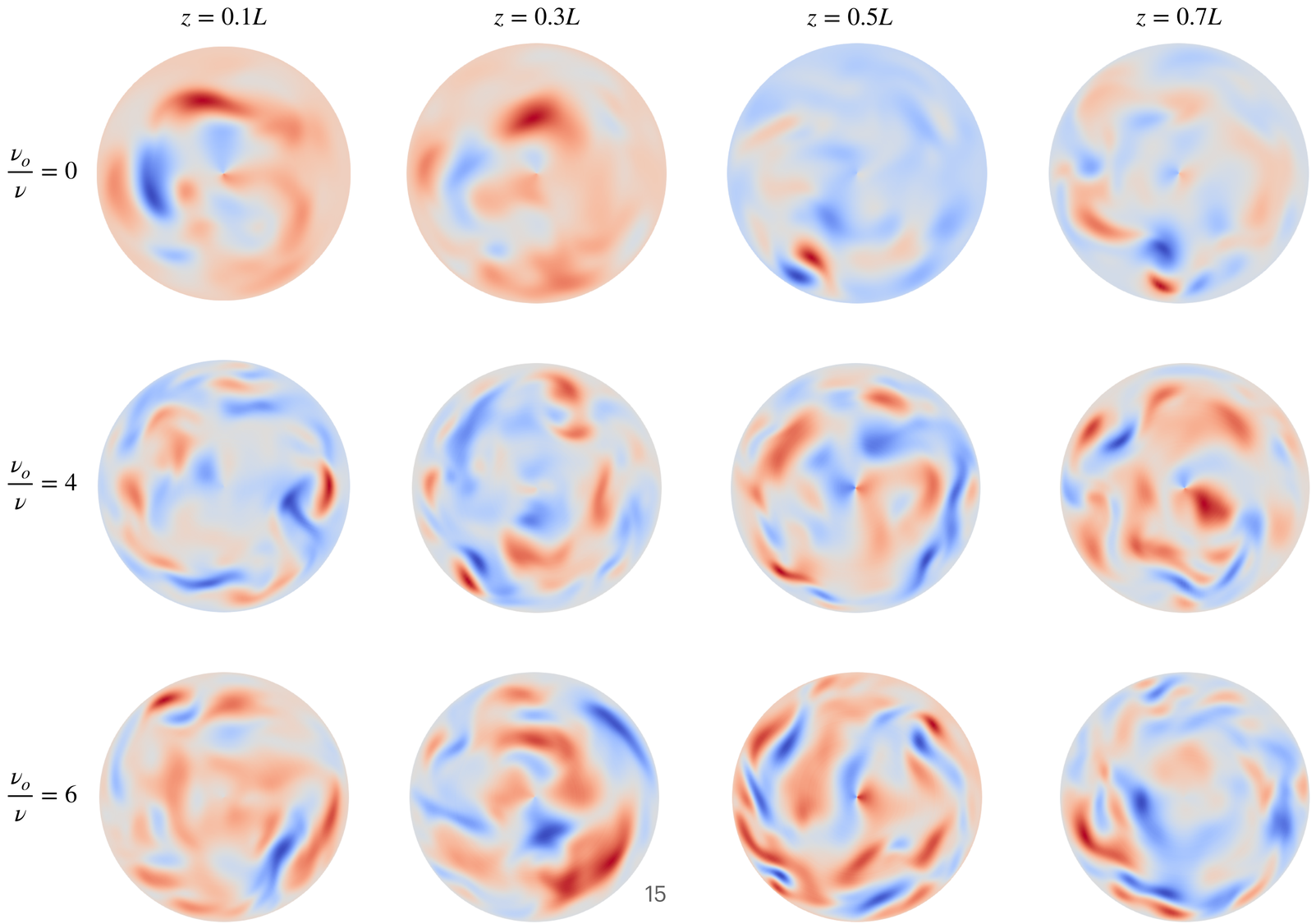
(t1): u_θ field from DNS during exponential growth

