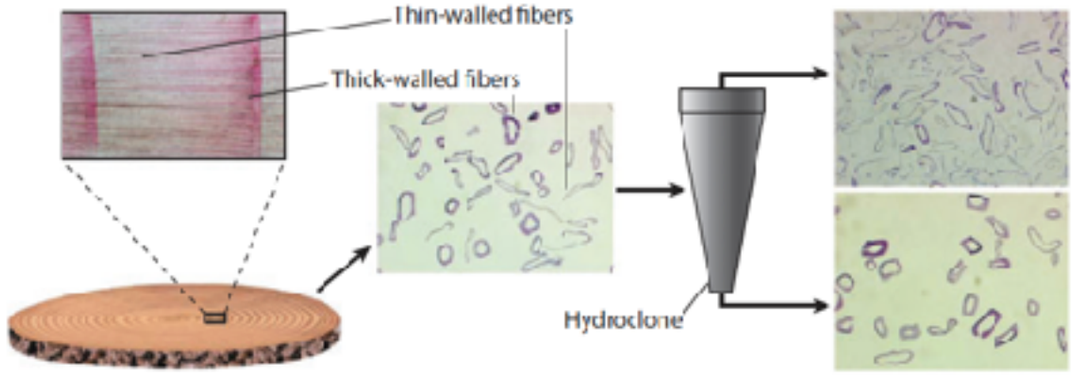
Dynamics of flexible fibers in turbulent flows

Hugo Poncelet, Gautier Verhille

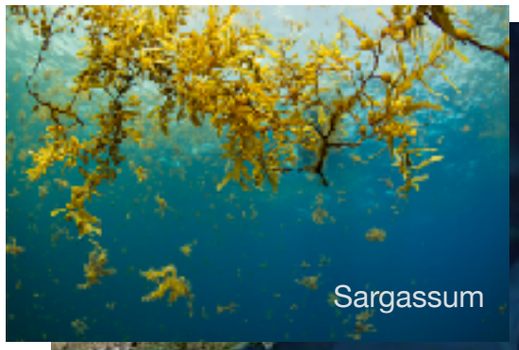
Motivation



Lundell *et al.*, *Ann. Rev. Fluid Mech.*, 2011

Paper industry

Volcanic eruption



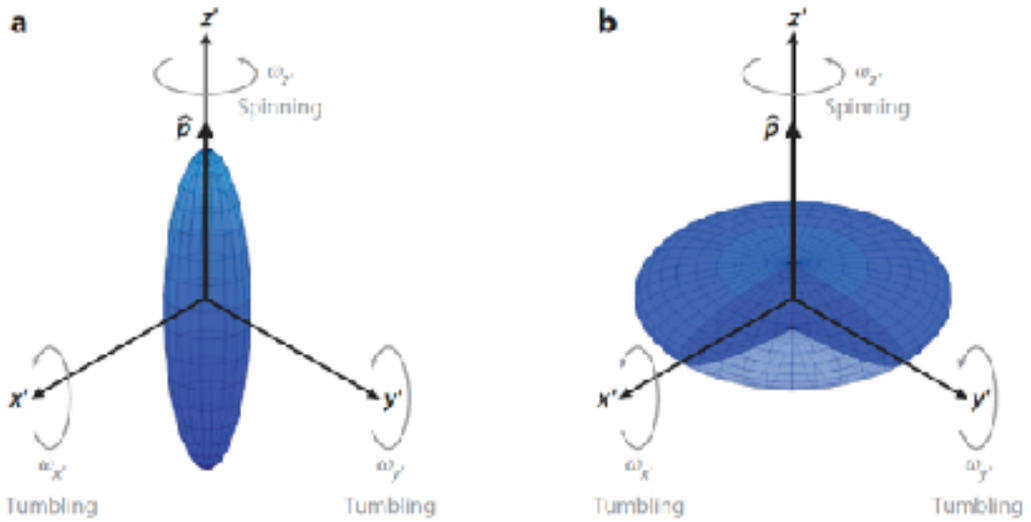
Plankton/algae

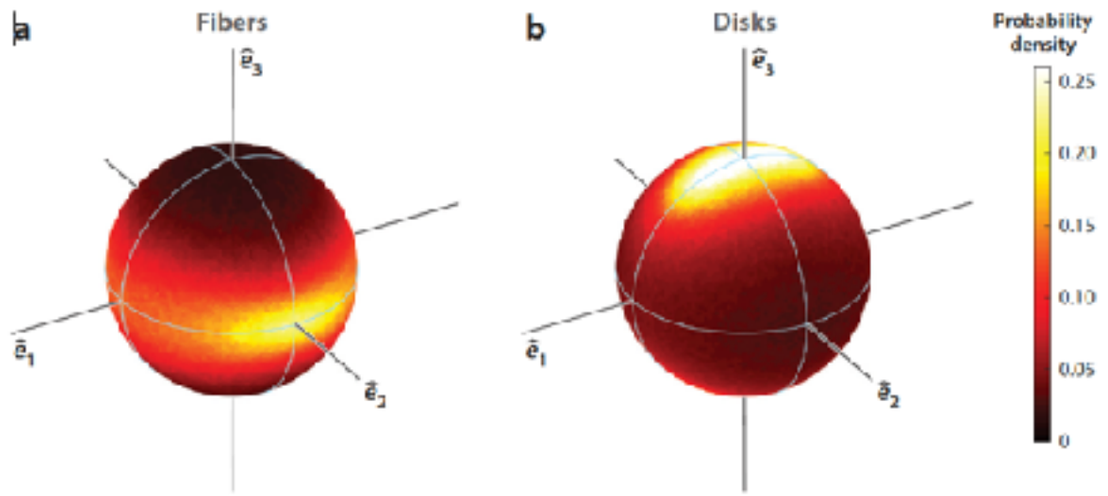
Plastic pollution

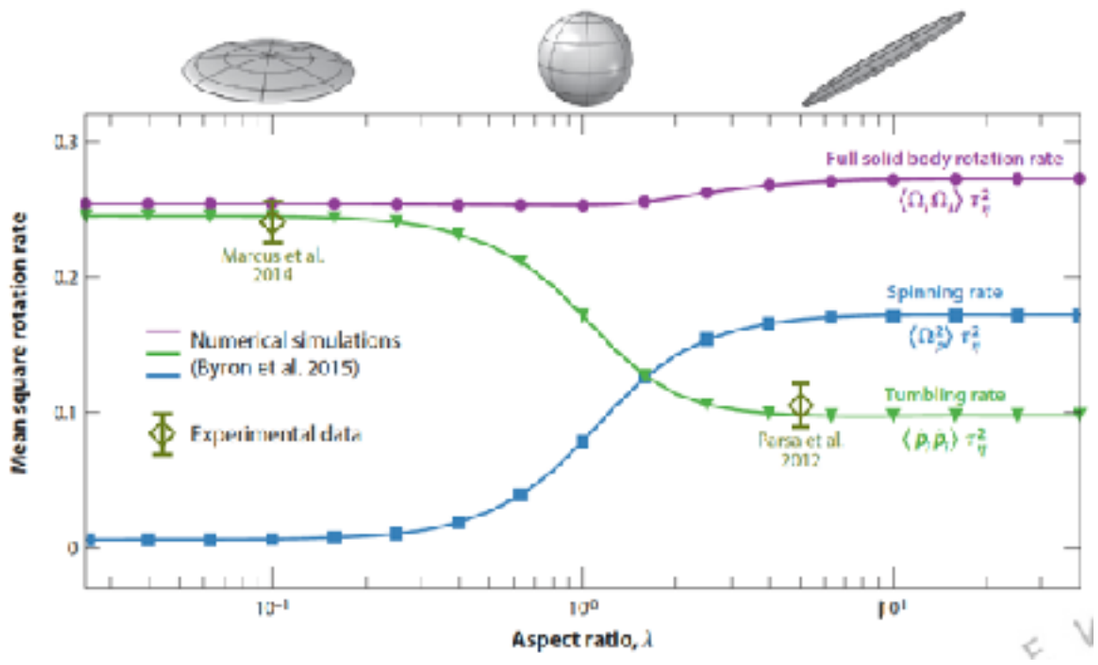
Small inertialess particles in turbulence

Jeffery torque => $\dot{p} = \mathbb{O} \cdot p + \Lambda(\mathbb{S} \cdot p - p(p \cdot \mathbb{S} \cdot p))$

vorticity strain



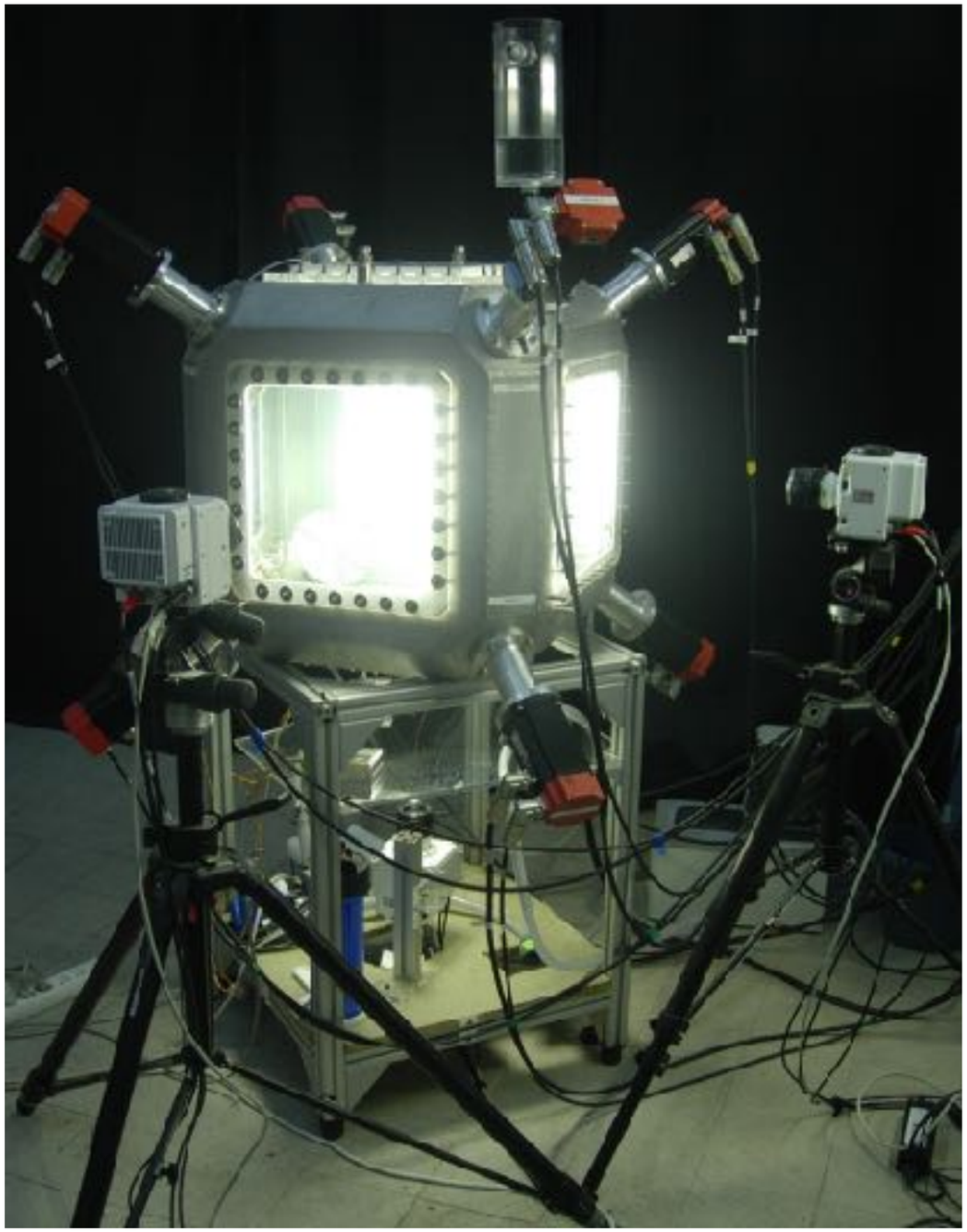




Preferential orientation: major axis tends to align with the local vorticity

Voth and Soldati, Annu. Rev. Fluid, 2017

Experimental setup



Turbulence properties

Water or Water+Ucon mixture

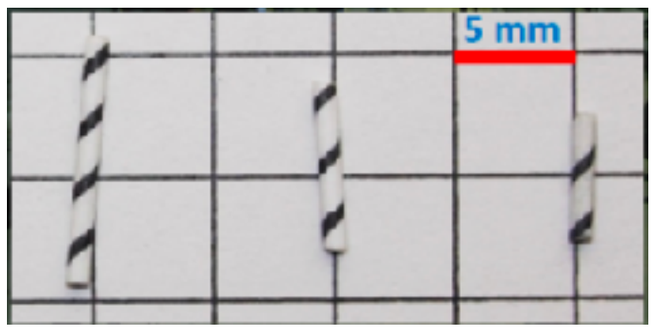
$R_\lambda \in [90; 630]$ $\quad$ $\lambda \in [1.7; 9.7]$ mm

$\eta_K \in [34; 434]$ μm $\quad$ $\tau_K \in [1.2; 17.1]$ ms

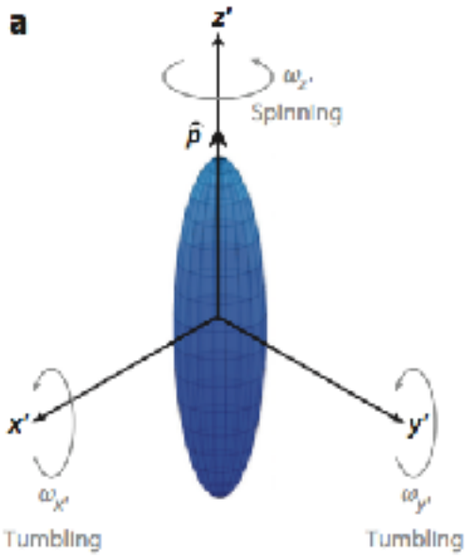
Fiber properties

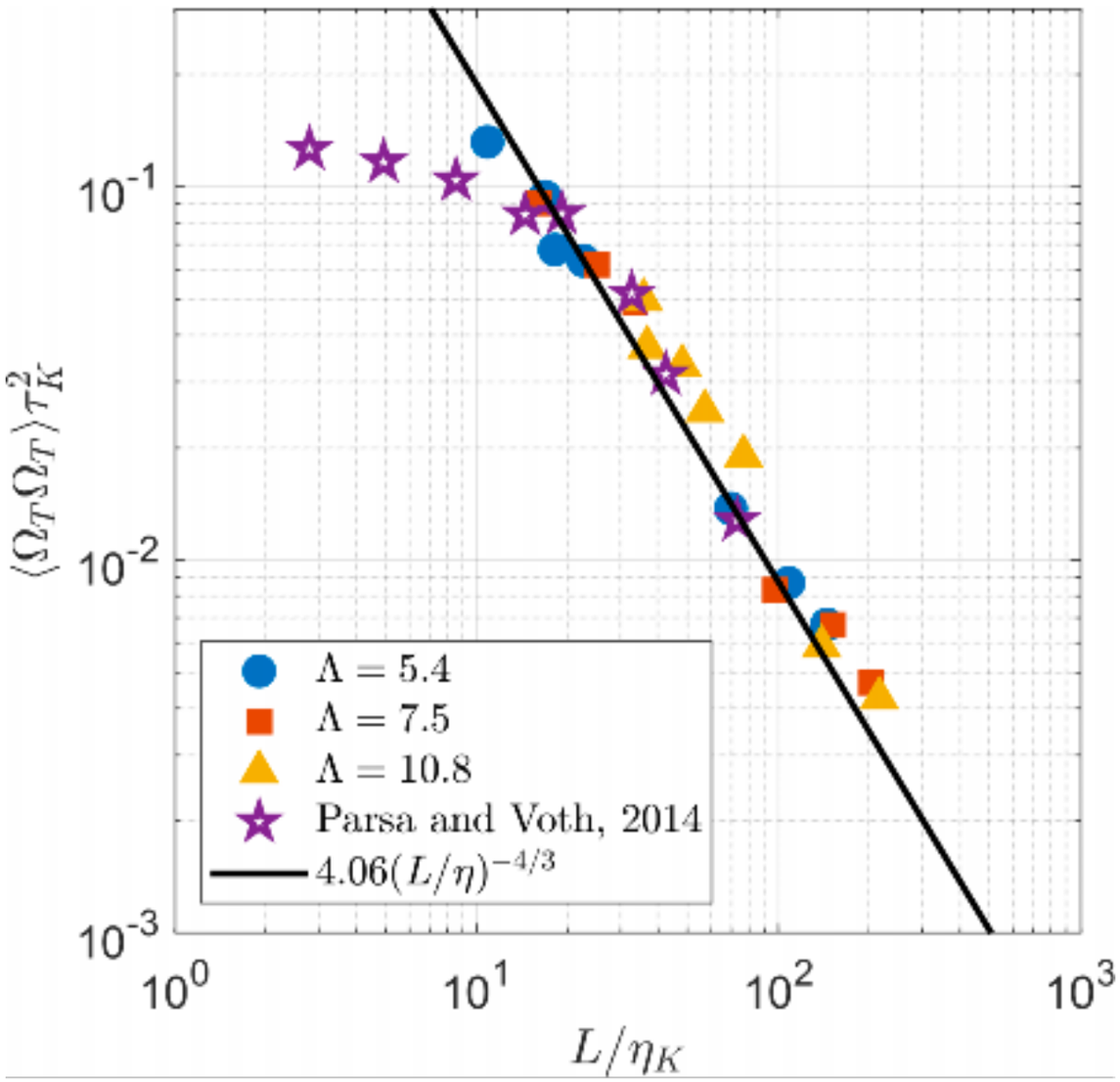
$L = 5; 7; 10$ mm $\quad$ $d = 0.93$ mm

$\rho = 1.04\ 10^3$ kg.m^{-3}



Long rigid fibers: Tumbling rate

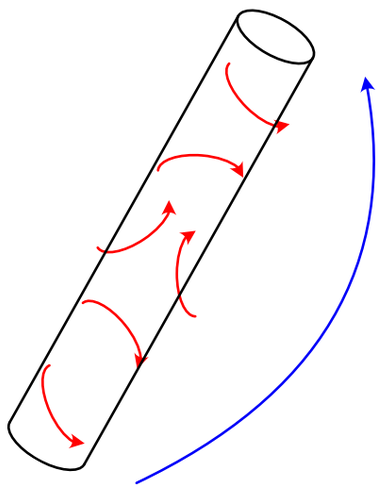




$$\Omega_T \sim \frac{u_L}{L} \sim \epsilon^{1/3} L^{-2/3} \sim \tau_K^{-1} \left(\frac{L}{\eta_K} \right)^{-2/3}$$

