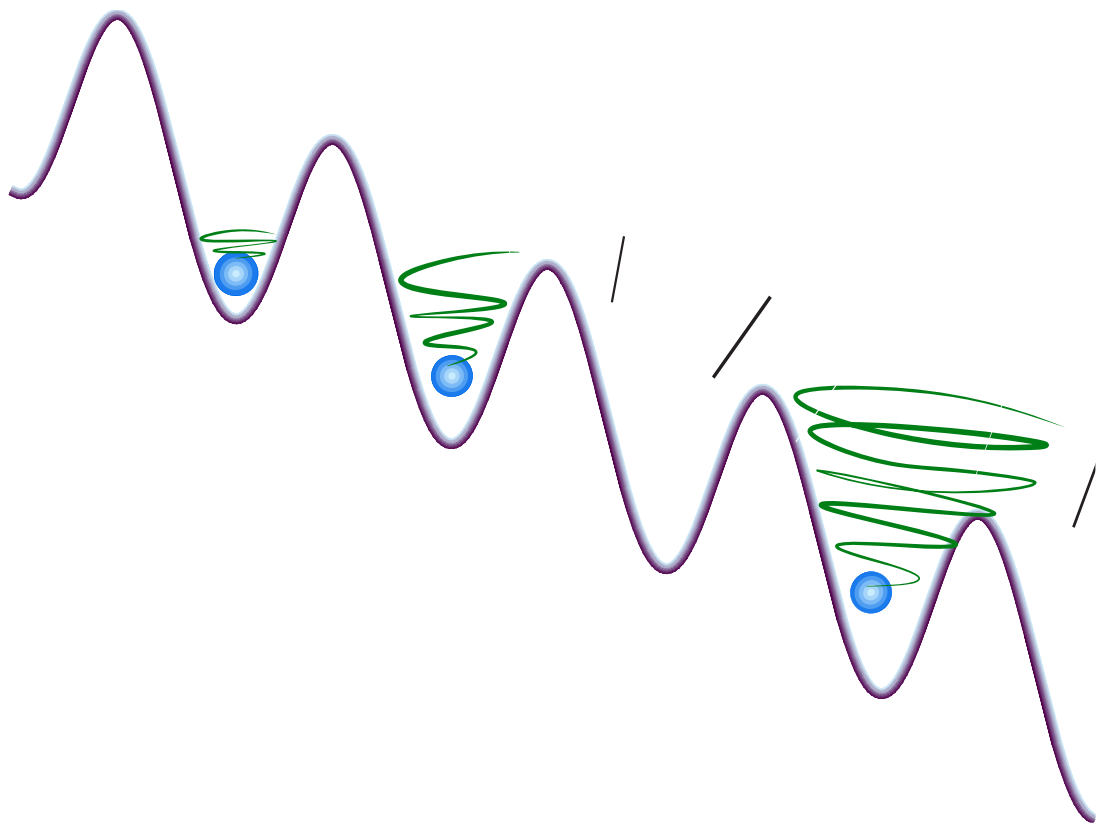
From localization to stochastic resonance: controlling particle dynamics in turbulence



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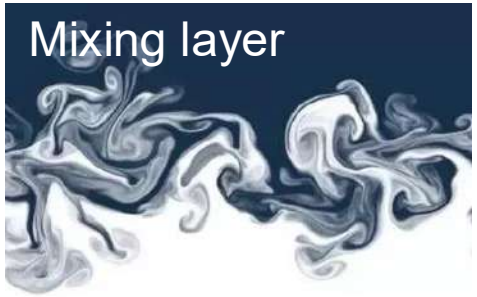
Vortex in turbulence

- Small-scale vortices exist in all kinds of turbulent flows.



- Share similar physical features (e.g., radius of the vortex)
  Independent of the nature of the flow considered
- Crucial for understanding & controlling turbulence
  Energy transfer / Mixing / Coherent structure evolution
- To capture and measure: a major conceptual and experimental challenge.

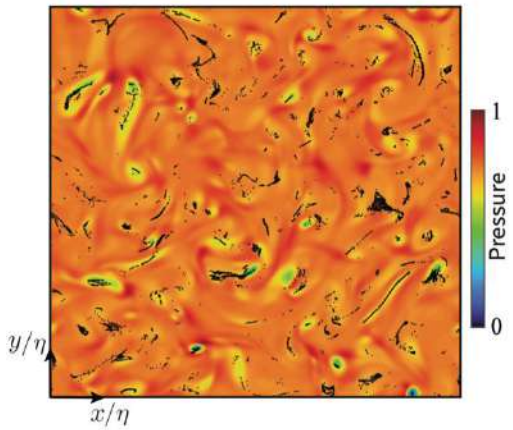
Difficult!