(t2,t3): u_θ field from DNS at later times



DNS: Flow structures in u_θ

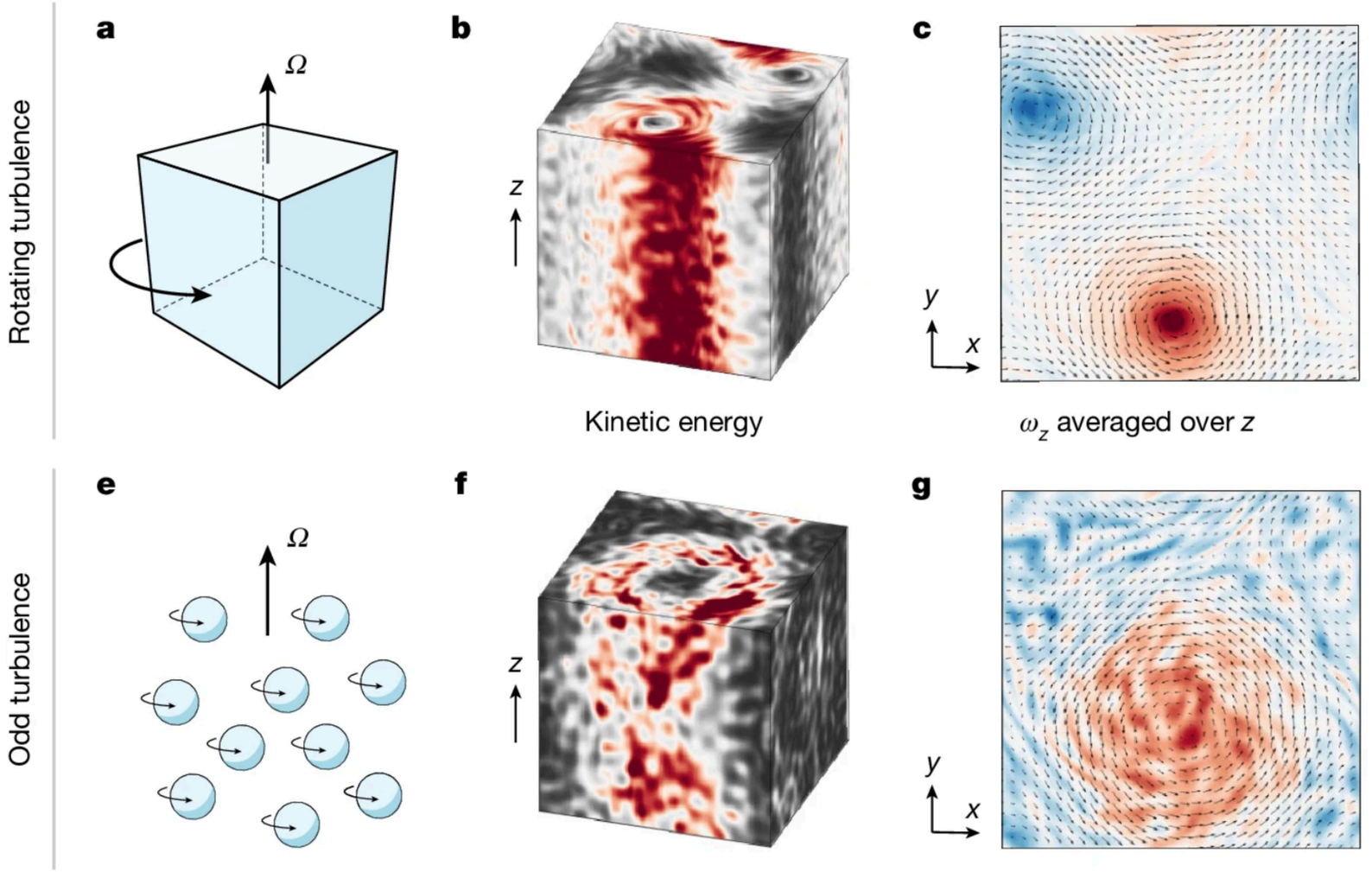
$Re = 5000$



Comparison with global rotation

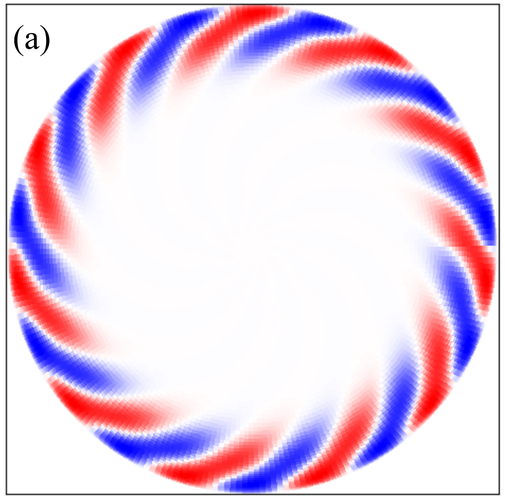
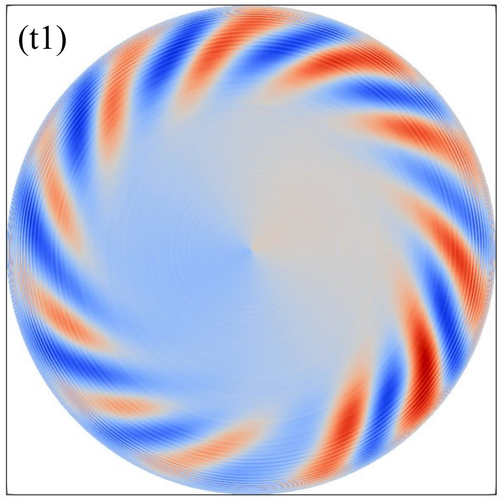
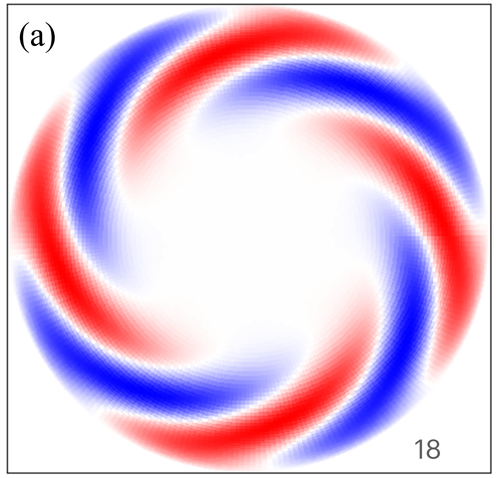
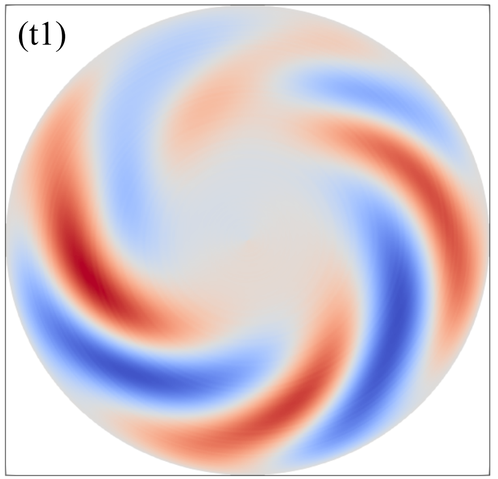
	Global rotation	Odd viscosity
Anisotropic at large scales	✓	✓
Isotropic at small scales	✓	✗

Comparison with global rotation



de Wit, X.M., et. al., *Nature* **627**, 515–521 (2024)

Comparison with global rotation, Re=2000

	LSA	DNS
Odd viscosity		
Global rotation		

Comparison with global rotation, Re=2000

Global rotation

$$\frac{\Omega R^2}{\nu} = 400$$

Odd viscosity

$$\frac{\nu_o}{\nu} = 6$$

- A relatively small odd viscosity leads to linear instability!