Long rigid fibers: Spinning rate

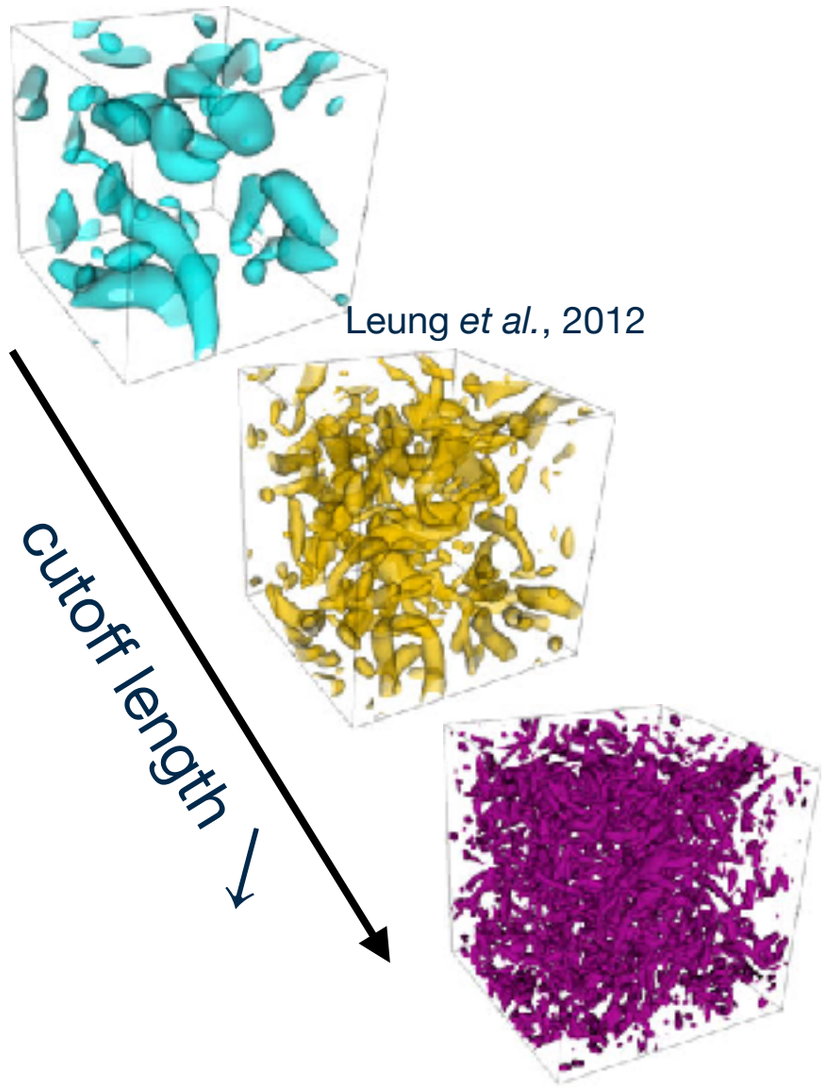
No spinning



scale separation

no coherent forcing

Spinning due to
large scale vorticity

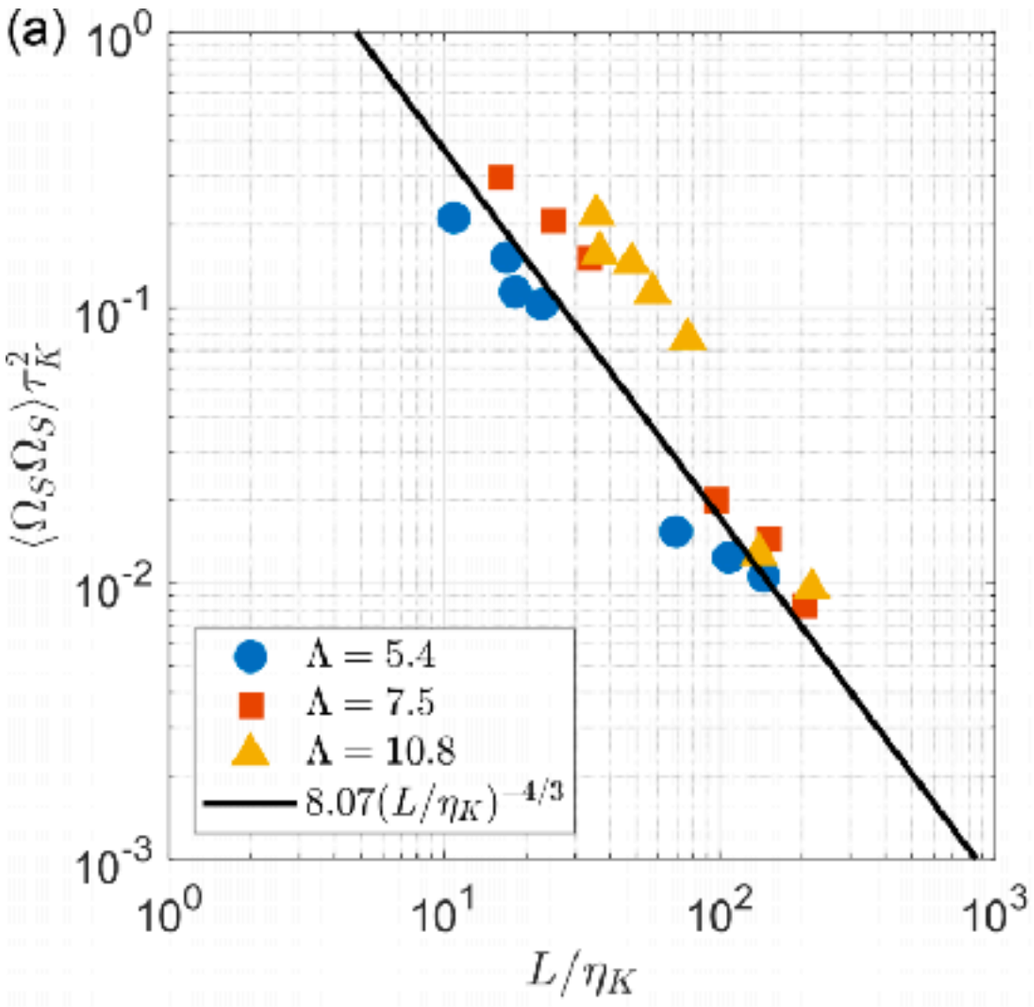


Leung *et al.*, 2012

Other?

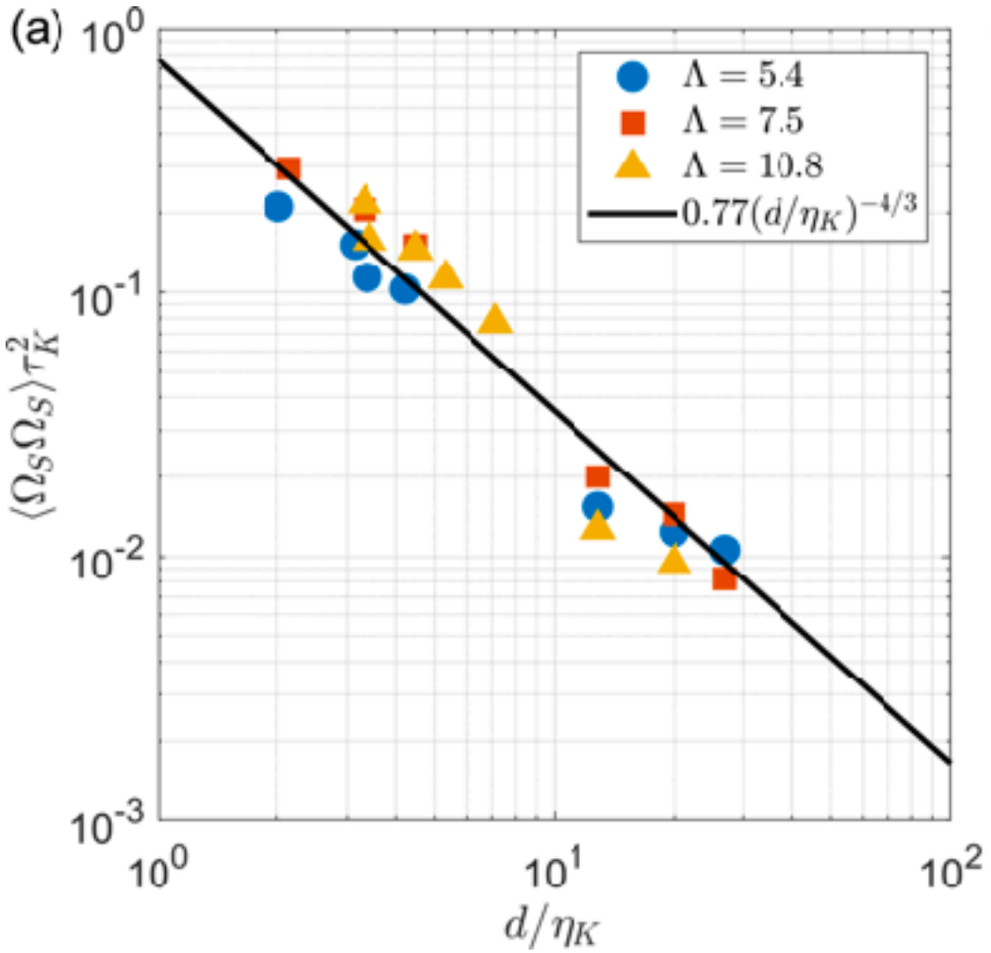
spinning given by fiber length

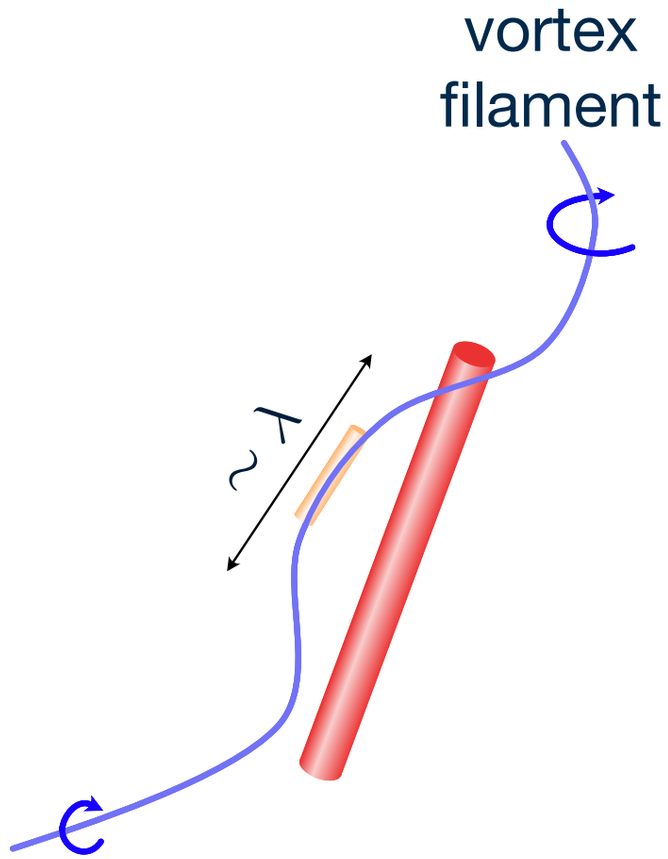
Spinning rate



Spinning rate higher than tumbling rate
Length is not the control parameter

Modelling the spinning rate



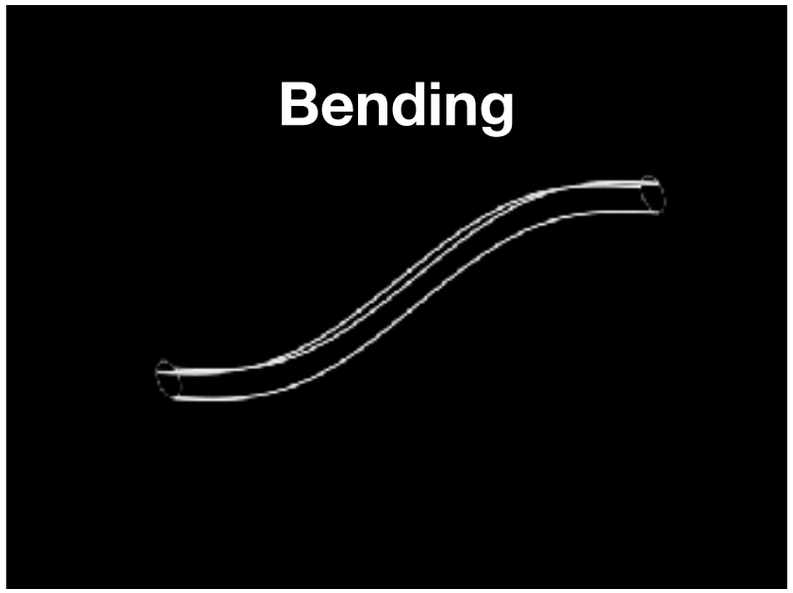


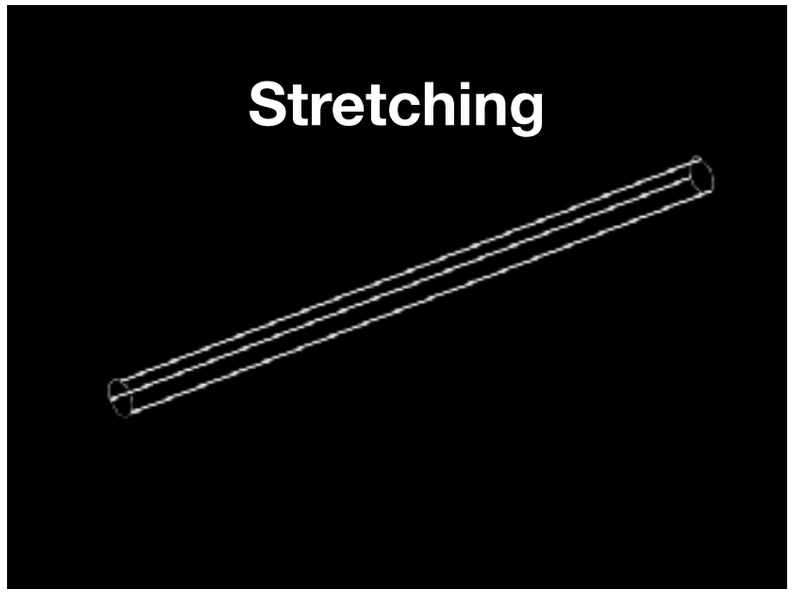
$$\frac{\langle \Omega_S \Omega_S \rangle}{\langle \Omega_T \Omega_T \rangle} \sim \left(\frac{L}{d}\right)^{4/3} f\left(\frac{L}{\lambda}\right)$$

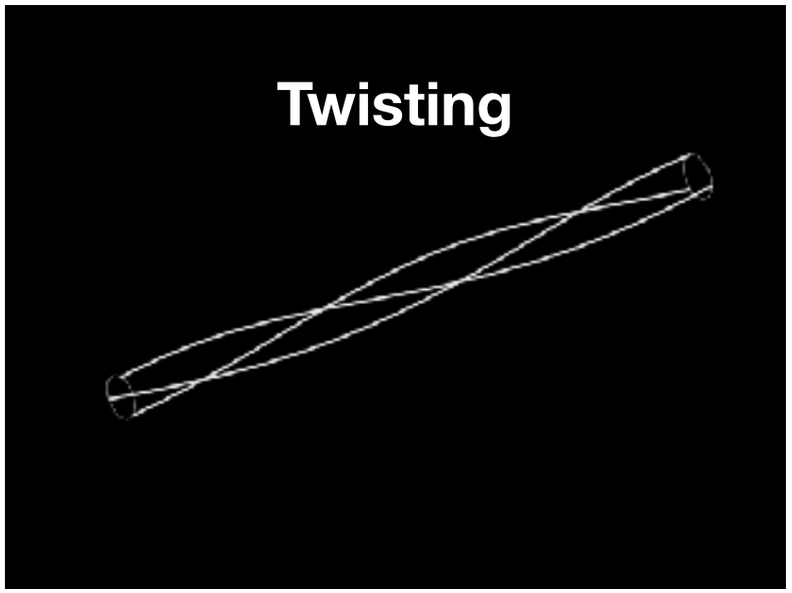
What about flexible fibers

Timescale of the deformation

Timescale of the deformation







$$EI\partial_s^4 \boldsymbol{r} \sim \eta \partial_t \boldsymbol{r}$$

$$\tau_b \sim \frac{\eta L^4}{EI}$$

$$Ed^2 \partial_{ss} \boldsymbol{r} \sim \eta \partial_t \boldsymbol{r}$$

$$\tau_s \sim \frac{\eta L^2}{Ed^2}$$

$$C\partial_{ss}\theta \sim \eta d \partial_t d\theta$$

$$\tau_t \sim \frac{\eta d^2 L^2}{C}$$

$$C \sim EI$$

For fibers $L \gg d$ $\tau_s \sim \tau_t \ll \tau_b$

At first order fibers are inextensible and untwistable

Marchioli *et al.*, *Annu. Rev. Fluid Mech.*, 2025

The Euler Bernoulli equation: in practice

$$\sigma \partial_{tt} \boldsymbol{r} - \partial_s (T \partial_s \boldsymbol{r}) + EI \partial_s^4 \boldsymbol{r} = \boldsymbol{\xi} \quad , \quad |\partial_s \boldsymbol{r}|^2 = 1$$