- Small scale
- Fast motion
- Transient (finite lifetime)

- **Question: How to measure or even control the small-scale vortices in turbulence?**

A promising candidate: **Small light** particles

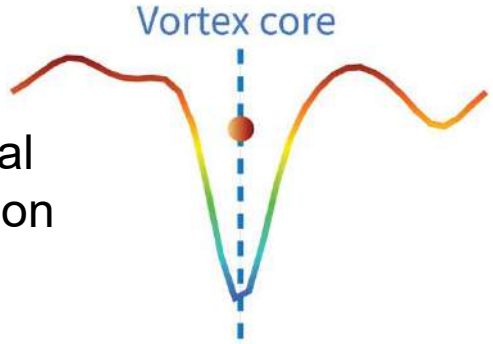
- We can add some light particles! → **Easy to "see" the vortex!**



Turbulence +
Light particles

⇒

Preferential
concentration



Vortex core: local pressure minimum

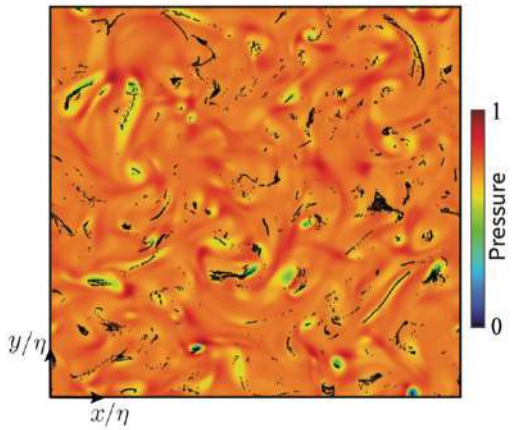
Spatial structure?
Size?

- Let's see what we can learn from nature!

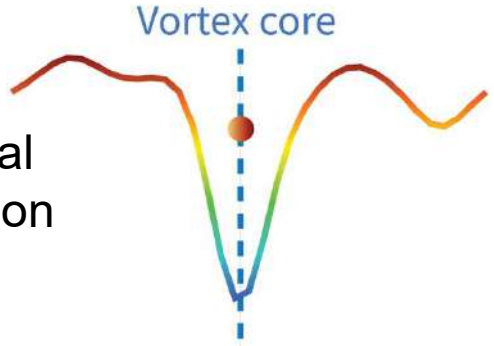
Turbulence +
Light particles +
External force

A promising candidate: **Small light** particles

- We can add some light particles! → **Easy to "see" the vortex!**



Turbulence + Light particles ⇒ Preferential concentration

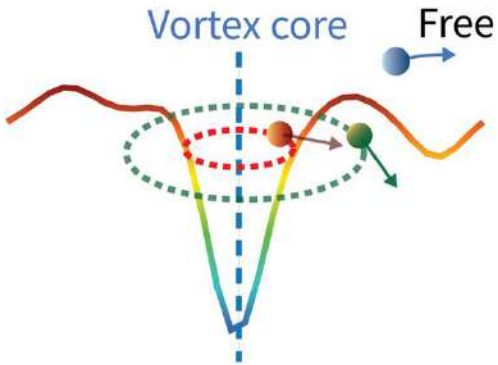


Vortex core: local pressure minimum

Spatial structure?
Size?

- Let's see what we can learn from nature!

What if we take this idea and actually *control it*?



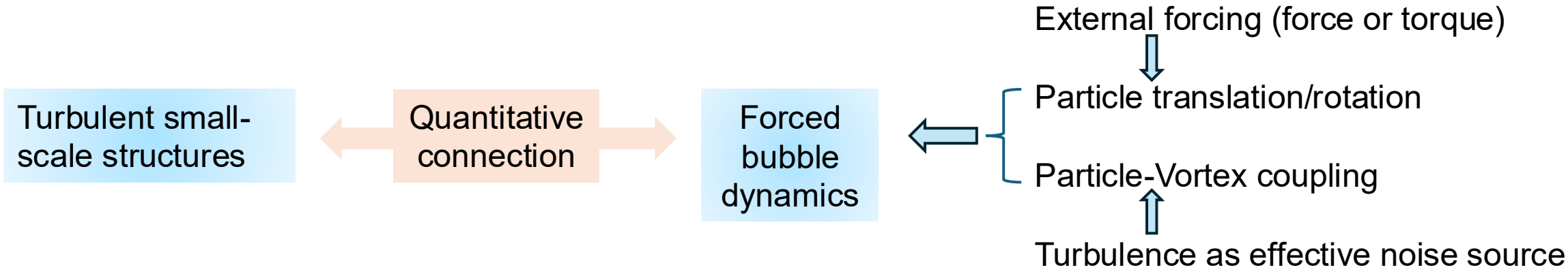
Forced light particles

Turbulent small-scale structures ⟷ Forced bubble dynamics

Forced light particles as probes?