Comparison with global rotation

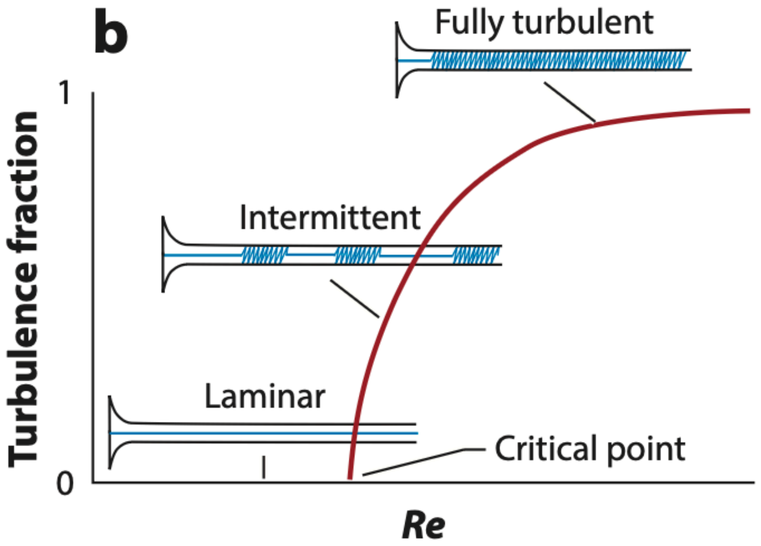
Global rotation

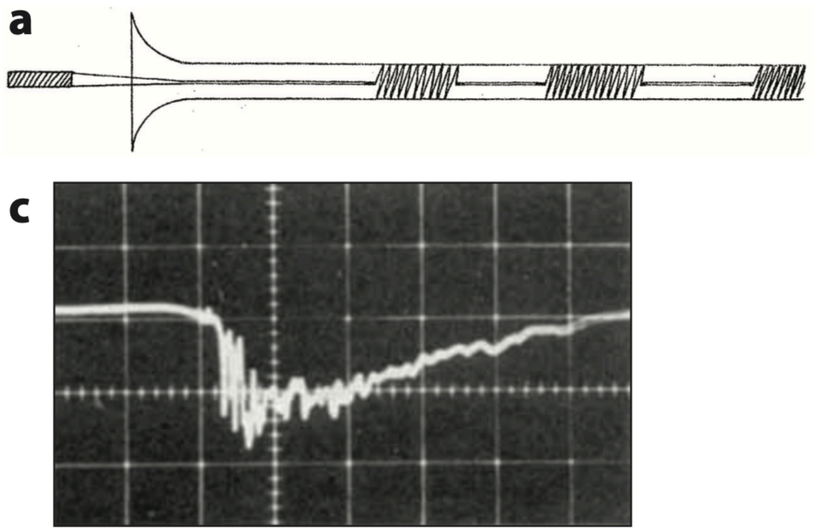
$$\frac{\Omega R^2}{\nu} \sim n$$

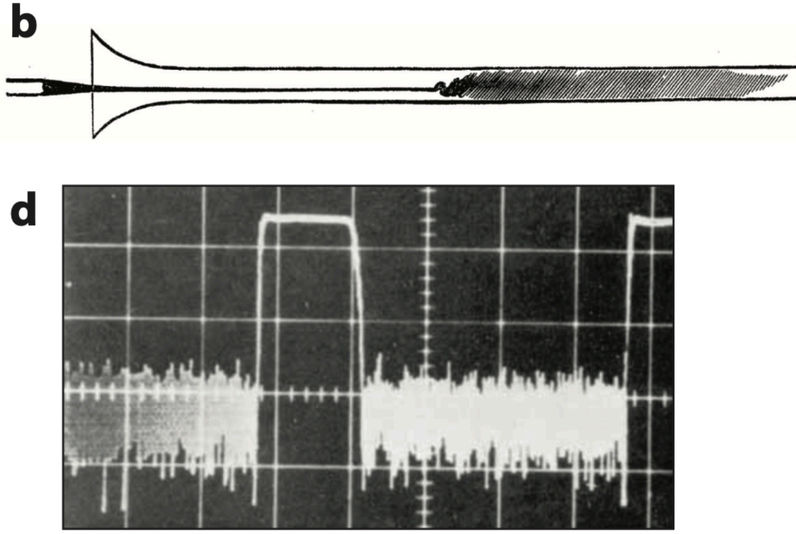
Odd viscosity

$$\frac{\nu_o}{\nu} \sim \frac{n}{Re}$$