Modeling the forcing term

Slender body theory

Small fibers $L \ll \eta_K$ No inertia

High aspect ratio (neglect flow disturbance) $\lambda = L/d \gg 1$

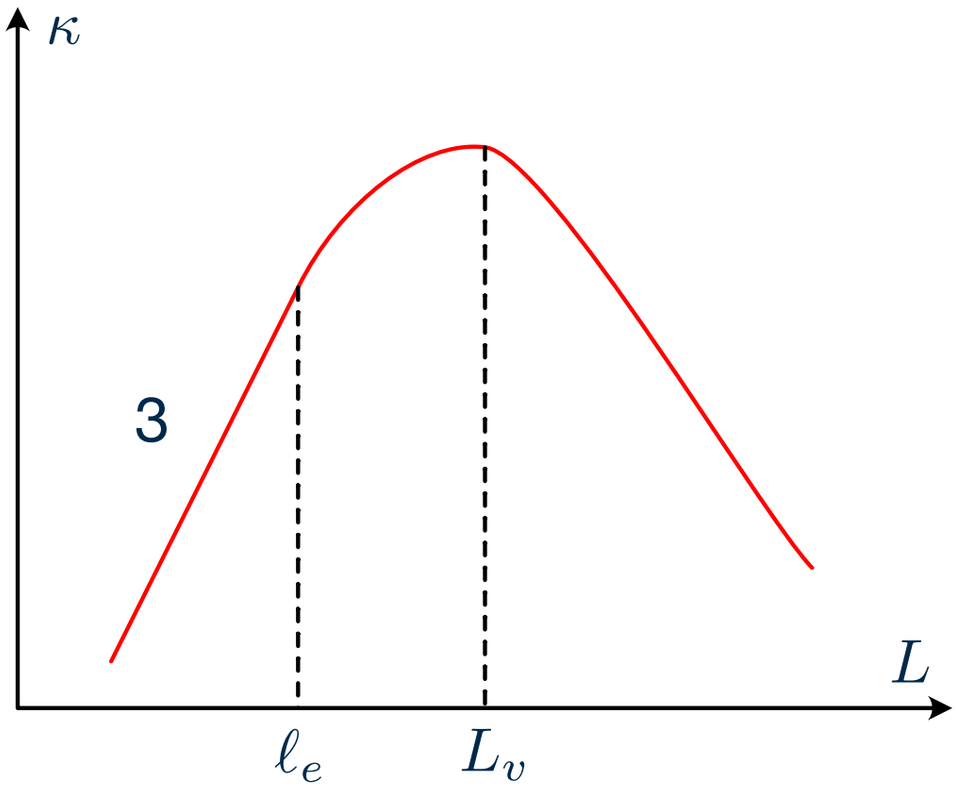
$$\boldsymbol{\xi} = \underline{\eta}(\boldsymbol{u} - \partial_t \boldsymbol{r})$$

Overdamped or underdamped regime?

$$\frac{|\sigma \partial_{tt} \boldsymbol{r}|}{|\underline{\eta} \partial_t \boldsymbol{r}|} \sim \frac{\rho}{\rho_f} \frac{\pi d^2}{\nu \tau}$$

Deformation of flexible fibers

$$\ell_e = \frac{(EI)^{1/4}}{(\rho_f \eta \epsilon)^{1/8}}$$

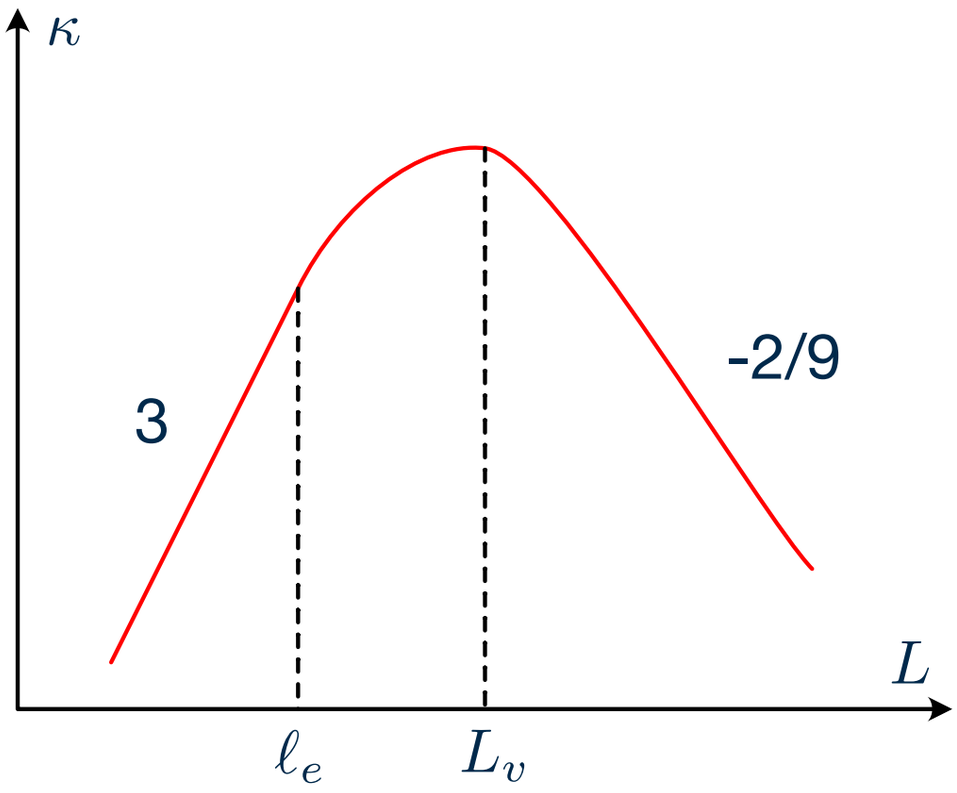


Power balance

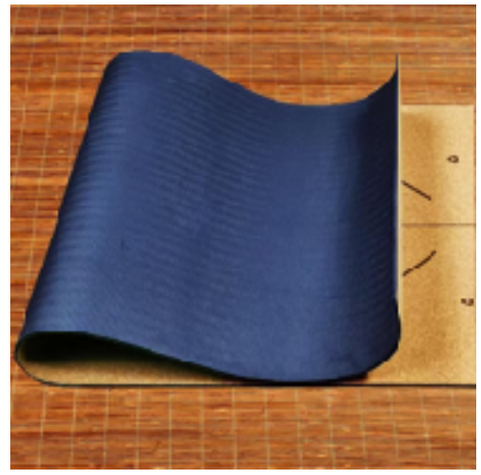
$$\rho_f L^3 \epsilon \sim \frac{LEI\kappa^2}{\tau_b} = \frac{(EI)^2\kappa^2}{\eta L^3} \Rightarrow \kappa \sim \frac{(\rho_f \eta \epsilon)^{1/2}}{EI} L^3$$

Deformation of flexible fibers

$$\ell_e = \frac{(EI)^{1/4}}{(\rho_f \eta \epsilon)^{1/8}}$$



$$L_v = \frac{(EI)^{3/10}}{\eta \epsilon^{1/3}}$$

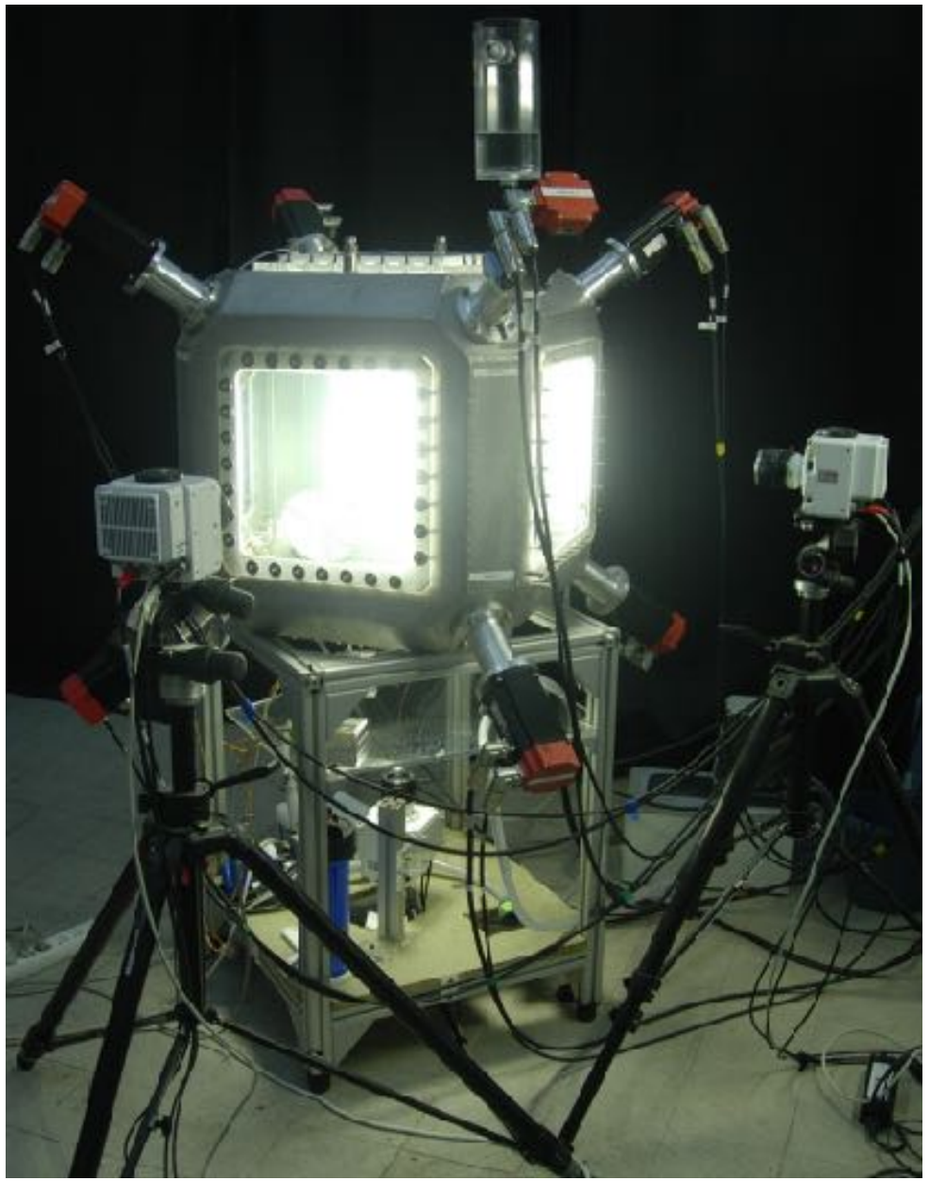


Power balance and fiber inextensibility

$$\rho \langle \kappa \rangle^{-3} \epsilon \sim \eta \int v \cdot u_{slip} \mathrm{d}s \Rightarrow \langle \kappa \rangle \sim \left(\frac{\rho \epsilon^{2/3}}{EI} \right)^{1/6} L^{-2/9}$$

Gay *et al.*, Characterisation of flexible fibre deformations in turbulence, *EPL*, 123(2), 24001, 2018

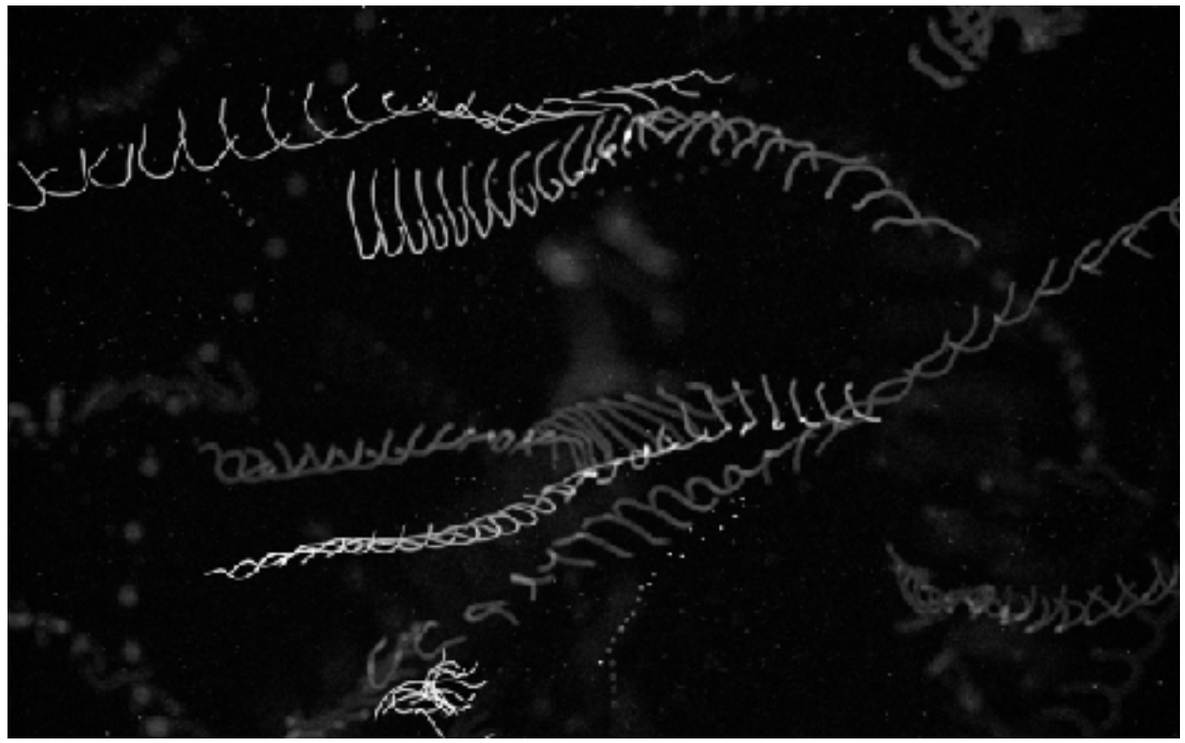
Flexible fibers: experimental setup



$$R_\lambda \in [300; 710]$$

$$\eta_K \in [30; 70]\ \mu\text{m}$$

$$L_I \sim 7\ \text{cm}$$



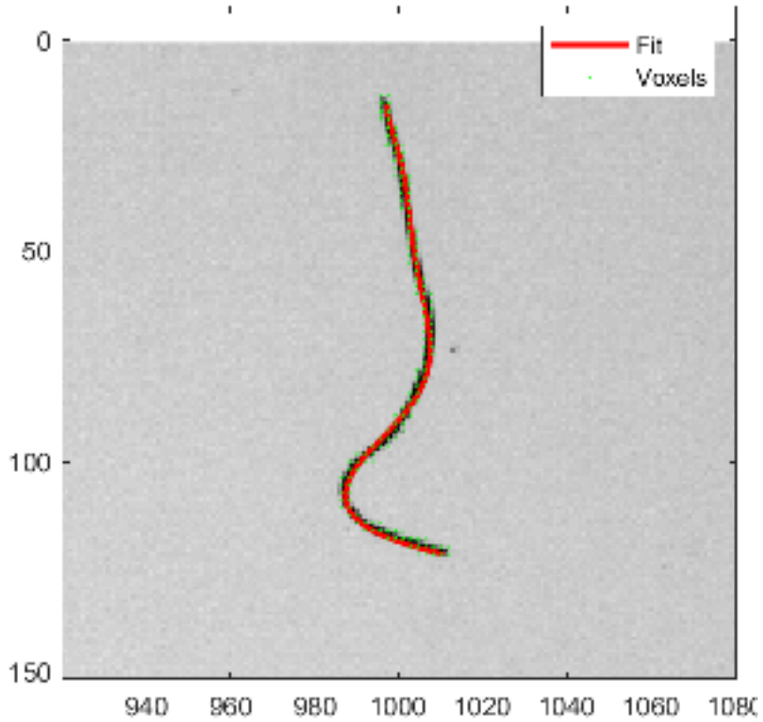
$$L = [10; 15; 20]\ \text{mm}$$

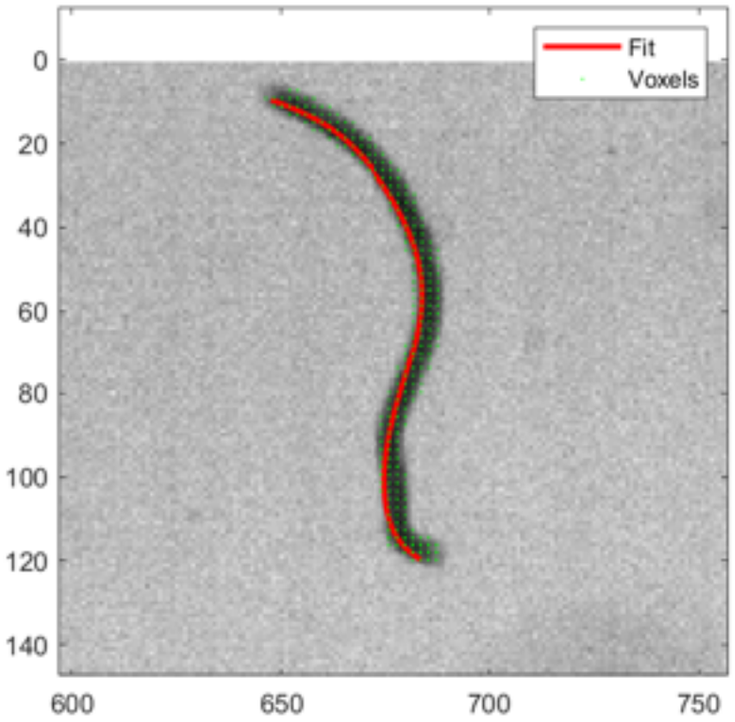
$$d = [130; 400; 990]\ \mu\text{m}$$

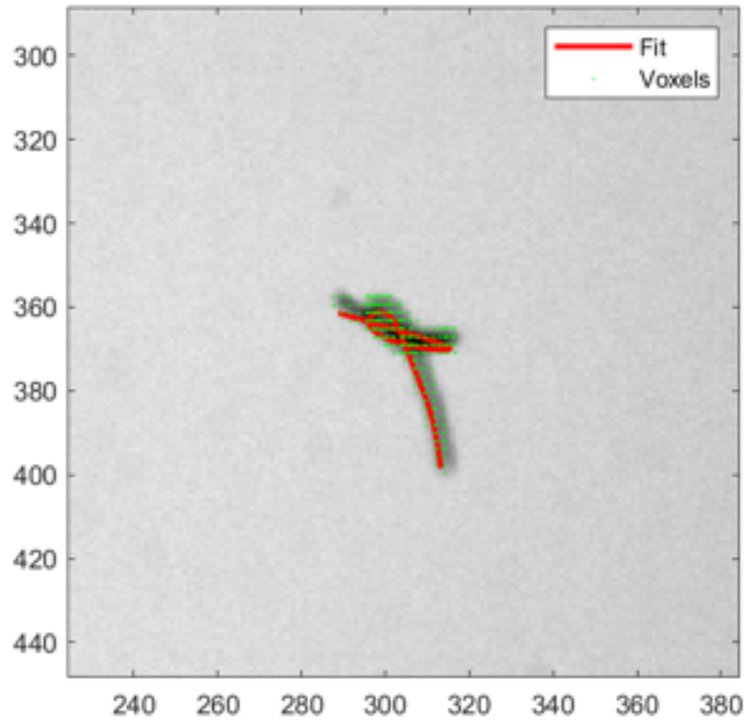
$$E = [5; 70; 2000]\ \text{kPa}$$

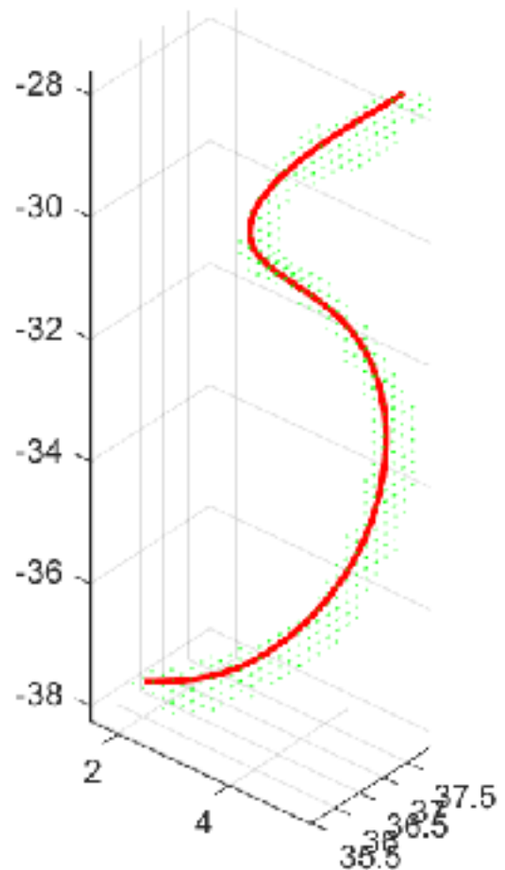
$$\rho \sim 1.06 \cdot 10^3\ \text{kg.m}^{-3}$$