Translational Forcing
- "Pushing the particles back and forth in space".
- Measure **position** of the particles.
- Control particle position → **Probe the pressure fluctuations** in turbulence.

Rotational Forcing
- "Applying a torque and making particles rotate".
- Measure **rotation** angles of the particles.
- Control particle rotation → **Probe the vorticity** in turbulence.



Focus of today

TOPIC 1

Oscillatory forcing
+
Translational dynamics

Localization-delocalization transition for light particles in turbulence

TOPIC 2

Rotating magnetic field
+
Rotational dynamics

Stochastic resonance of rotating particles in turbulence

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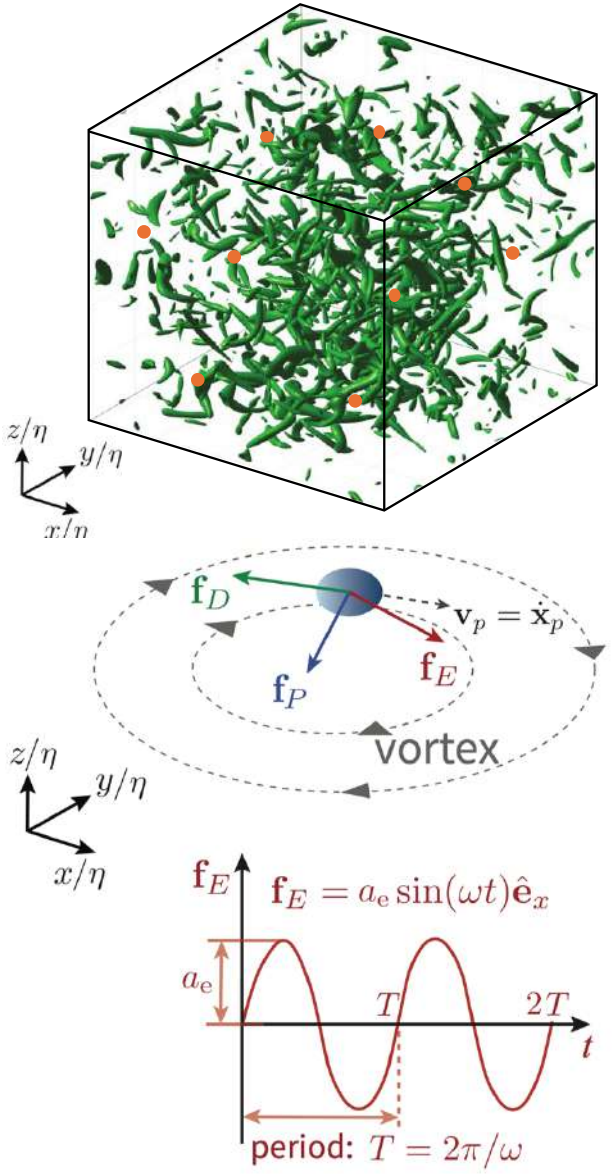
Chosen system



- Light particles in Homogeneous isotropic turbulence (HIT)
- Numerical simulation: 3D fully resolved DNS of Navier-Stokes equation.
  - Point particles (β, St)
    - $\beta = 3/(1 + 2\rho_p/\rho_f)$, ideal light particle: $\beta = 3$
    - $St = \tau_p/\tau_\eta$, strongest preferential concentration: $St = 1$
  - Forced by a periodic external force in x direction

One way coupling

- Simplified Maxey & Riley equation for small particles

$$\ddot{\mathbf{x}}_p = \beta\frac{\mathrm{D}\mathbf{u}_f}{\mathrm{D}t} - \frac{1}{\mathrm{St}}(\dot{\mathbf{x}}_p - \mathbf{u}_f) + a_\mathrm{e}\sin(\omega t)\hat{\mathbf{e}}_x$$

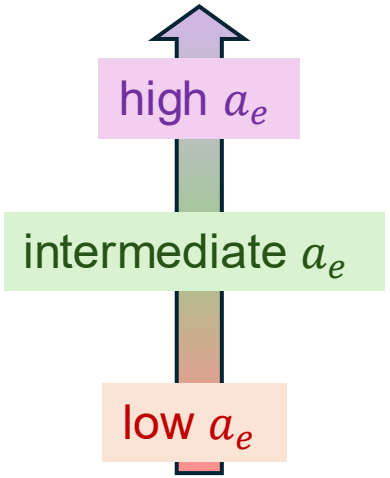
$\boldsymbol{f}_P$ Pressure force induced by vortex filaments