Transition in pipe: Puffs and slugs



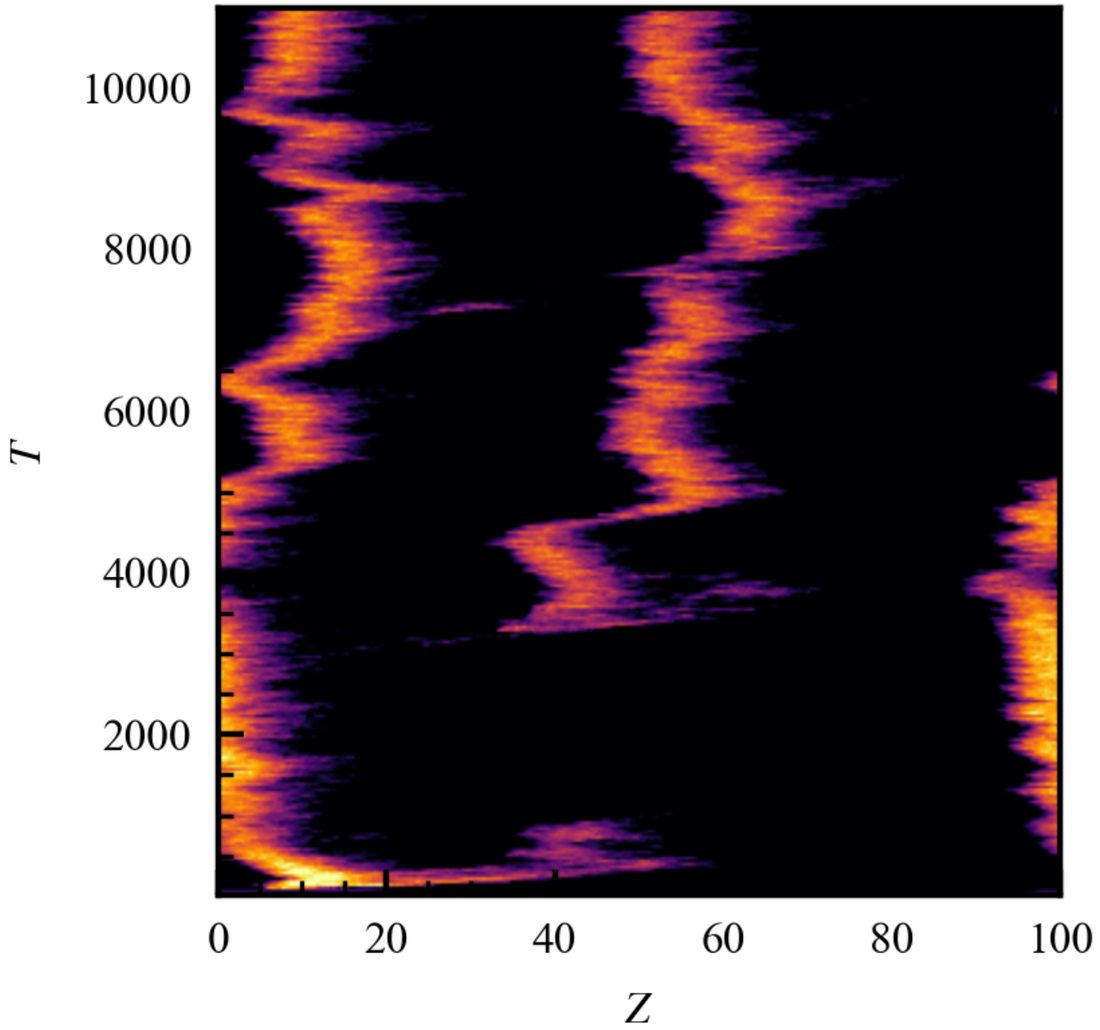




M. Avila, et. al., Annual Review of Fluid Mechanics (2022)

Transition in pipe: Puffs and slugs

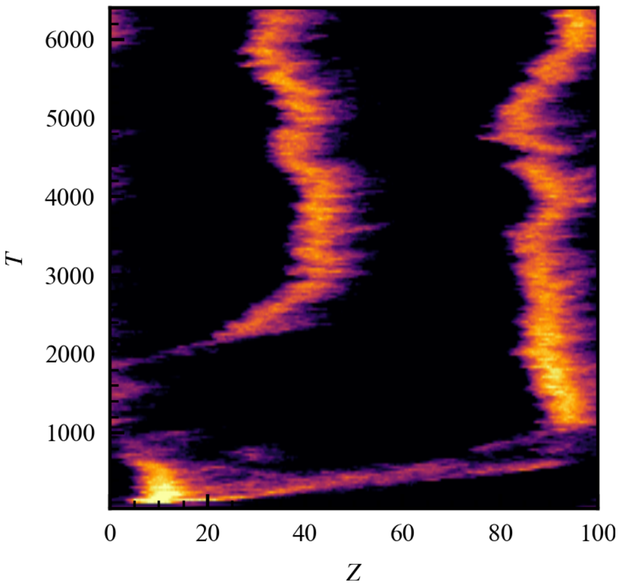
$Re = 3000; \qquad \frac{\nu_o}{\nu} = 0$