3D reconstruction

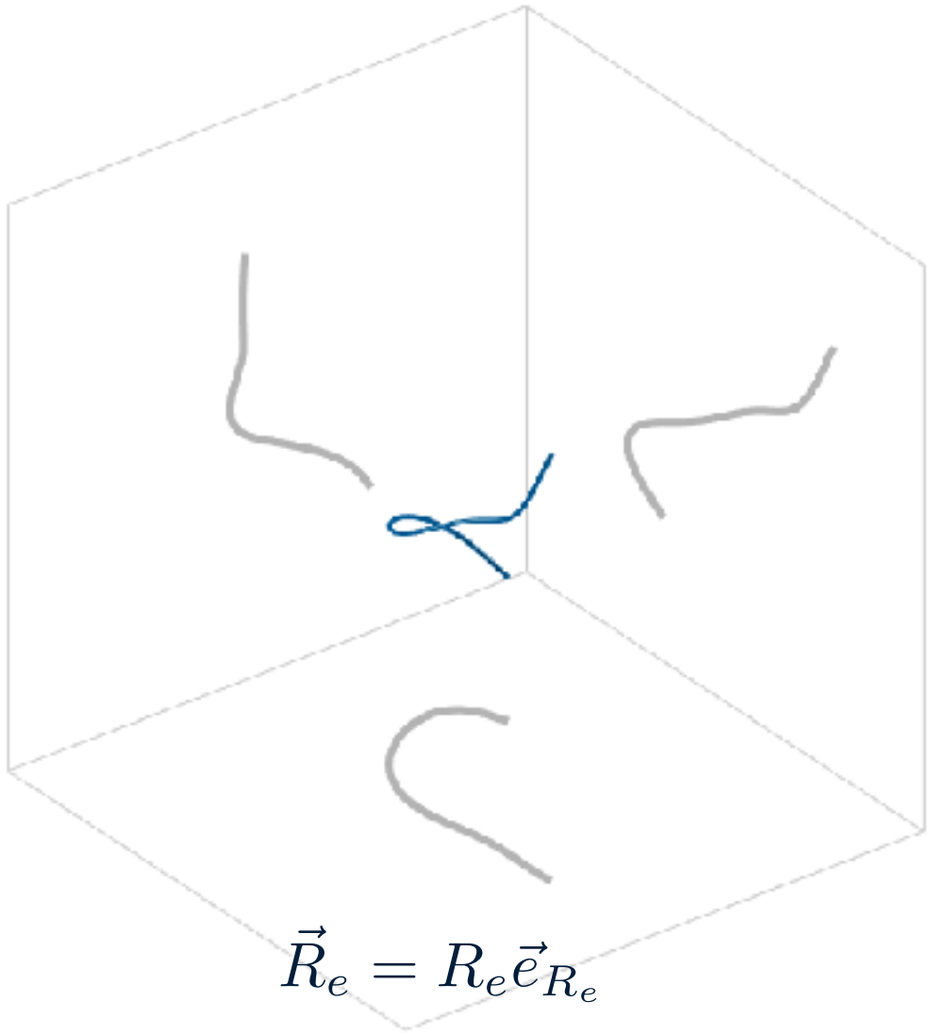




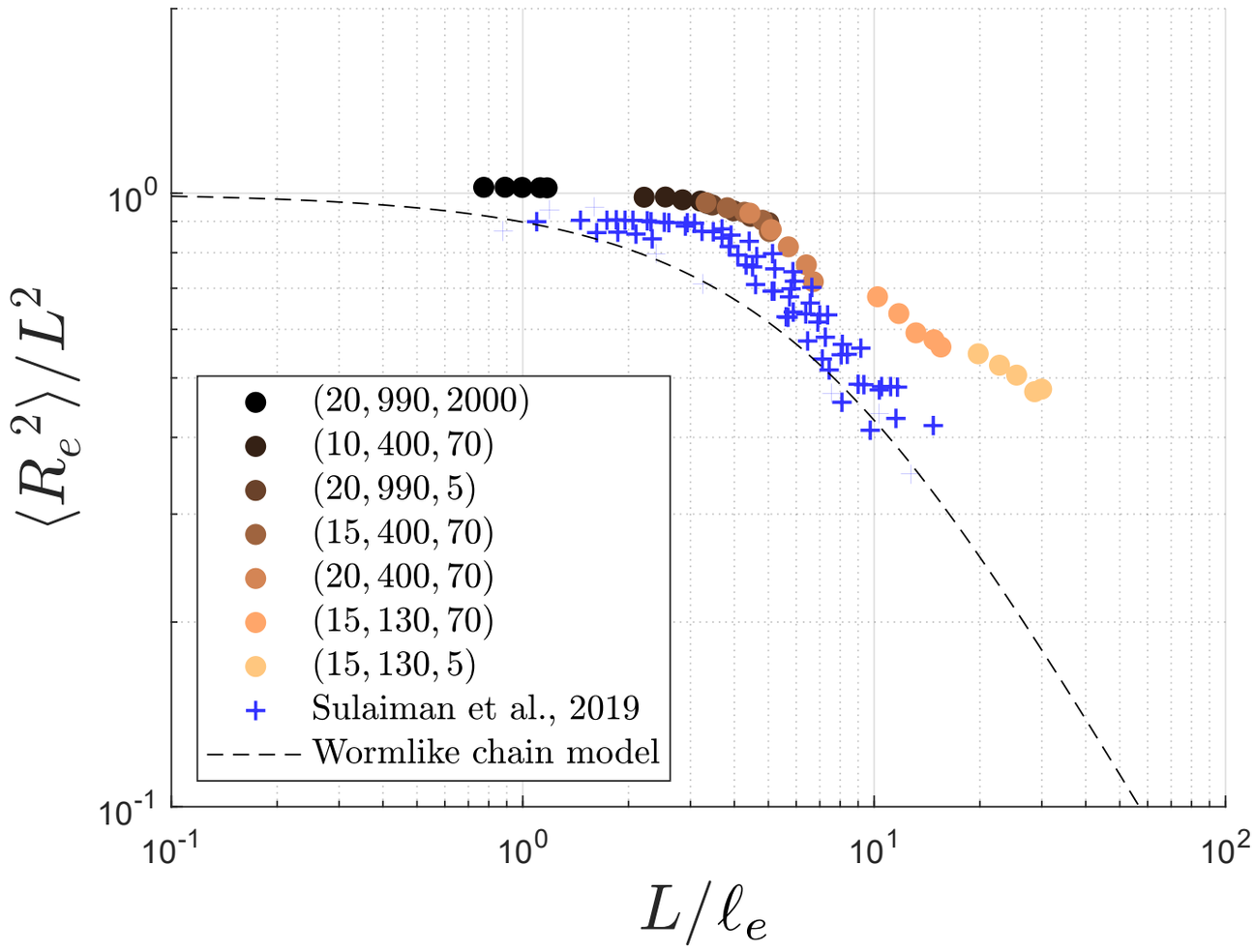




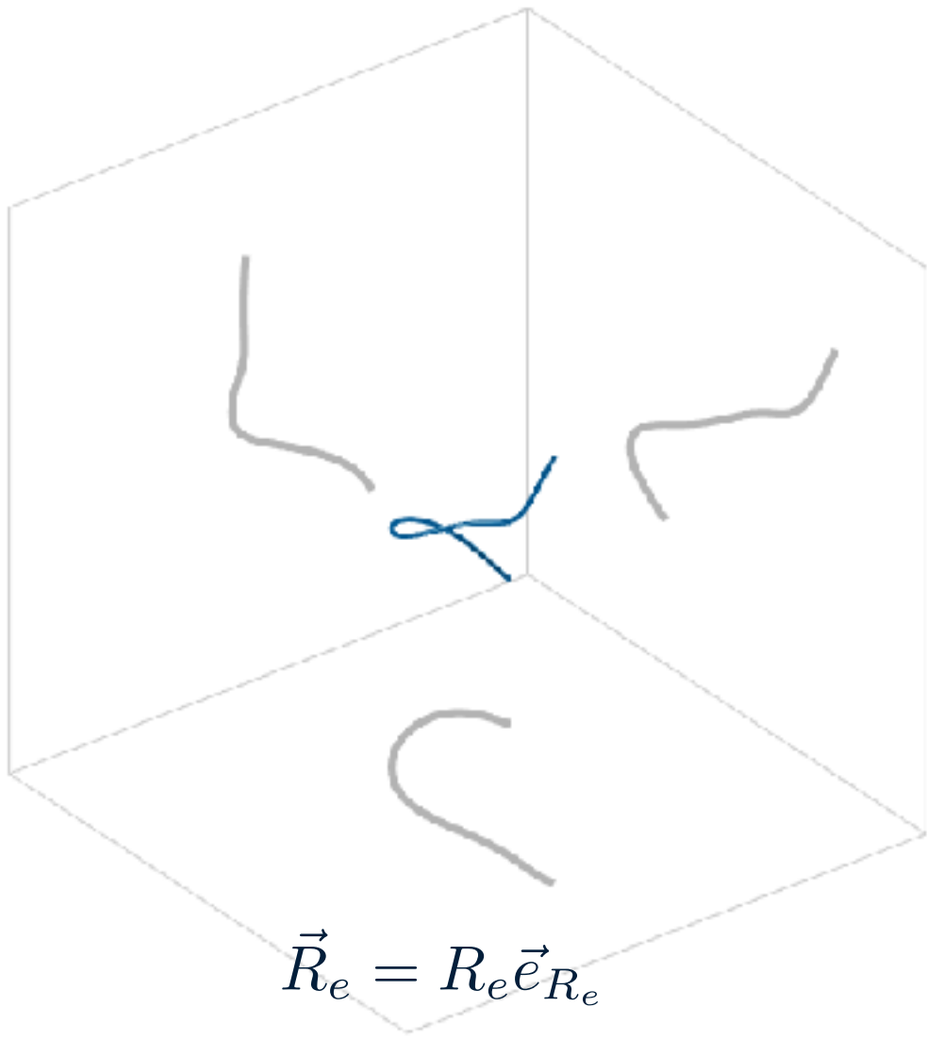
Norm of the end to end vector



$$\vec{R}_e = R_e \vec{e}_{R_e}$$



Dynamics of the end to end vector



$$\vec{R}_e = R_e \vec{e}_{R_e}$$

Rotational dynamics

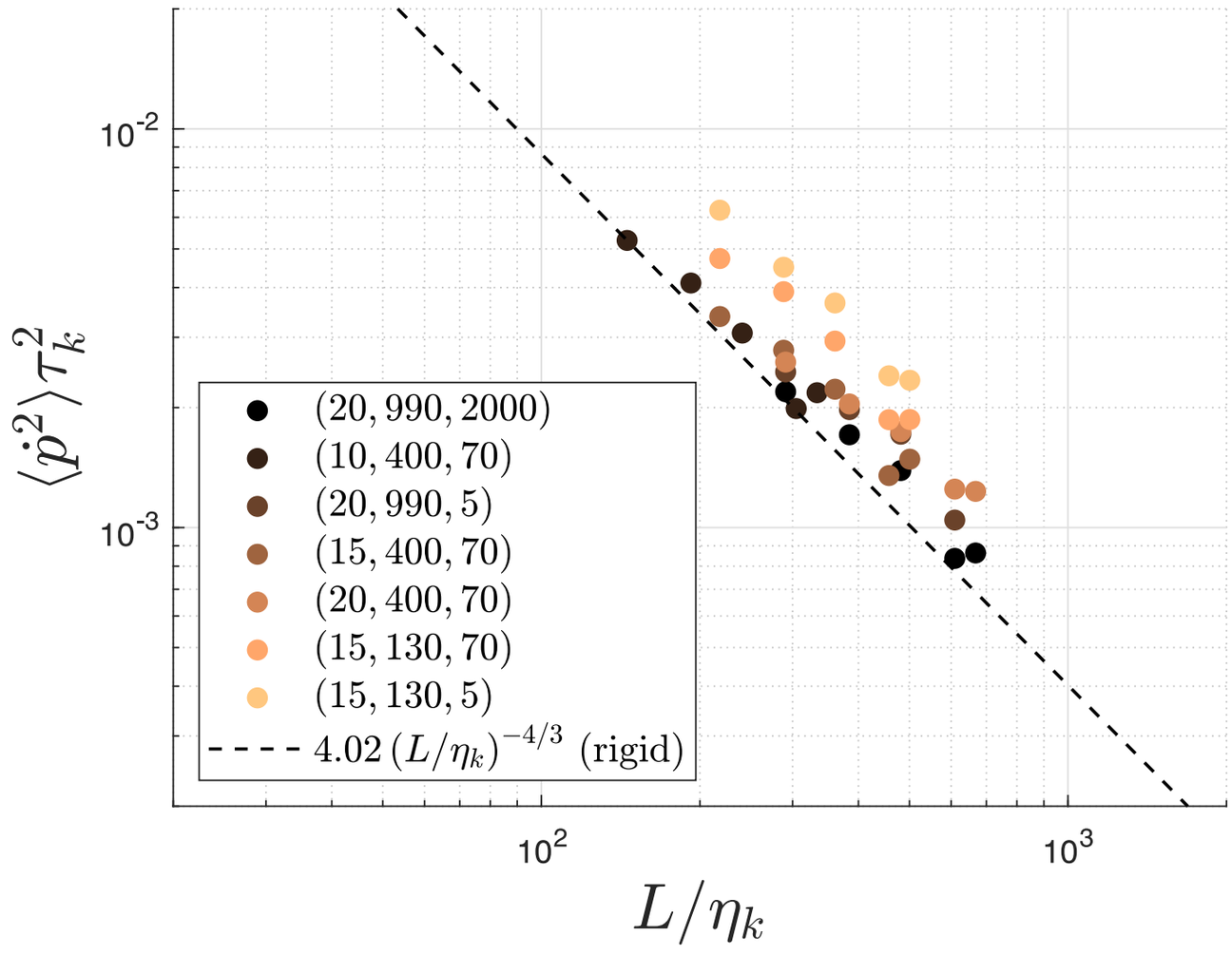
Evolution of $\vec{e}_{R_e}$

Deformation dynamics

Evolution of R_e

Does flexibility impact the rotational dynamics?