$\boldsymbol{f}_D$ Drag force

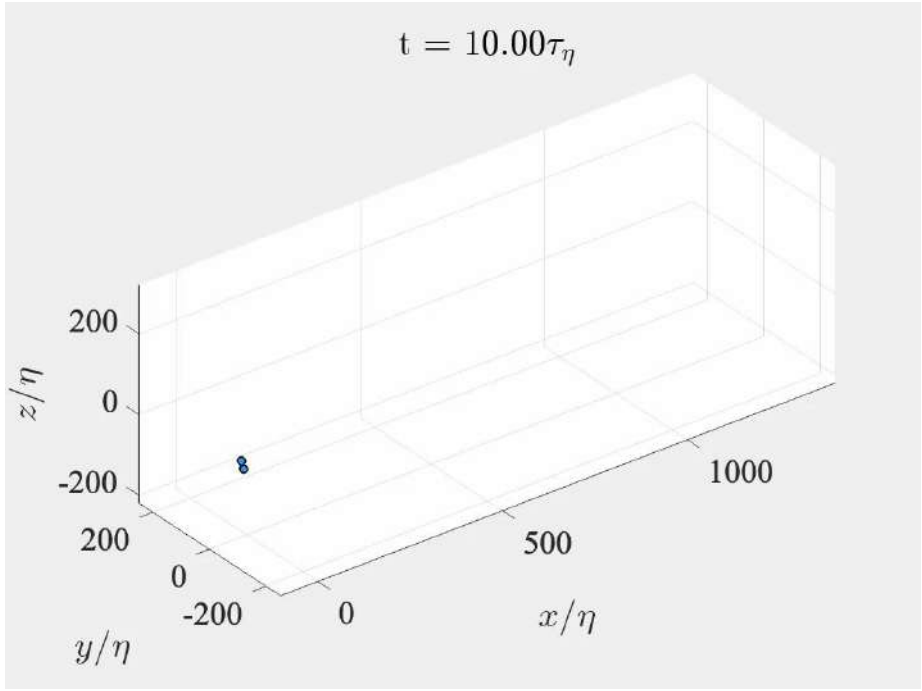
$\boldsymbol{f}_E$ Periodic external force (x direction)

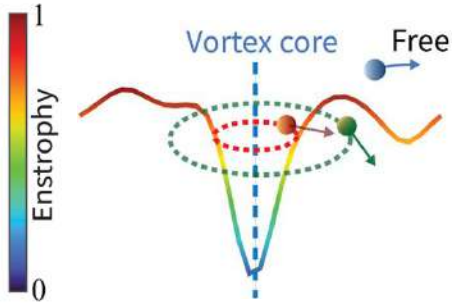
Three regimes

Three different bubble dynamic regimes as the external forcing amplitude increases.