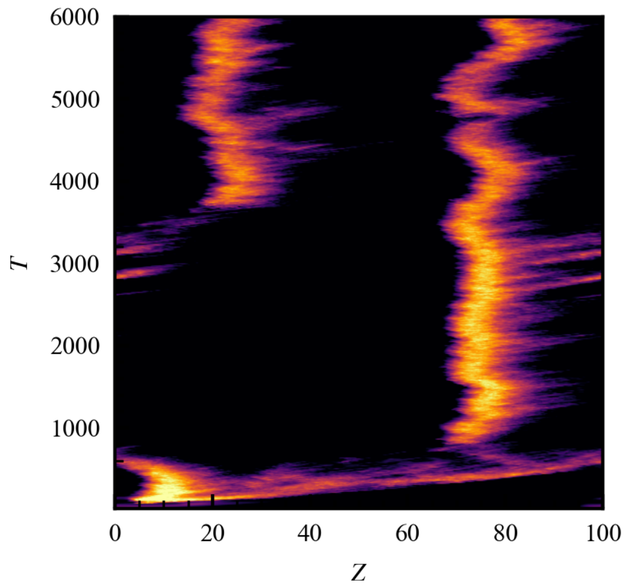
Streamwise vorticity in the co-moving frame

Transition in pipe: Puffs and slugs, $Re = 3000$

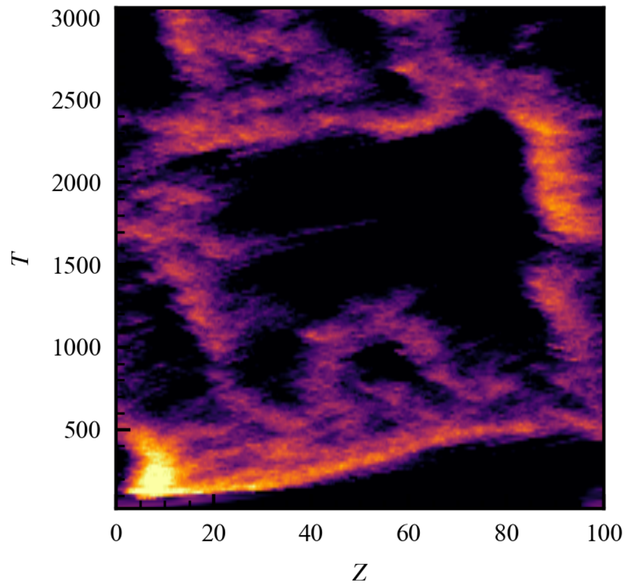
$\frac{\nu_o}{\nu} = 1$



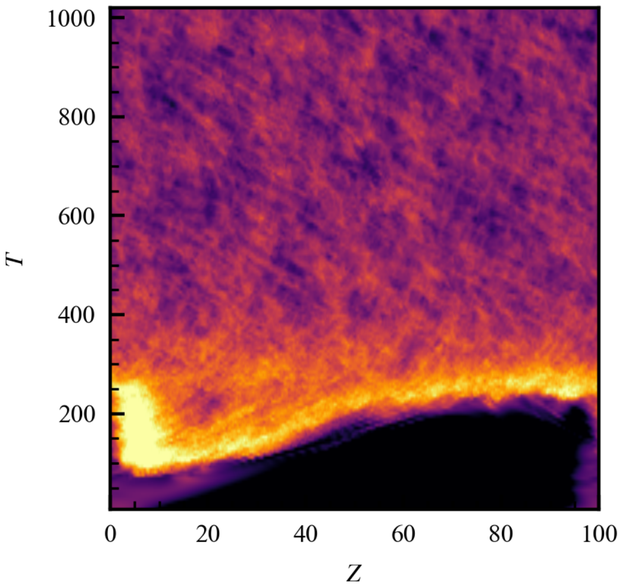
$\frac{\nu_o}{\nu} = 2$



$\frac{\nu_o}{\nu} = 4$

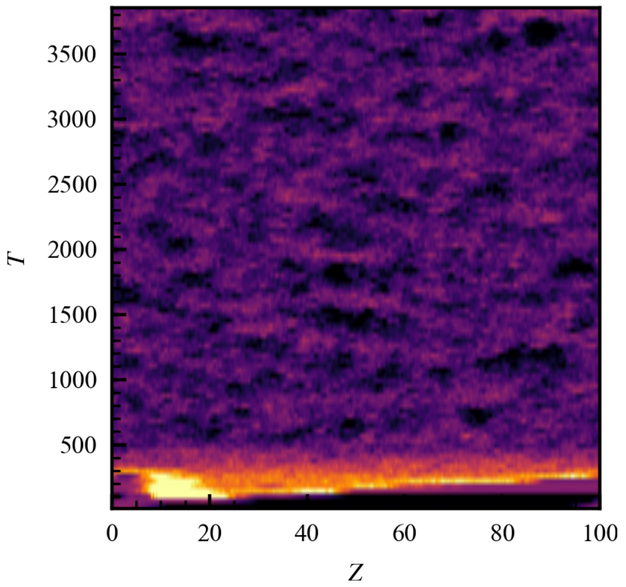


$\frac{\nu_o}{\nu} = 6$

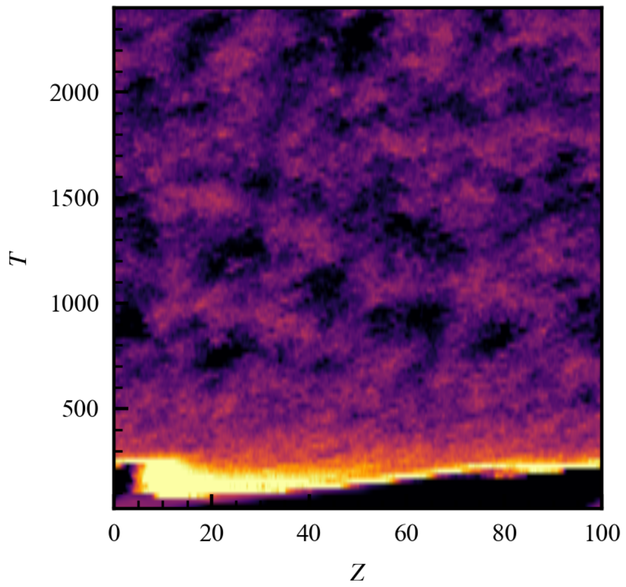


Transition in pipe: Puffs and slugs, $Re = 5000$

$\frac{\nu_o}{\nu} = 0$



$\frac{\nu_o}{\nu} = 2$