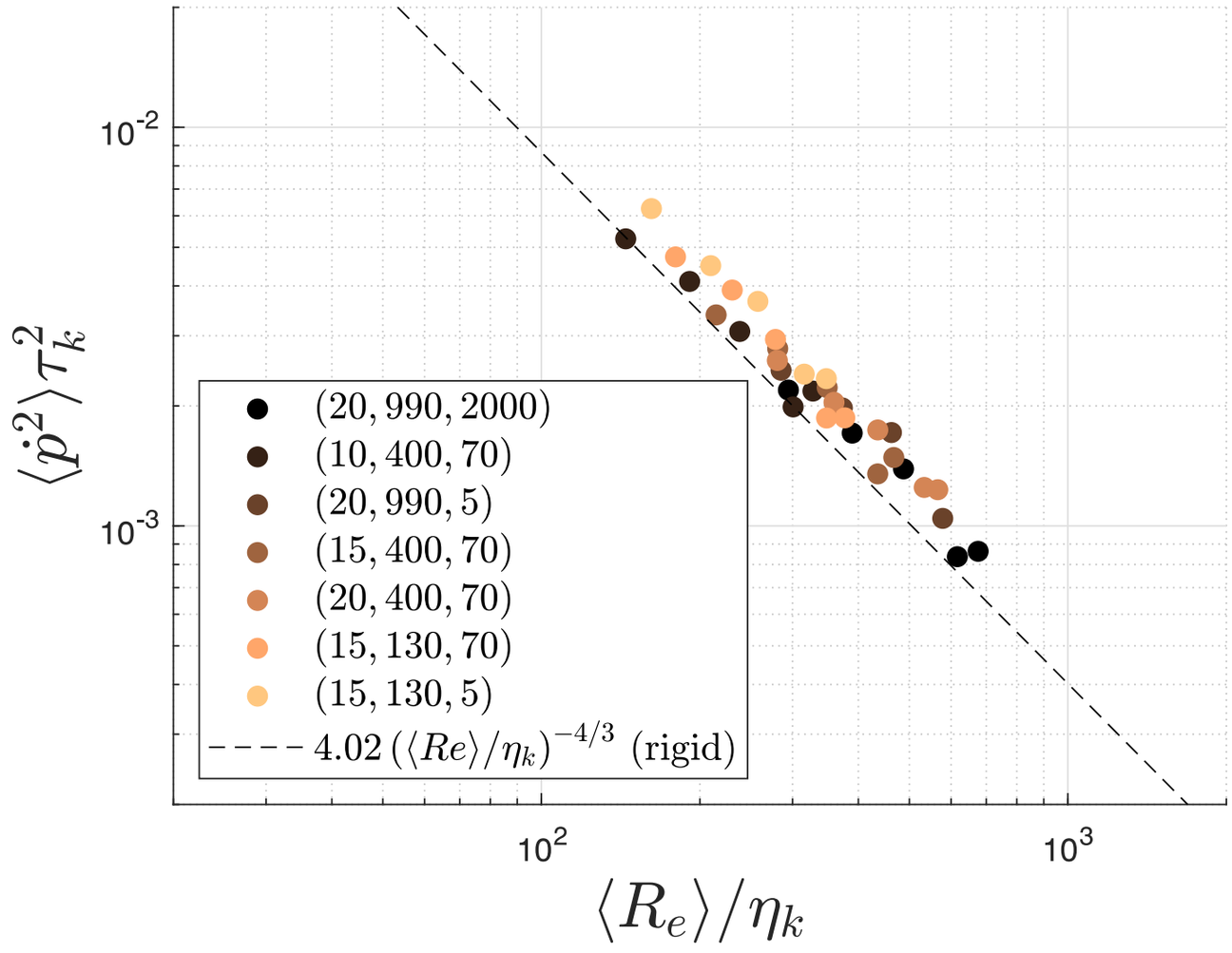
Tumbling rate



For rigid particles $\langle \dot{e}_{R_e} \dot{e}_{R_e} \rangle \sim \tau_K^{-2} \left(\frac{L}{\eta_K} \right)^{4/3}$

Flexible fibers tumble more than rigid fibers

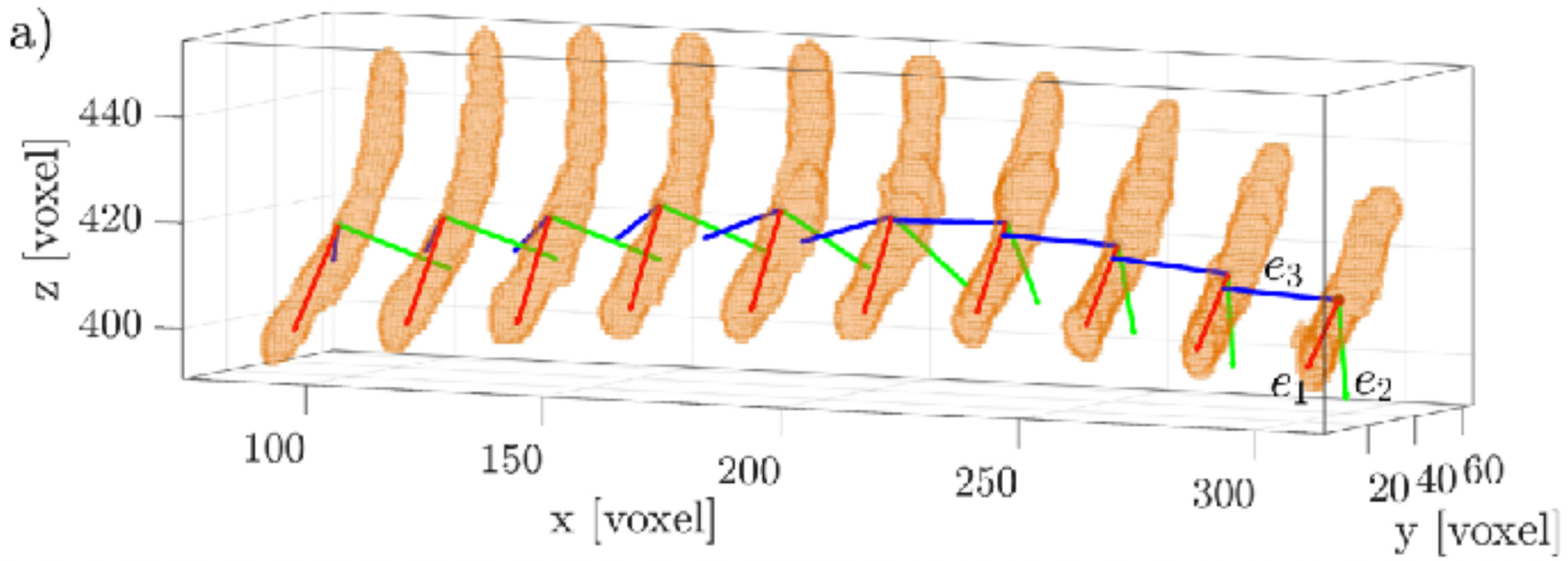
Tumbling rate

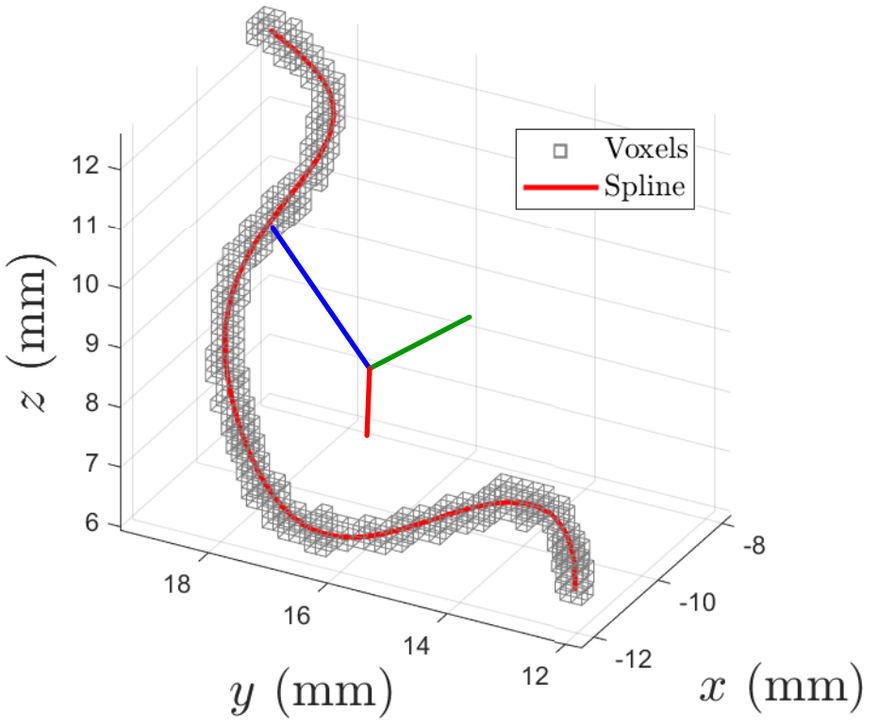


Seems to depend on the end to end distance

Here, fiber initially straight. Does not seem always true

Rotation of flexible fibers





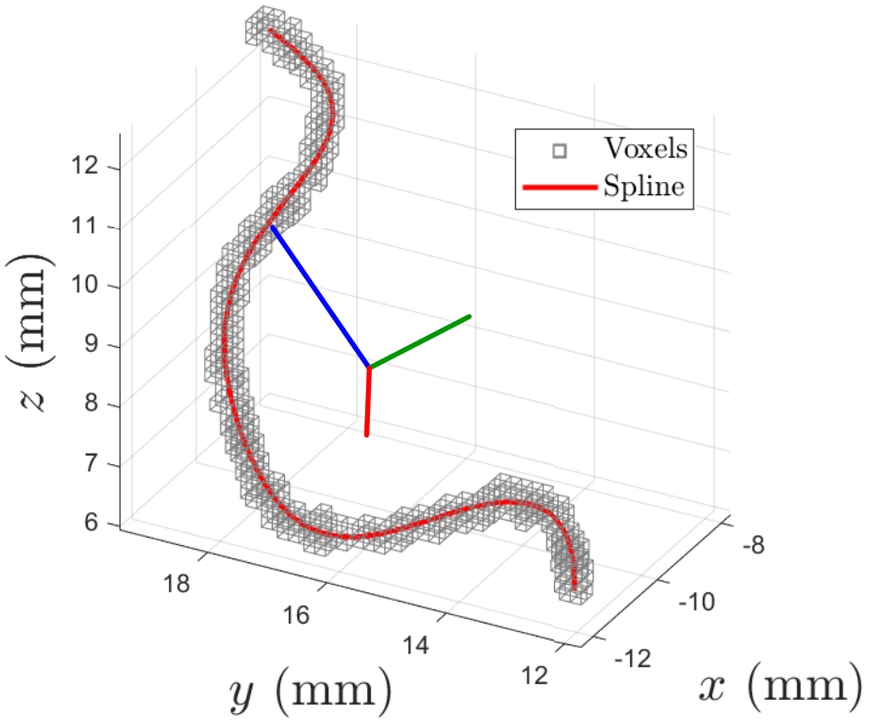
Eigen-frame based on the moment of inertia

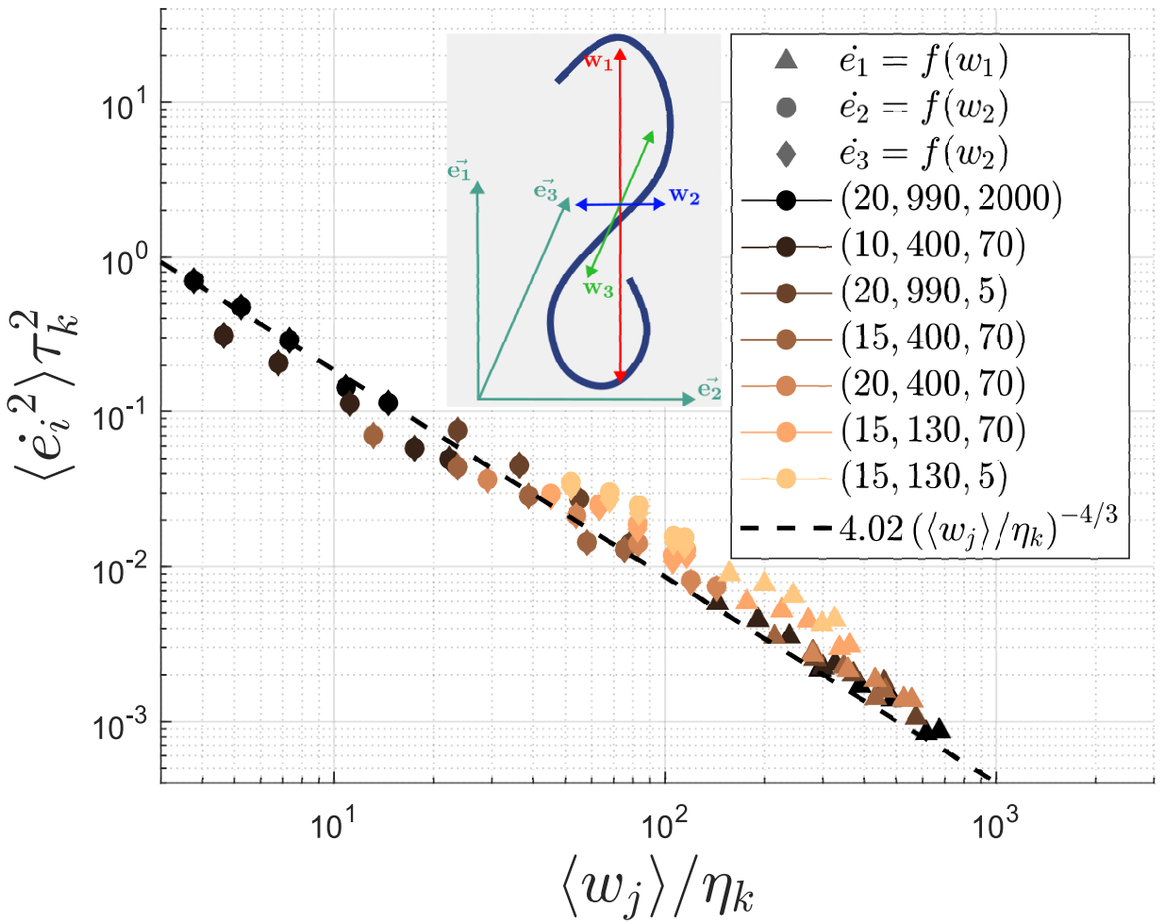
Rotation rate of the eigenvectors

Rotation of flexible fibers

Tumbling and spinning of long straight fibers are governed by length and diameter

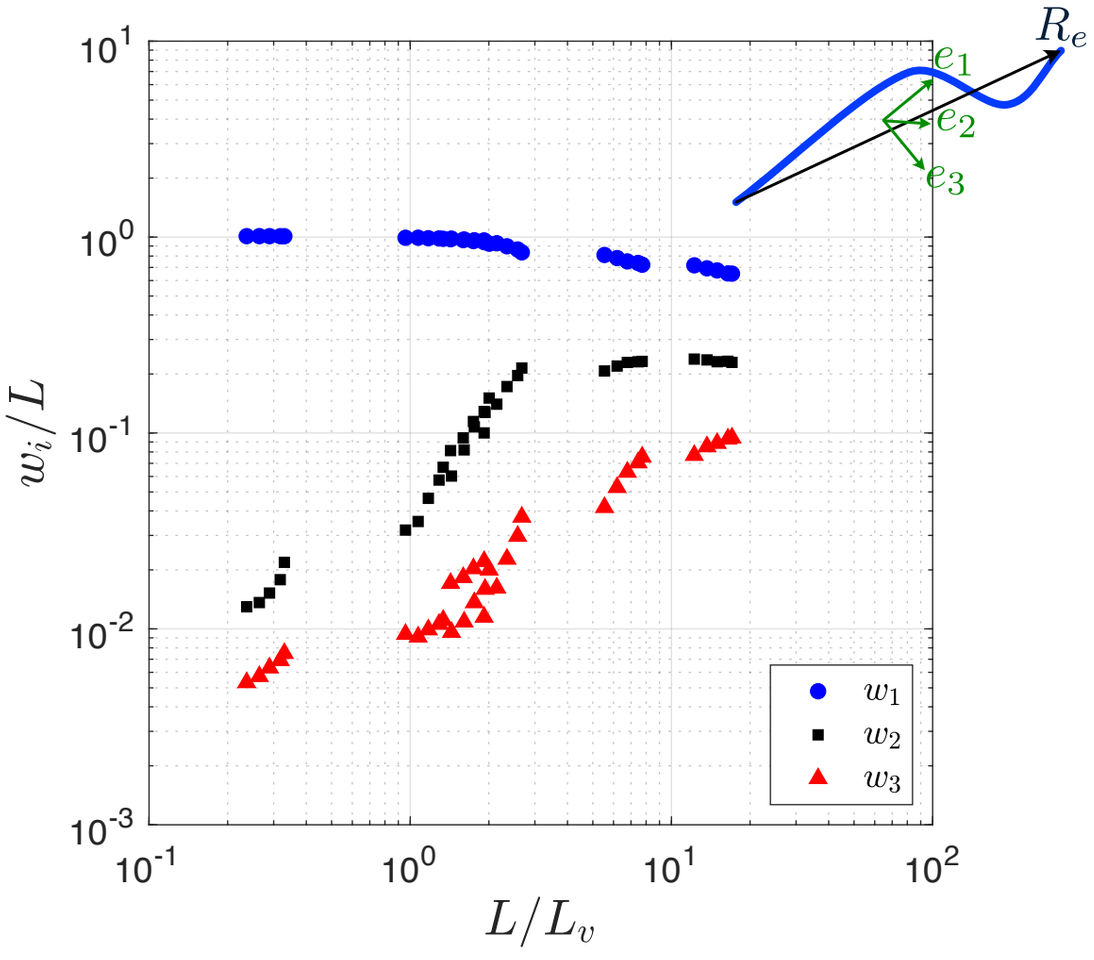
$$\langle \Omega_T \Omega_T \rangle \tau_k^2 \sim (L/\eta_K)^{-4/3} \qquad \langle \Omega_S \Omega_S \rangle \tau_k^2 \sim (d/\eta_K)^{-4/3}$$

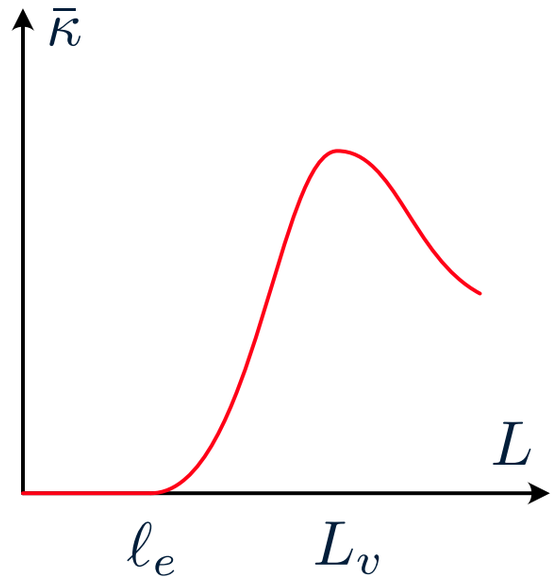




w_i = extent along the eigenvectors

Relation between mean extension and mean deformation

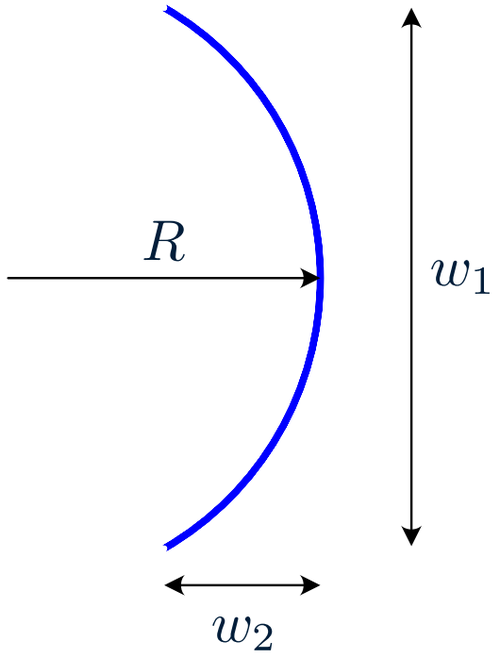




Simplest model
2D deformation (moderately flexible fibers)