Three regimes





high a_e

intermediate a_e

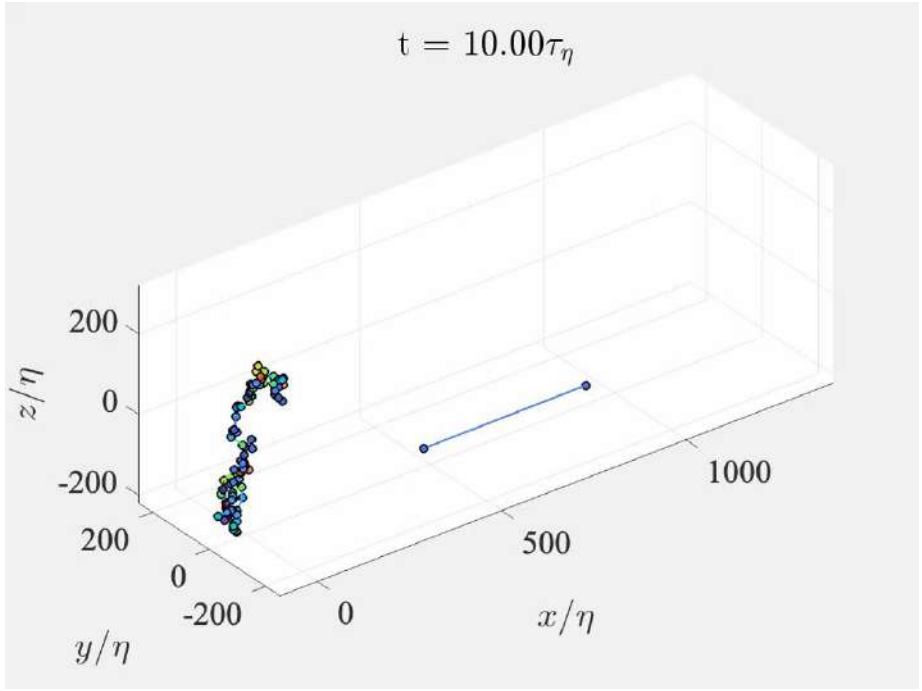
low a_e

Localized

- Drifting with vortices
- Oscillations limited in x

Vortex dominant

Three regimes



high a_e

intermediate a_e

low a_e

Delocalized

- Pronounced periodic oscillations

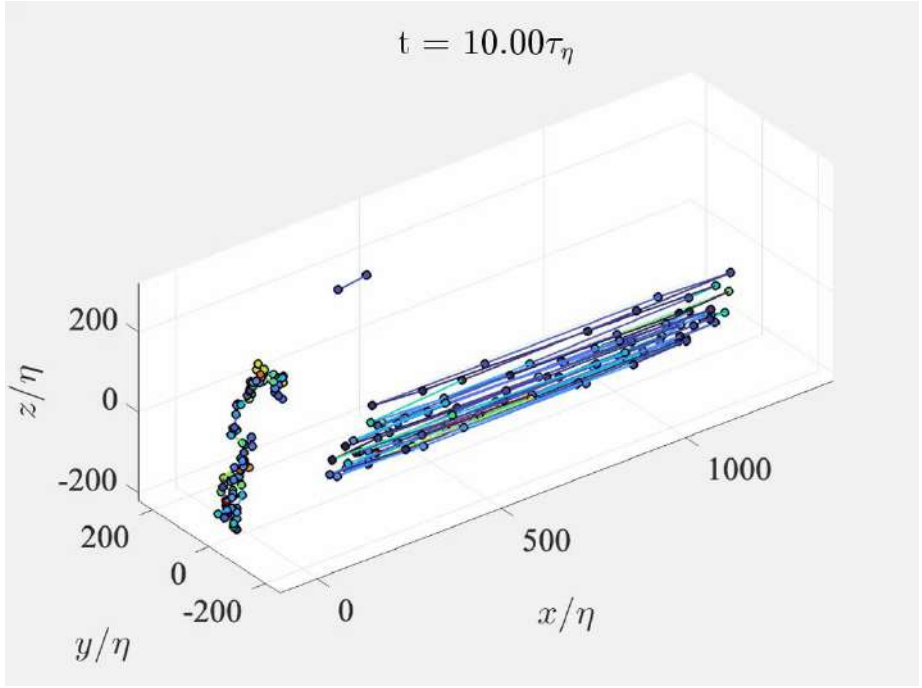
External force dominant

Localized

- Drifting with vortices
- Oscillations limited in x

Vortex dominant

Three regimes



high a_e — Delocalized
- Pronounced periodic oscillations

External force dominant

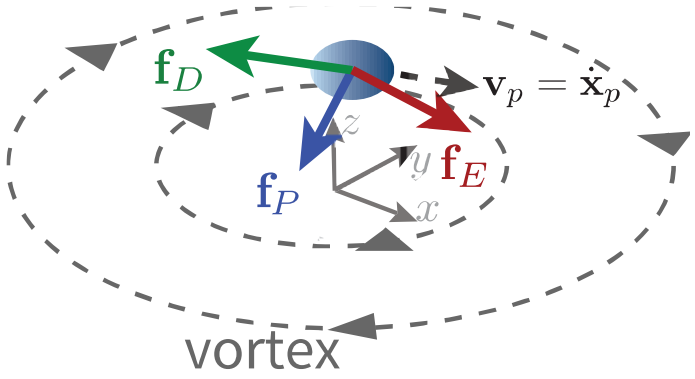
intermediate a_e — Transitional
- Combined feature of localized and delocalized regimes

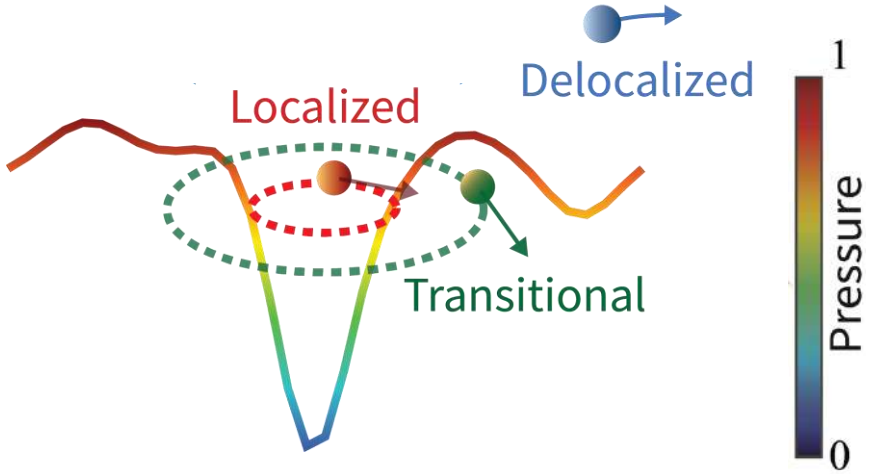
low a_e — Localized
- Drifting with vortices
- Oscillations limited in x

Vortex dominant

How to model particle dynamics?