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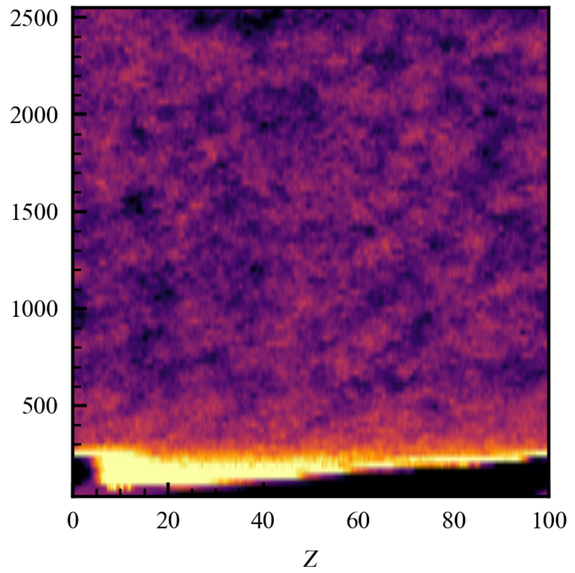

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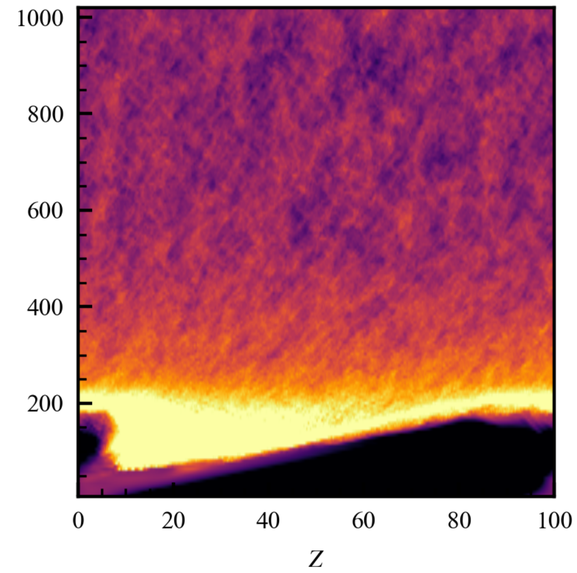

Summary and Outlook

- Odd viscosity provides a linear route to turbulence
- Instability dominates by the wall modes
- A good agreement between DNS and linear stability theory
- A small value of odd viscosity leads to instabilities compared to the case of global rotation

Prof. Federico Toschi

Prof. Vincenzo Vitelli

Dr. Michel Fruchart

Xander M. de Wit