Evolution of the extension

Assumption:
in average constant curvature



$$\frac{w_1}{L} = \mathrm{sinc}\frac{L}{2R}$$

$$R = \bar{\kappa}^{-1}$$

$$\frac{w_2}{L} = \frac{L}{R}\left(1 - \cos\frac{L}{2R}\right)$$

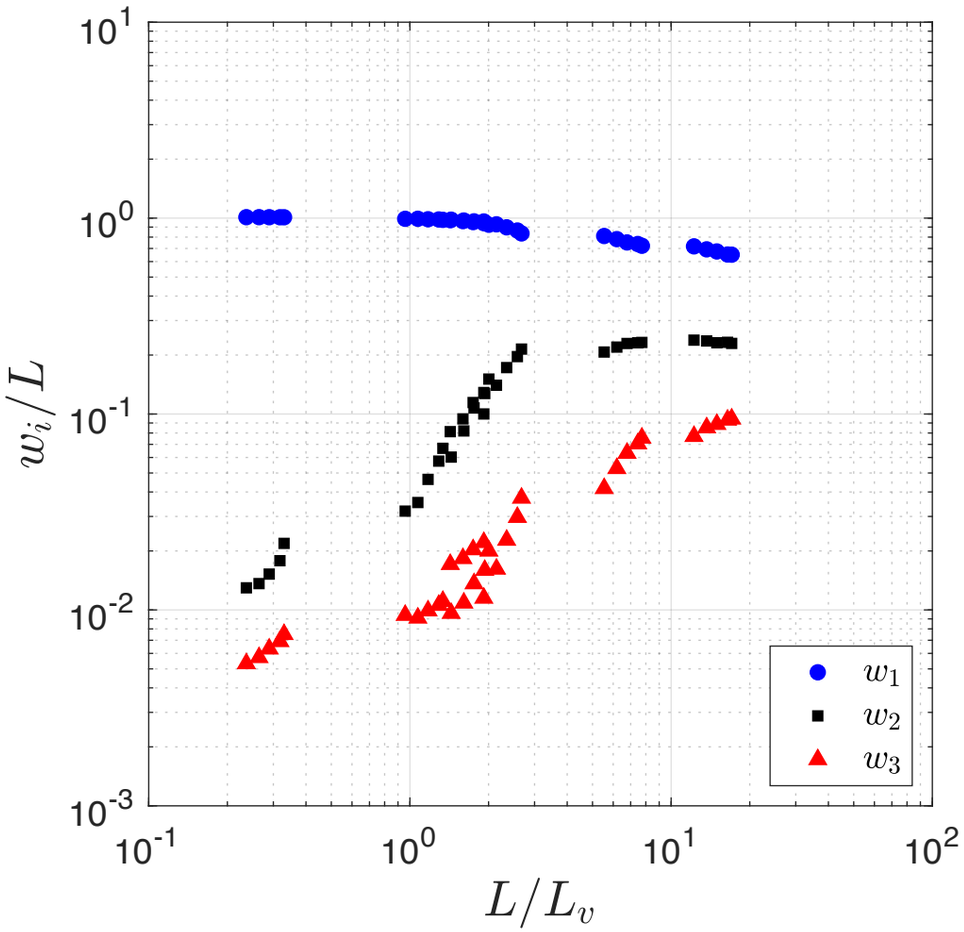
Model of the curvature

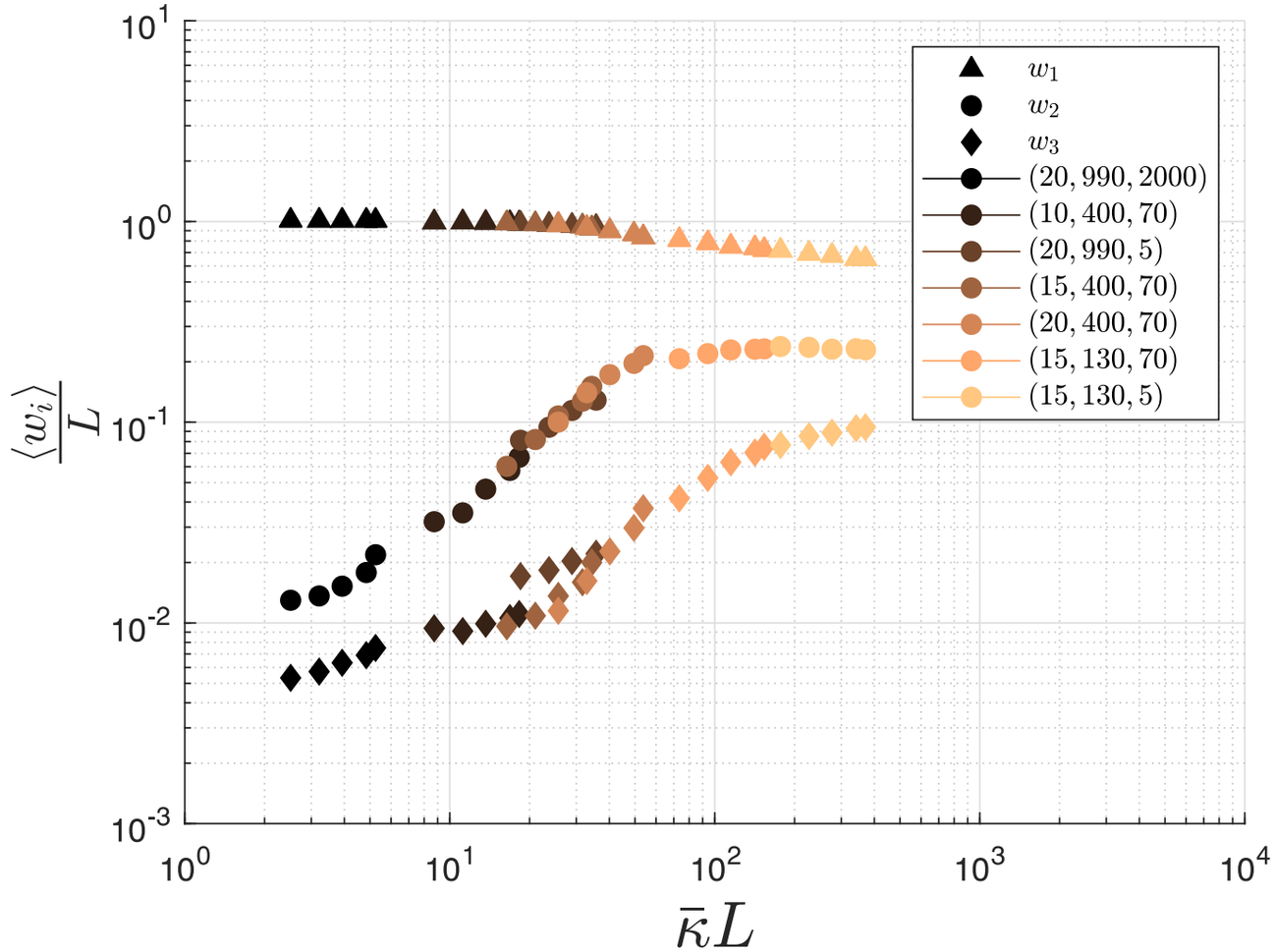
Long fibers => viscous dissipation higher than bending

not too long to remain 2D => structure responsible of deformation scale with the fiber length

Power budget gives $$\bar{\kappa}L \sim \left(\frac{L}{L_v}\right)^{10/9}\left(\frac{L}{\eta_K}\right)^{4/9}$$

Evolution of the extension

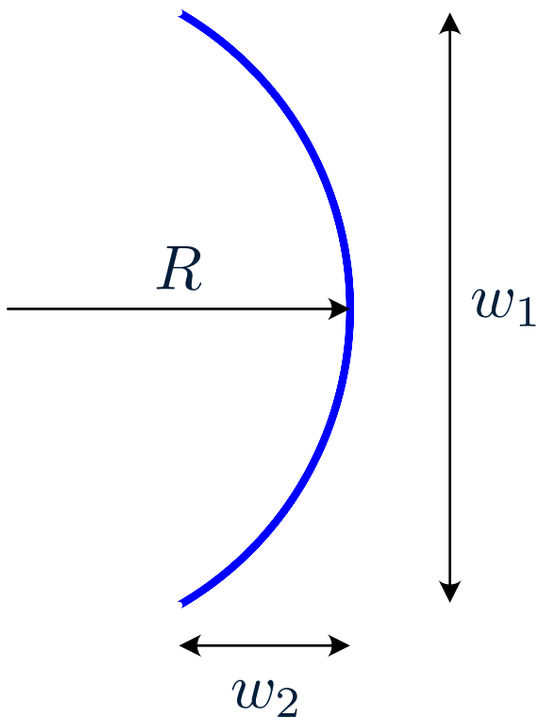




$$\bar{\kappa}L \sim \left(\frac{L}{L_v}\right)^{10/9} \left(\frac{L}{\eta_K}\right)^{4/9}$$