Vortex: pressure potential well



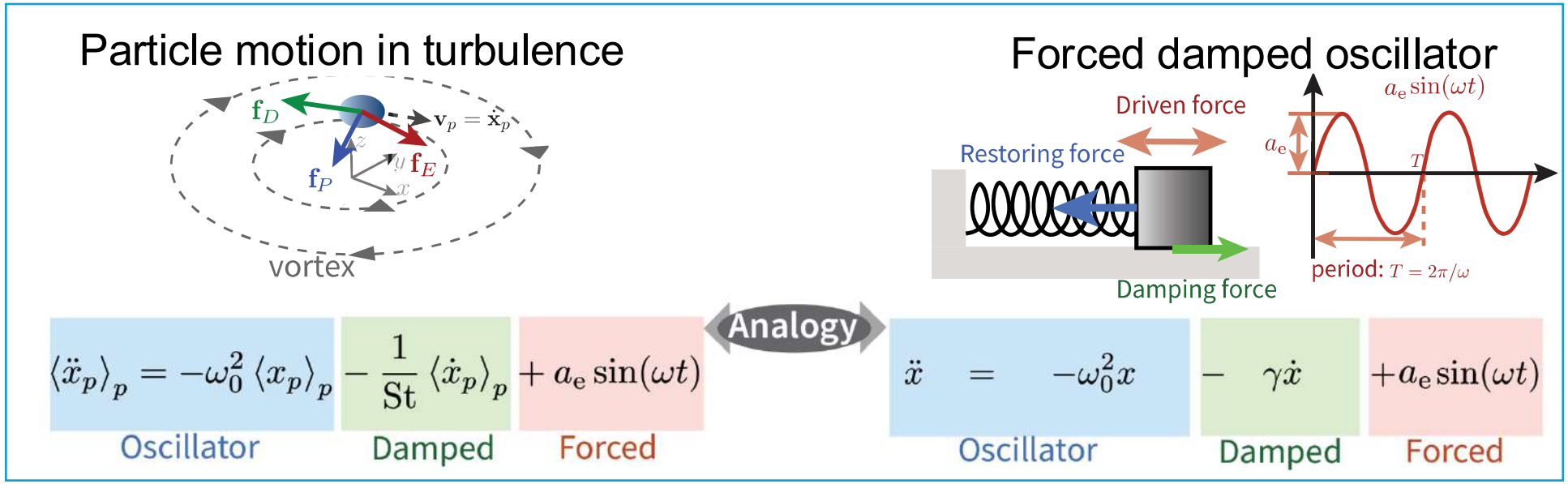


Pressure force: restoring force

The forced particles are moving under the confinement of the pressure potential well induced by the vortices in turbulence.

Model: Analogue forced damped oscillator

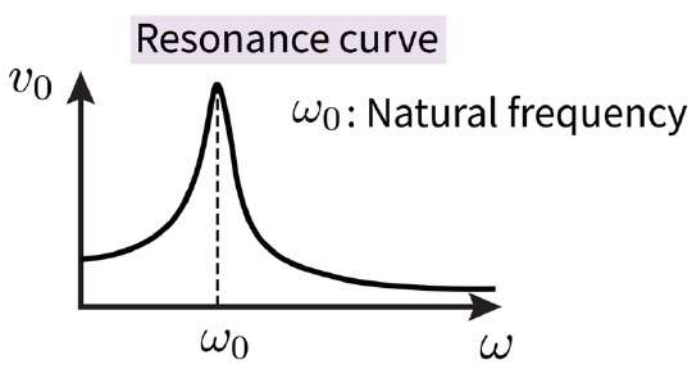
- The dynamics of the collective particle motion → a forced damped oscillator

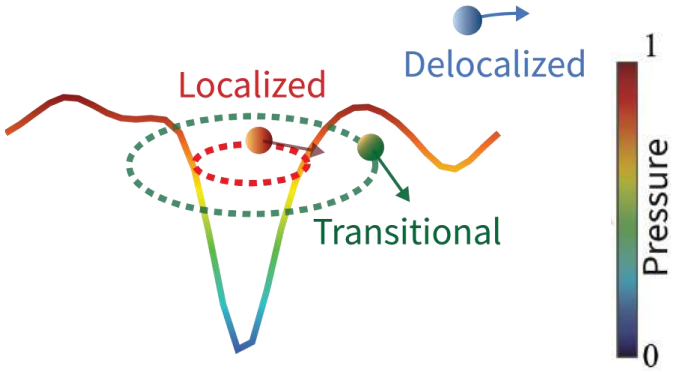


- This analogy suggests the possibility to find a resonant behavior in the collective particle dynamics

Natural frequncy: ω_0

Intrinsic property of background turbulence: the "curvature" of the pressure field.

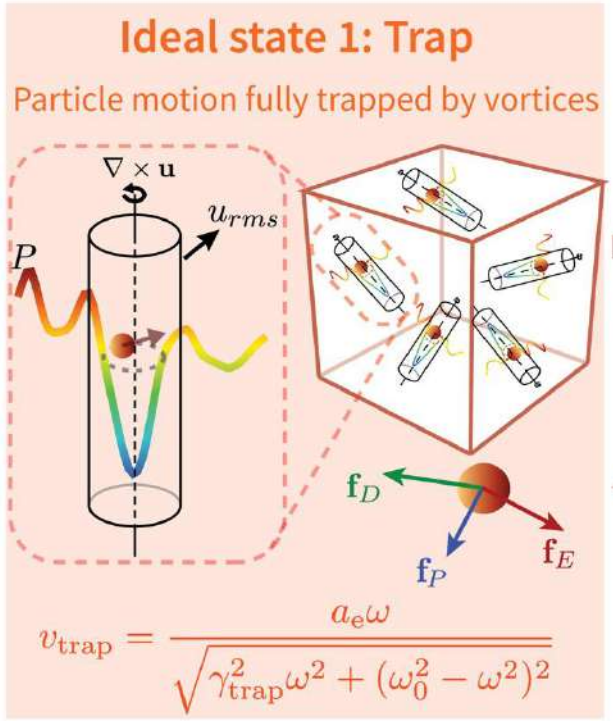




Model: Analogue forced damped oscillator

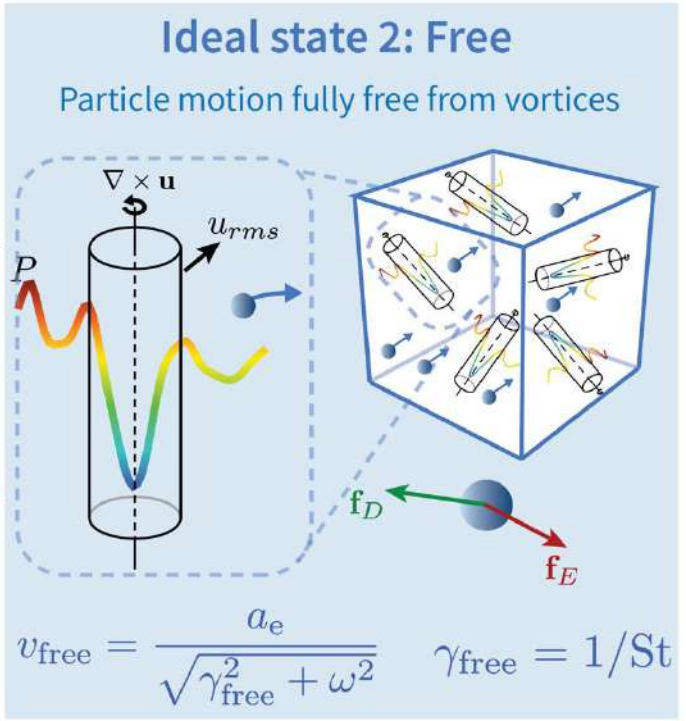
$$\langle \ddot{x}_p \rangle_p = -\omega_0^2 \langle x_p \rangle_p - \frac{1}{\mathrm{St}} \langle \dot{x}_p \rangle_p + a_e \sin(\omega t)$$