Better collapse

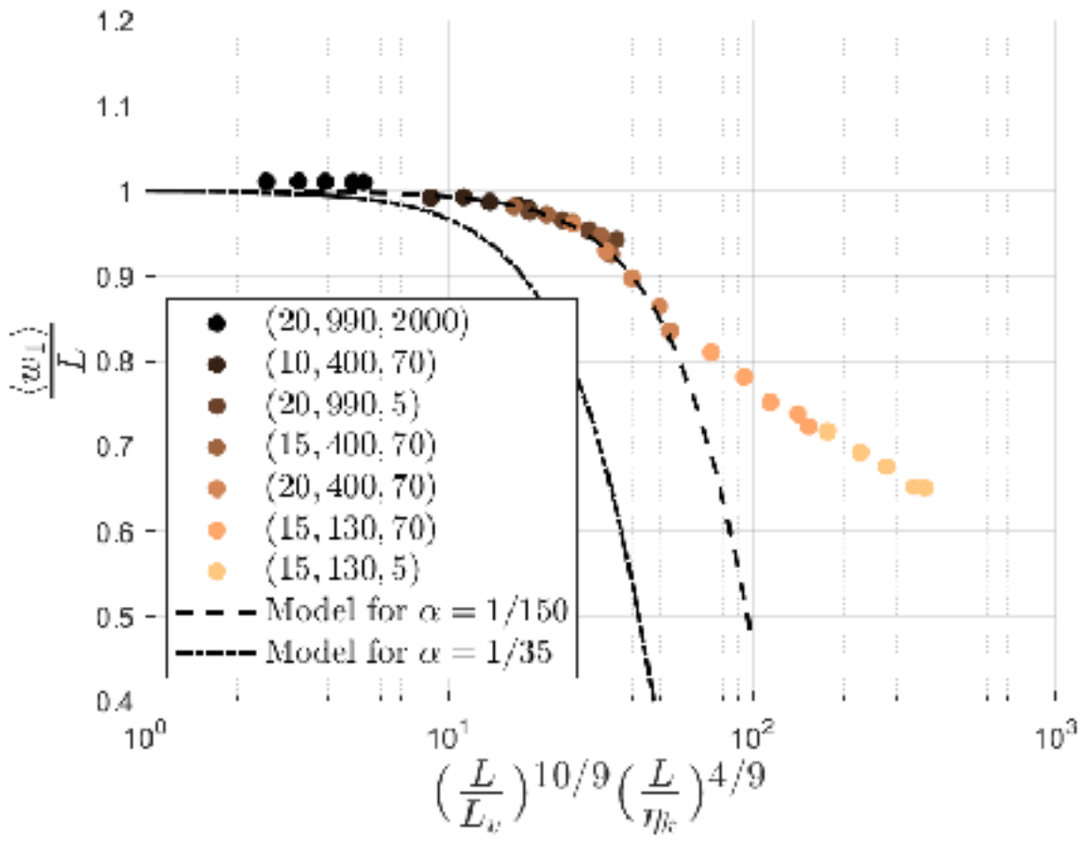
Evolution of the extension

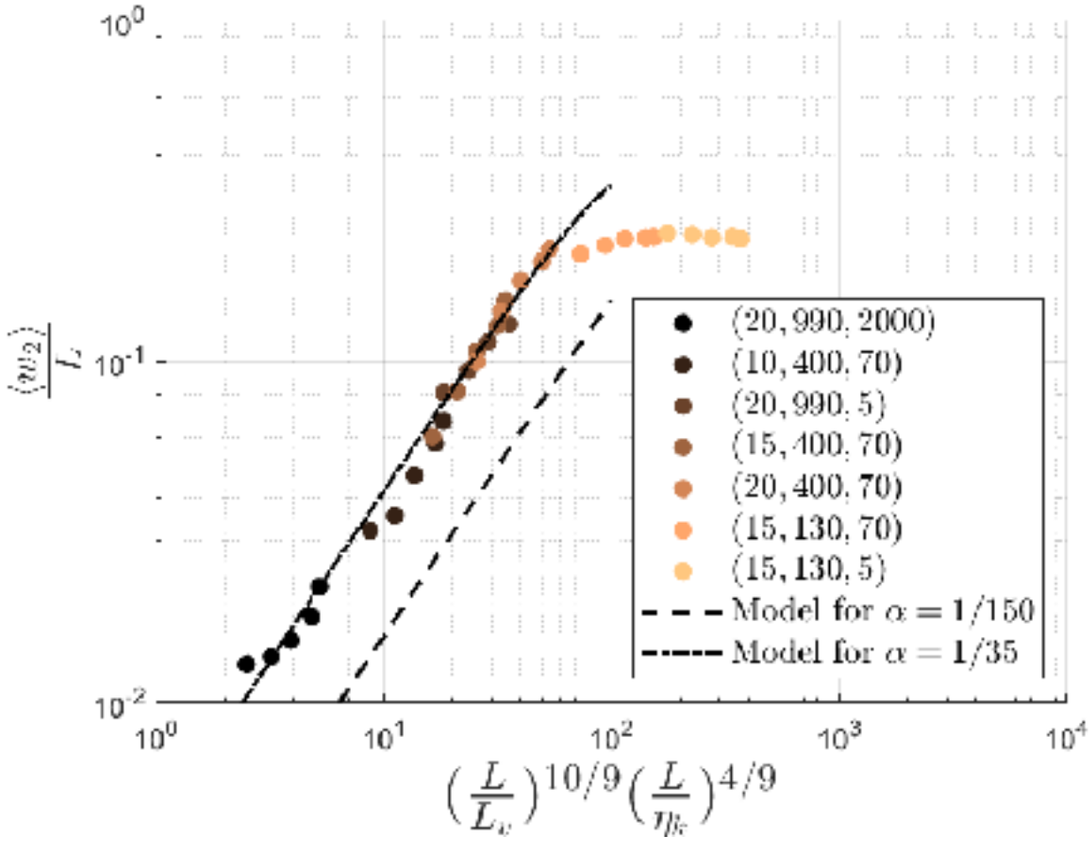


$$\frac{w_1}{L} = \operatorname{sinc}\frac{L}{2R}$$

$$R = \bar{\kappa}^{-1}$$

$$\frac{w_2}{L} = \frac{L}{R}\left(1 - \cos\frac{L}{2R}\right)$$

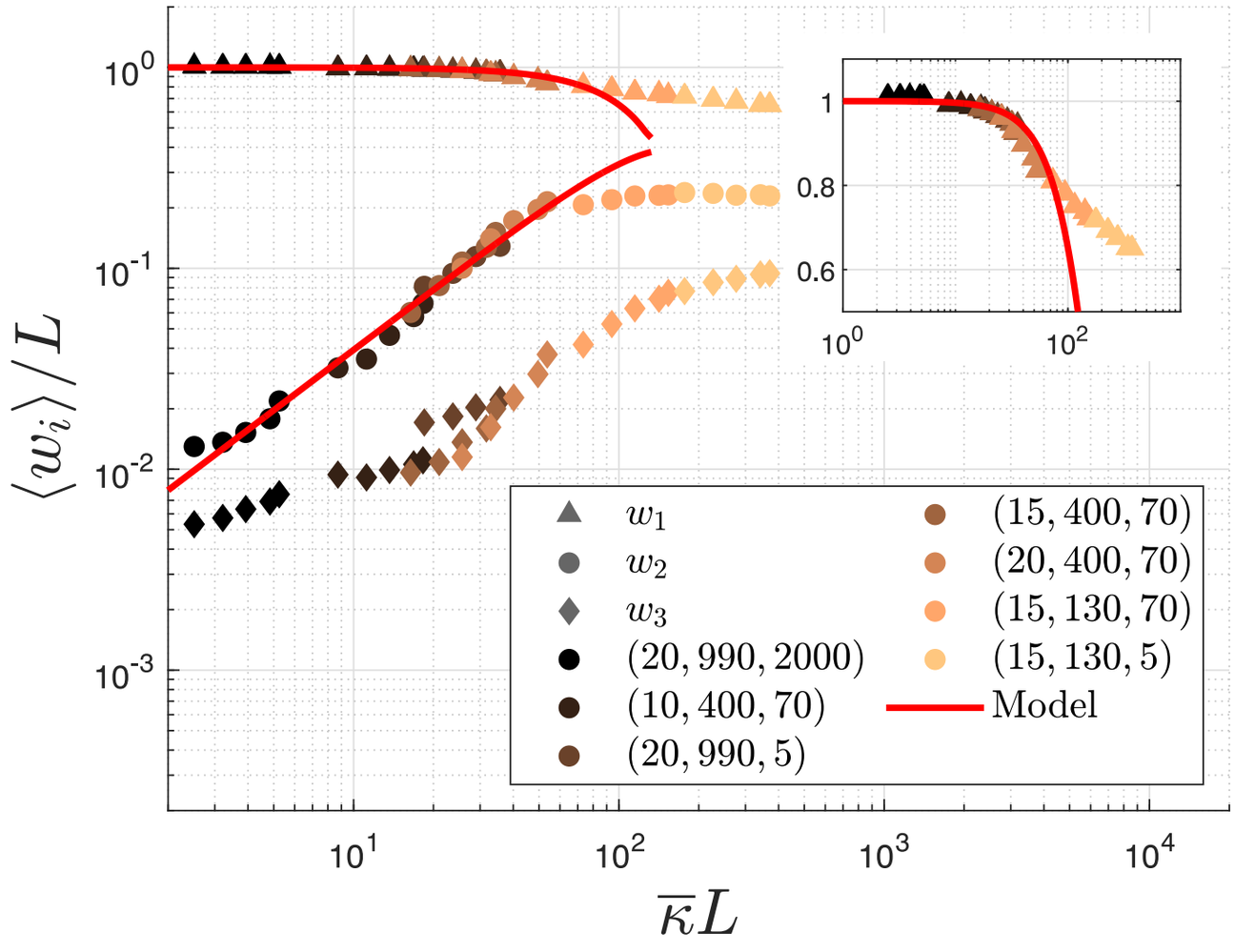




Evolution of the extension

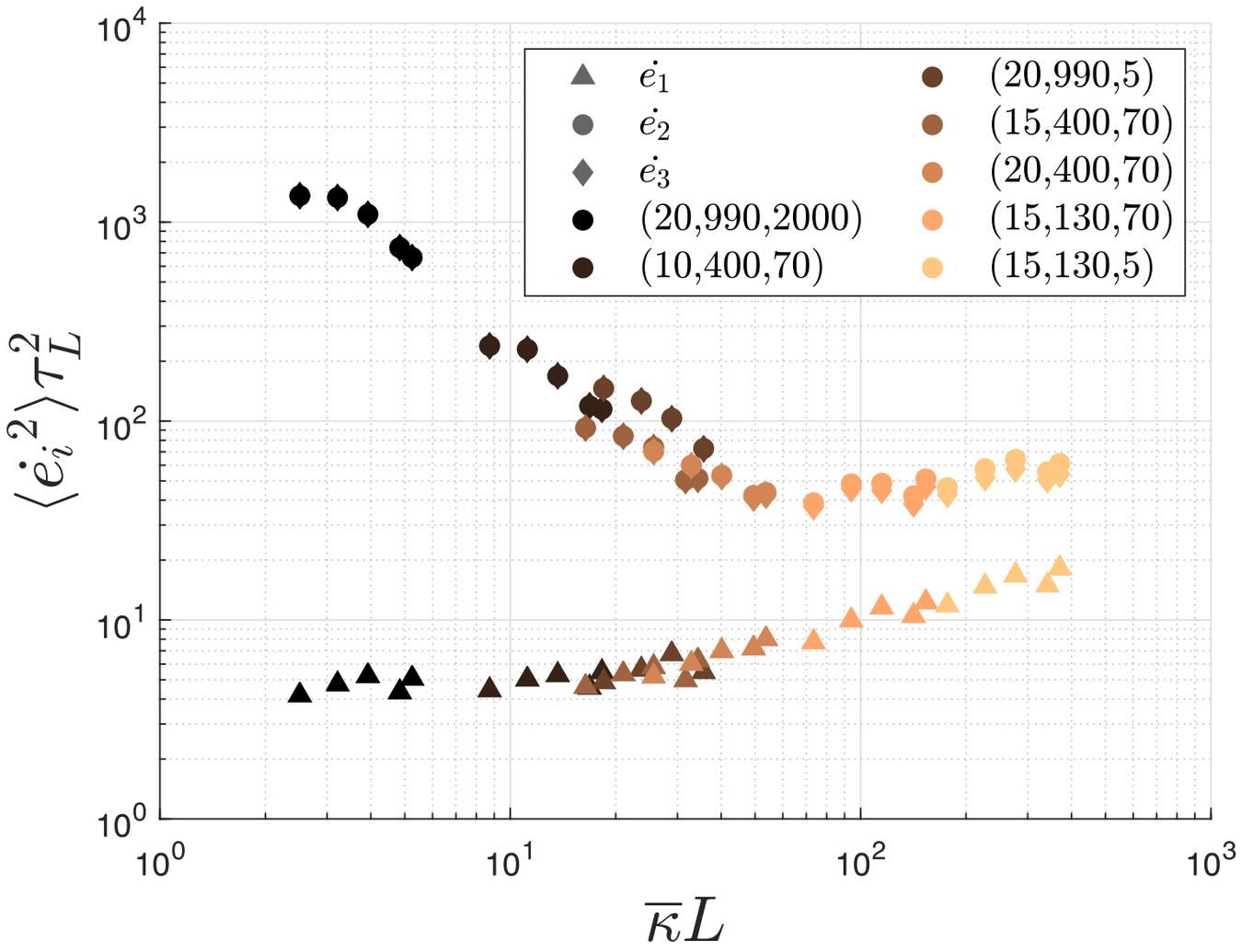
Boundary conditions: curvature = 0 at extremities

$$\bar{\kappa}(s) = \kappa_0 s(1-s)$$



$$\kappa_0 \simeq \frac{1}{25} \left(\frac{L}{L_v} \right)^{10/9} \left(\frac{L}{\eta_K} \right)^{4/9}$$

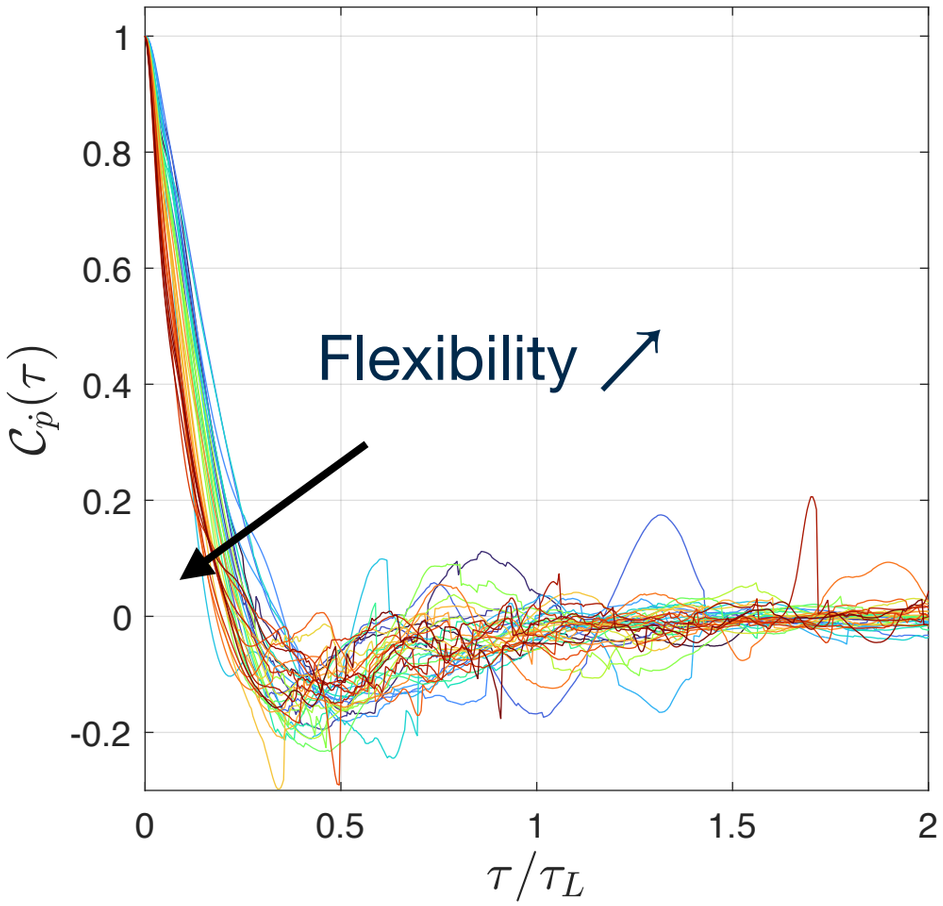
Influence of flexibility on the tumbling rate



$$\langle \dot{e}_{Re} \cdot \dot{e}_{Re} \rangle \tau_k^2 \sim \left(\frac{L}{\eta_K} \right)^{-4/3} (1 + f(\overline{\kappa} L))$$

Timescales of deformation and rotation

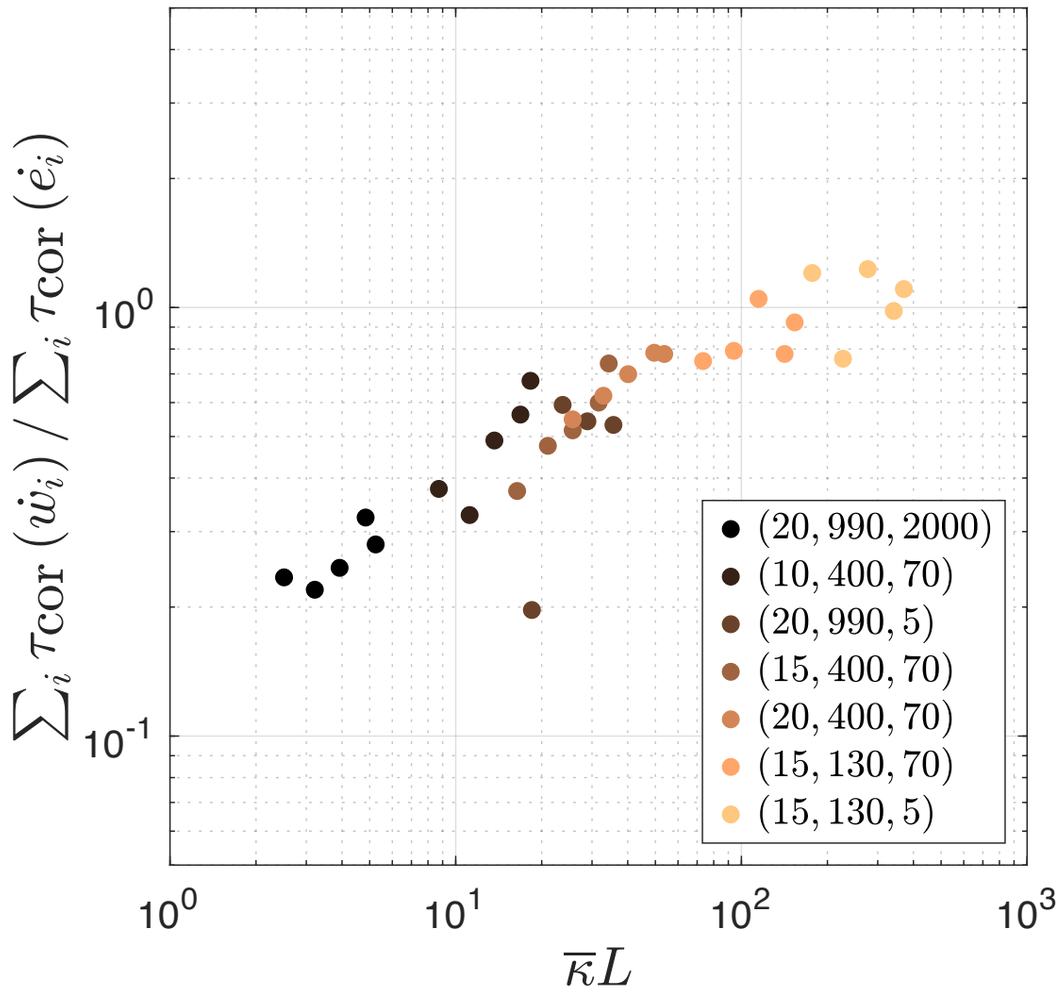
Validity of previous model: deformation slower than rotation



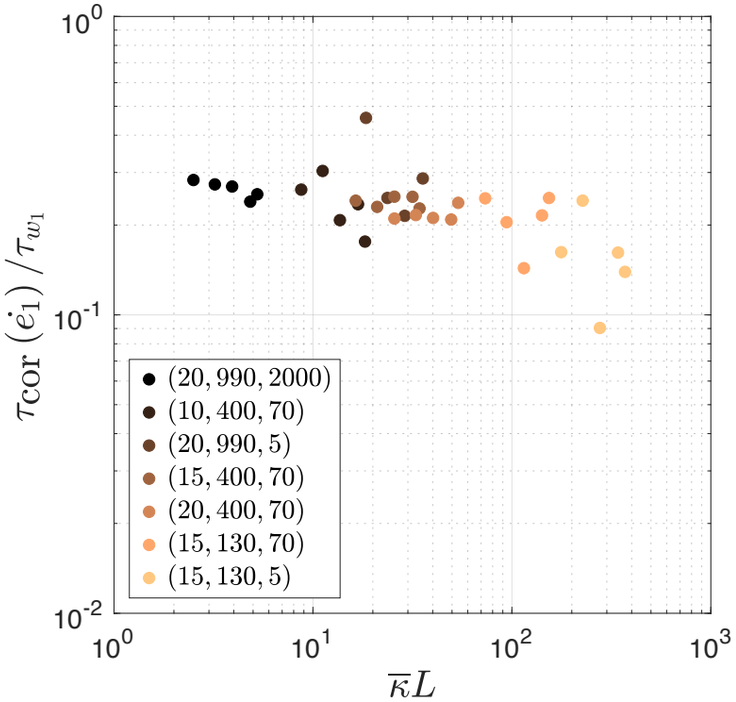
Definition of the correlation time by

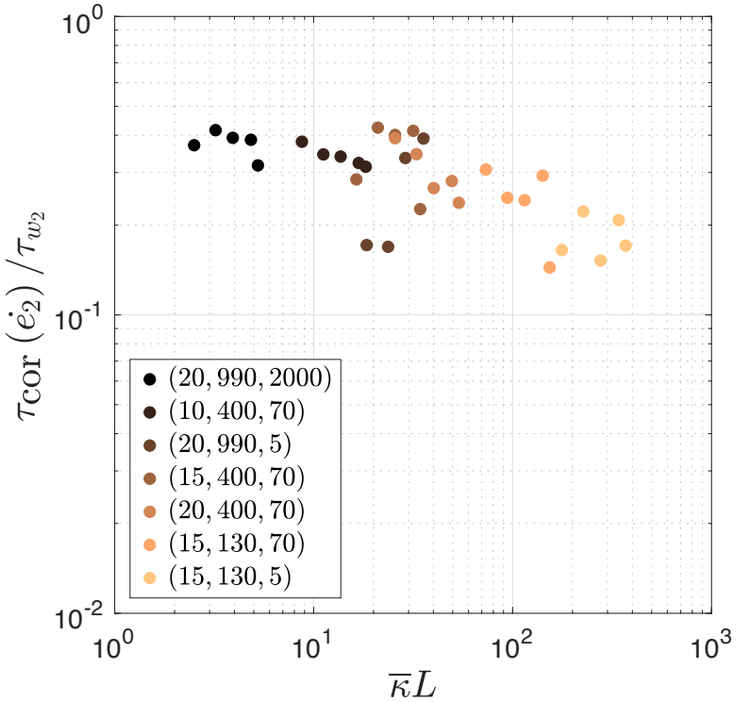
$$\mathcal{C}(\tau) = 0$$

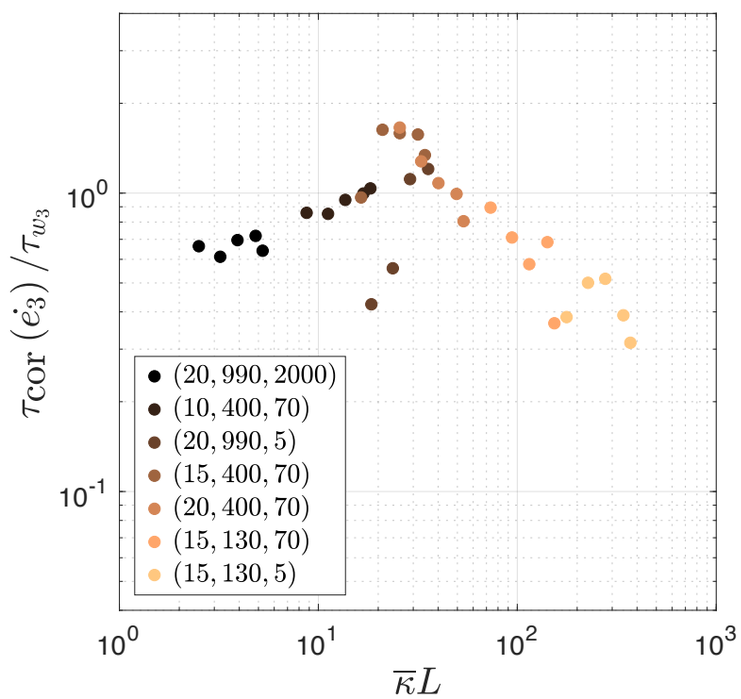
Timescales of deformation and rotation



Timescales of deformation and rotation

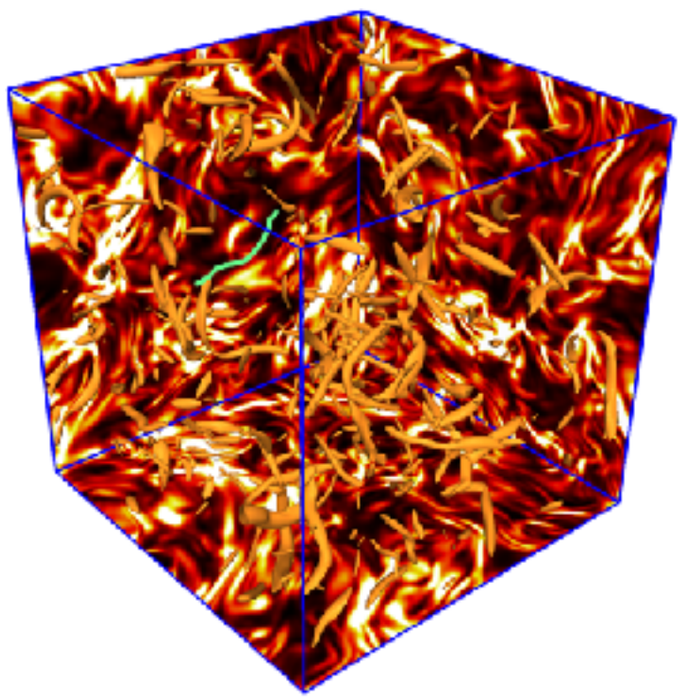






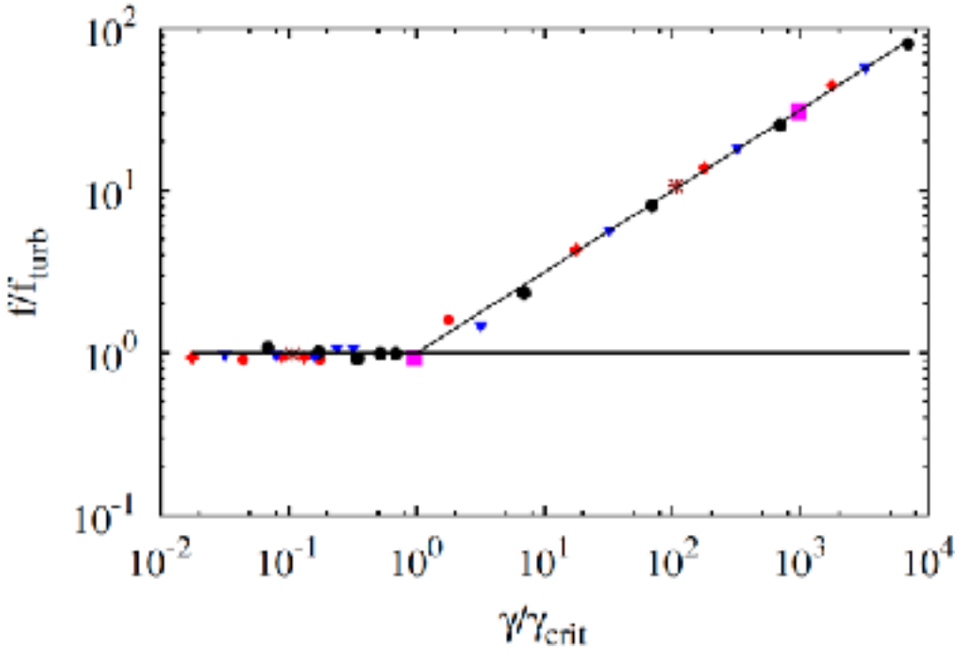
Correlation time of rotation given by the extent

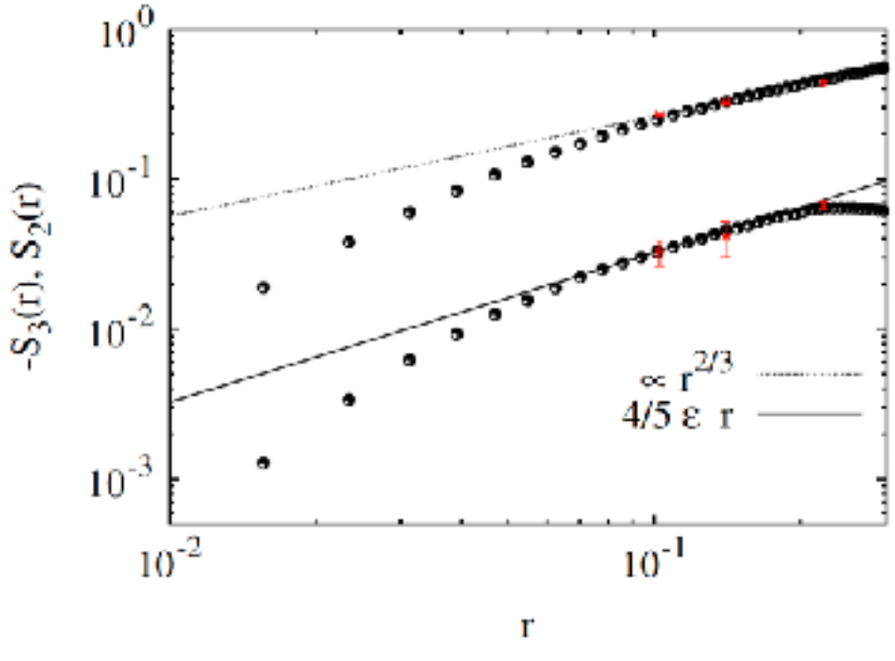
Flexible fibers as a proxy of turbulence



Underdamped regime

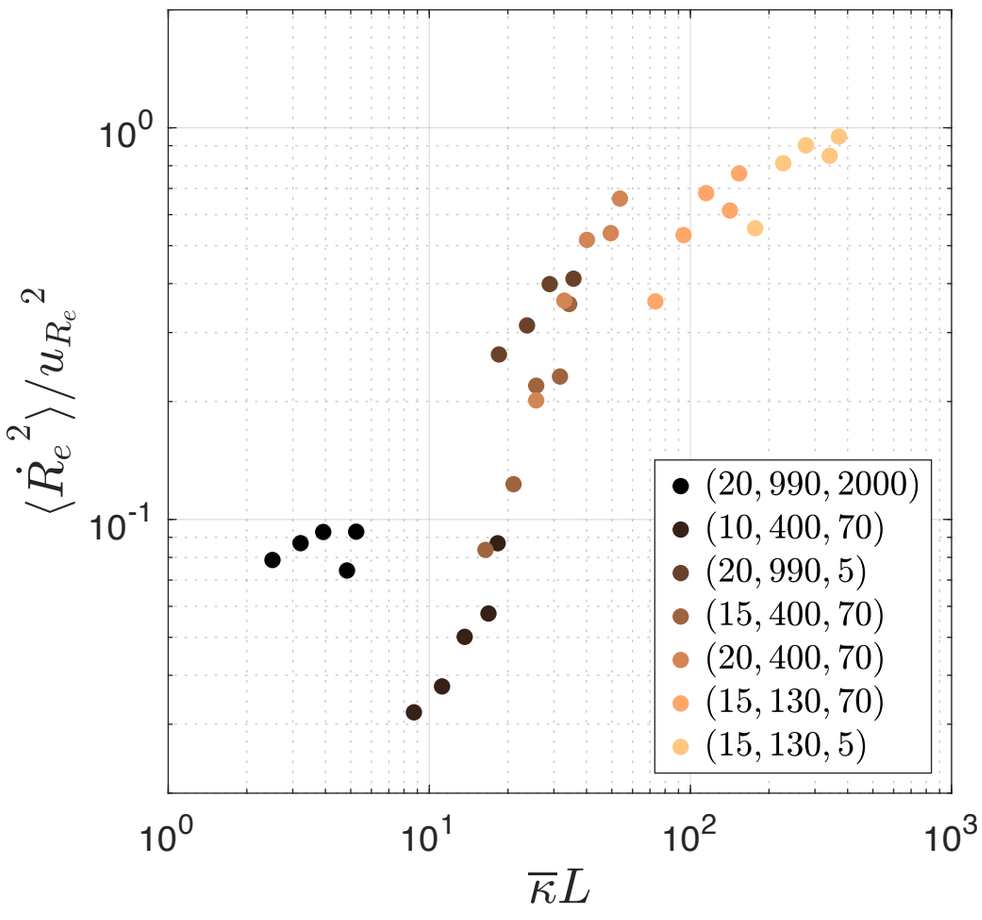
End to end dynamics gives access to the structure function

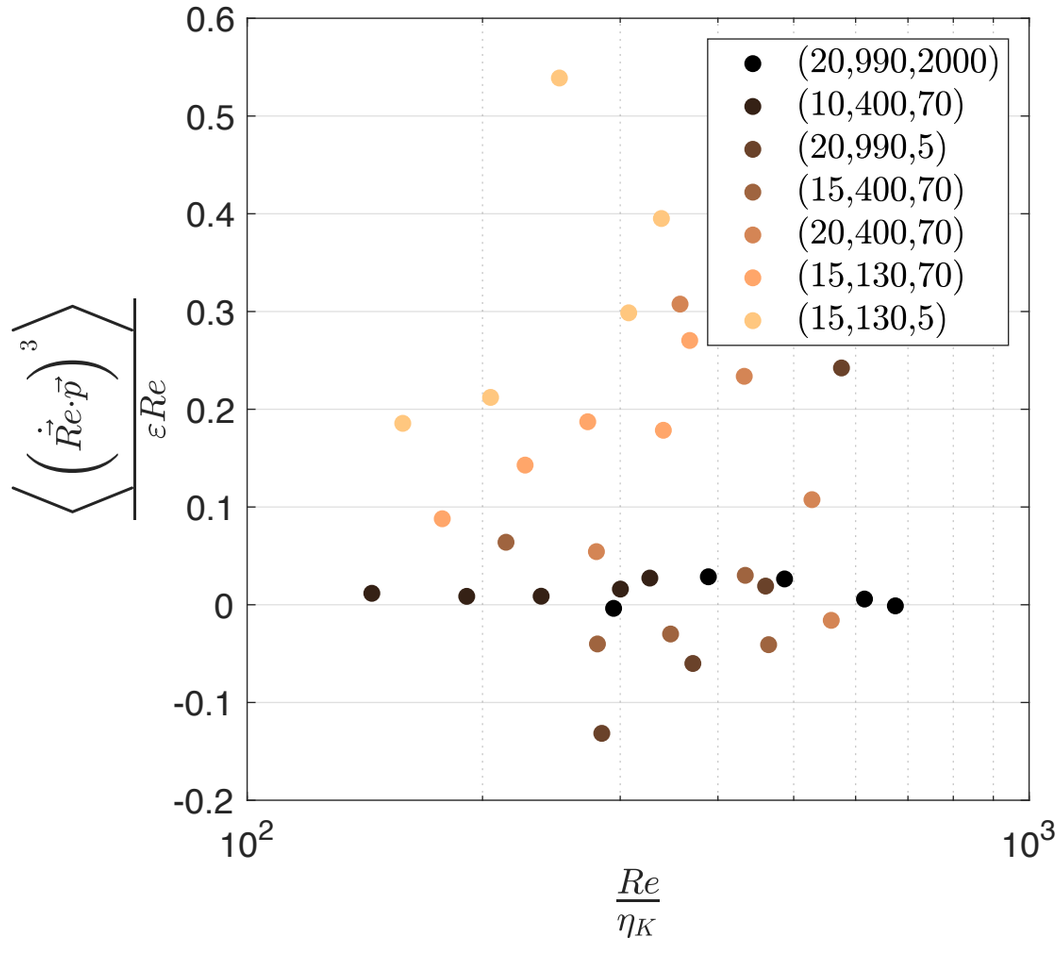




Rosti *et al.*, *Phys. Rev. Lett.*, 2018

Flexible fibers as a proxy of turbulence





$$\langle (\delta_r u \cdot \hat{e}_r)^3 \rangle = -\frac{4}{5}\epsilon r$$

Under- vs over-damped regime

$$\sigma\partial_{tt}r + \eta\partial_t r + B\partial_s^4 r - \partial_s(T\partial_s r) = \eta u$$

$$\sigma\partial_{tt}r + \eta\partial_t r + B\partial_s^4 r - (\partial_s T)\partial_s r - T\partial_{ss} r = \eta u$$

At the extremities $\kappa = \partial_{ss} r = 0$

Long (flexible) fibers $B\partial_s^4 r \ll \partial_s(T\partial_s r)$

$$\sigma\partial_{tt}r + \eta\partial_t r = (\partial_s T)\partial_s r + \eta u$$

Under-damped regime

$$\sigma\partial_{tt}r \gg \eta\partial_t r$$

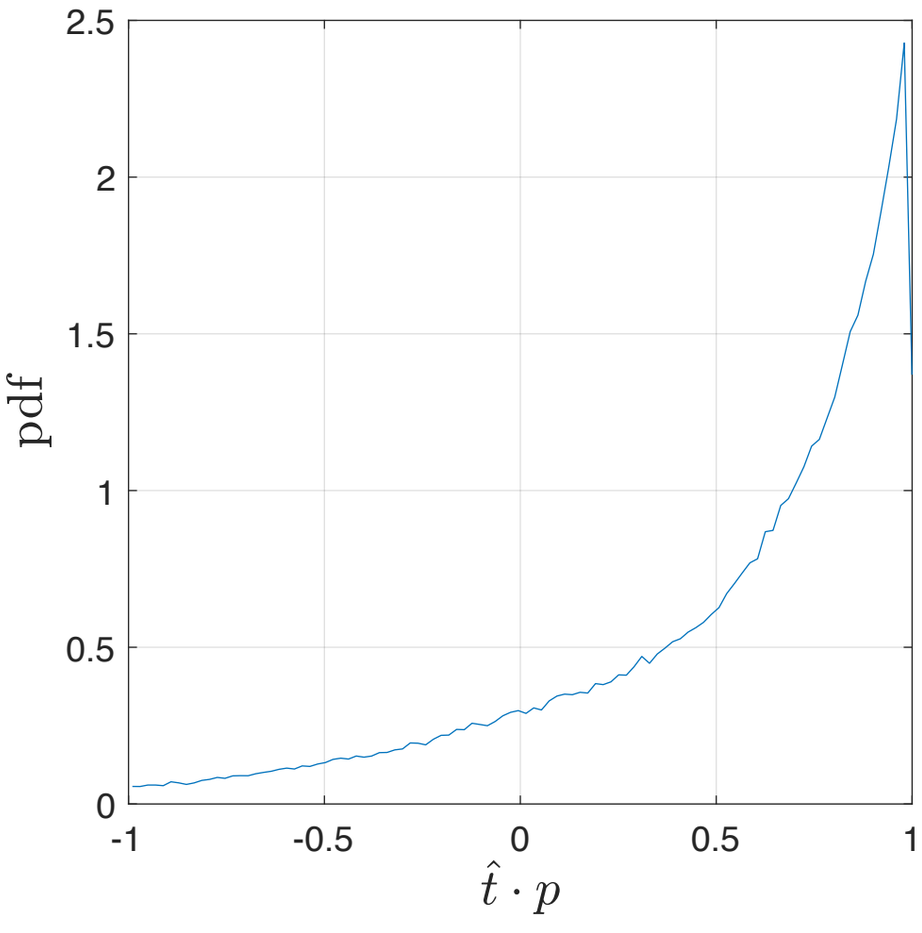
Over-damped regime

$$\sigma\partial_{tt}r \gg \eta\partial_t r$$

Velocity of the extremities gives access to structure function if tension term negligible

Over-damped regime

$$\partial_t r \cdot p = \frac{1}{\eta}(\partial_s T)\partial_s r \cdot p + u \cdot p$$

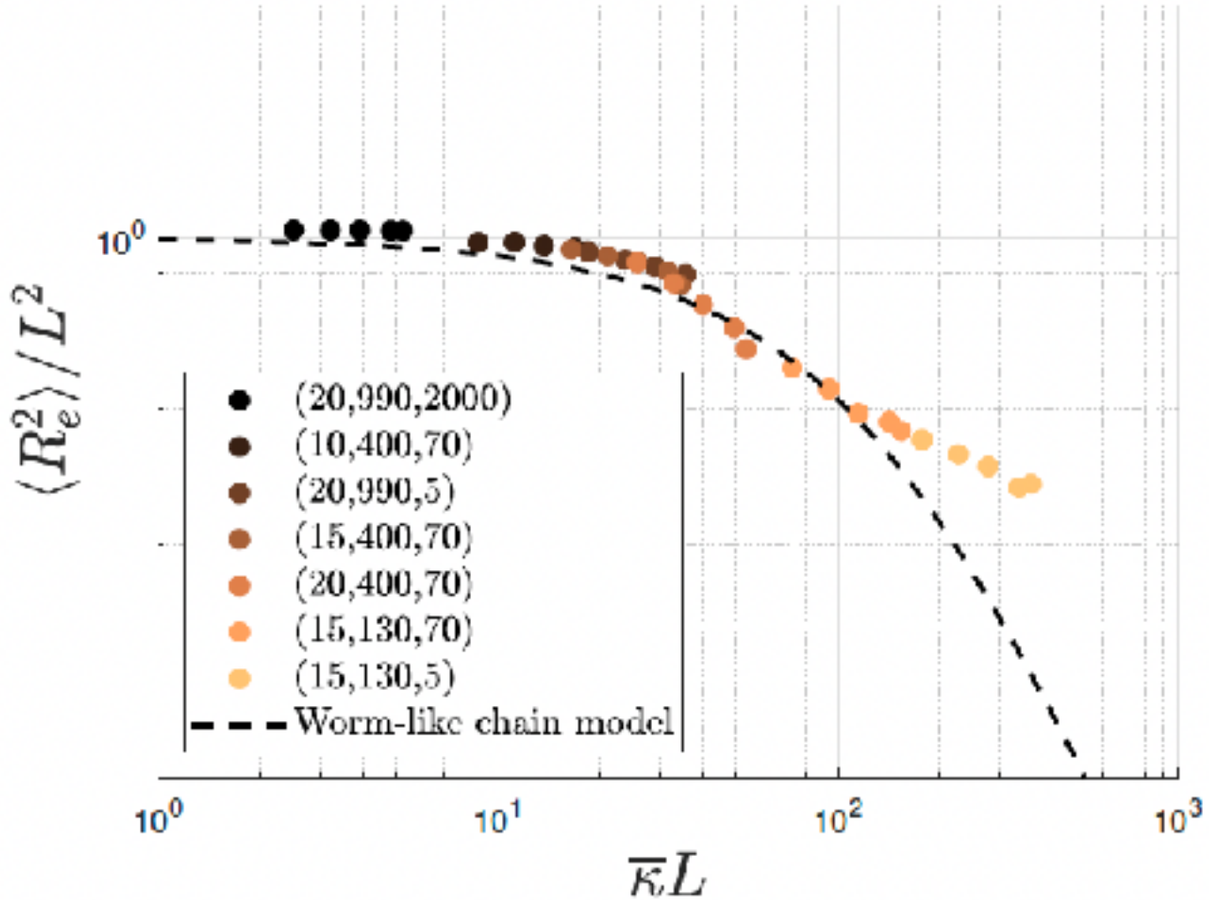


Preferential alignment

No reason to neglect tension

Conclusion

- Curvature is key to model deformation
- Deformation slower than rotation
    - Rotation rates related to the extent of the fibers
- Deformation same timescale than rotation
    - Overdamped regime: flexible fibers are not proxy of turbulence
    - Tension play a key role


$\langle R_e^2 \rangle / L^2$
10^0
(20,990,2000)
(10,400,70)
(20,990,5)
(15,400,70)
(20,400,70)
(15,130,70)
(15,130,5)
Worm-like chain model
10^0
10^1
10^2
10^3
$\overline{\kappa} L$