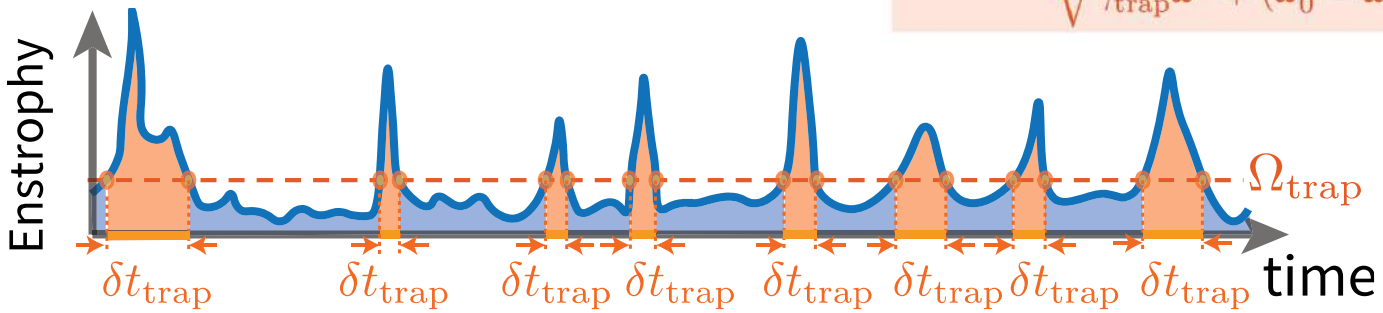
Oscillator Damped Forced



Vortex beakup

Meet vortex

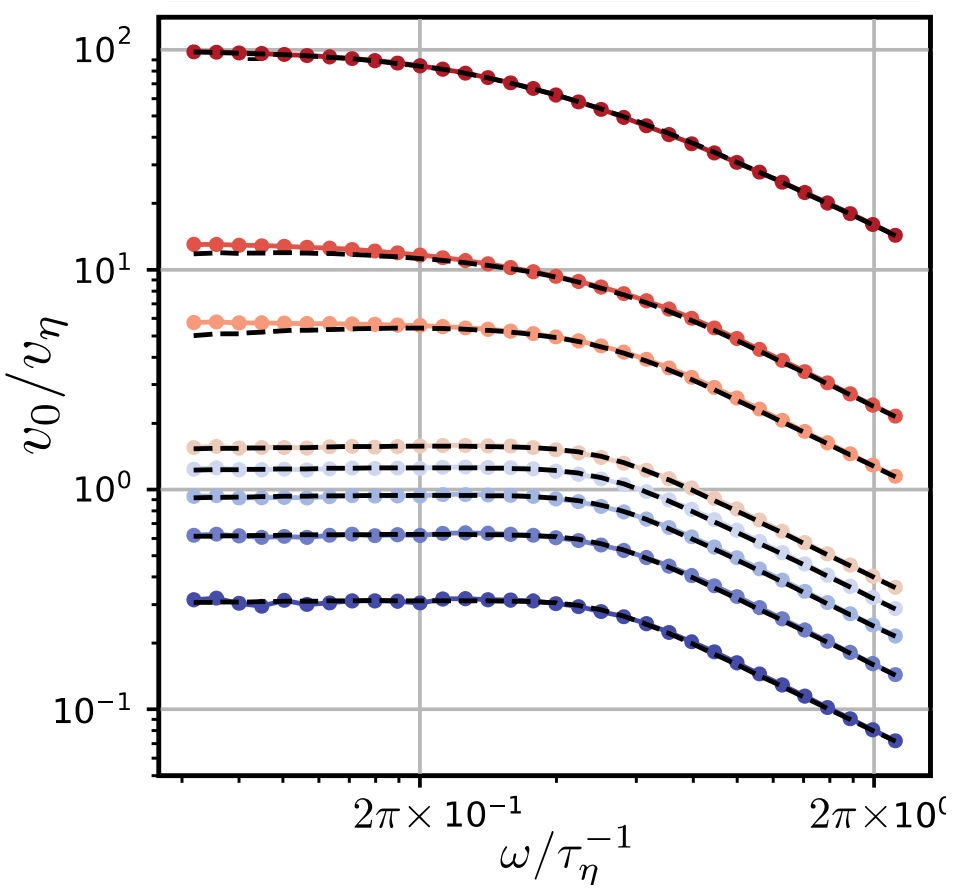




$$v_0^s = r v_{\mathrm{trap}} + (1 - r) v_{\mathrm{free}}$$

More information: Ziqi Wang, et al, *PNAS,* 121.38 (2024): e2405459121.

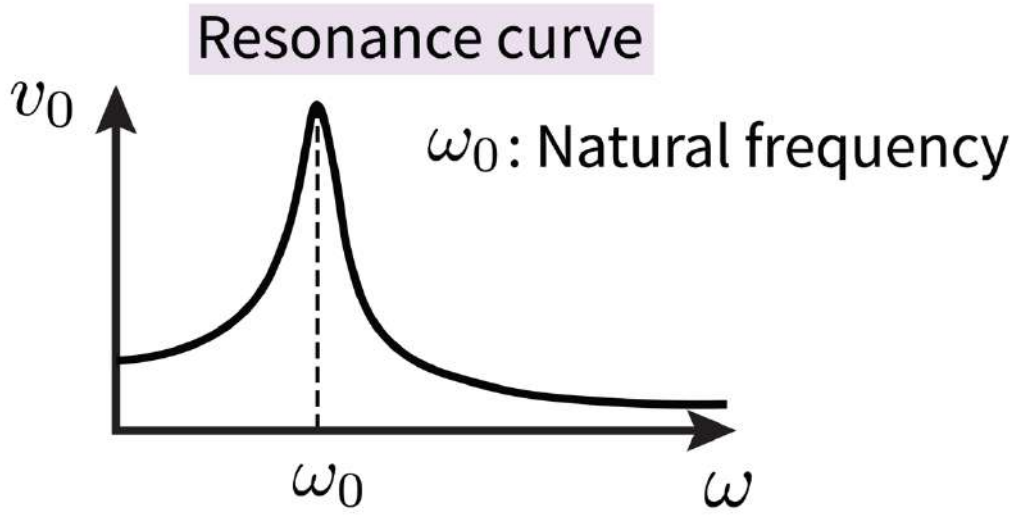
Model & Simulation: v_0/v_η v.s. ω



- An excellent agreement!

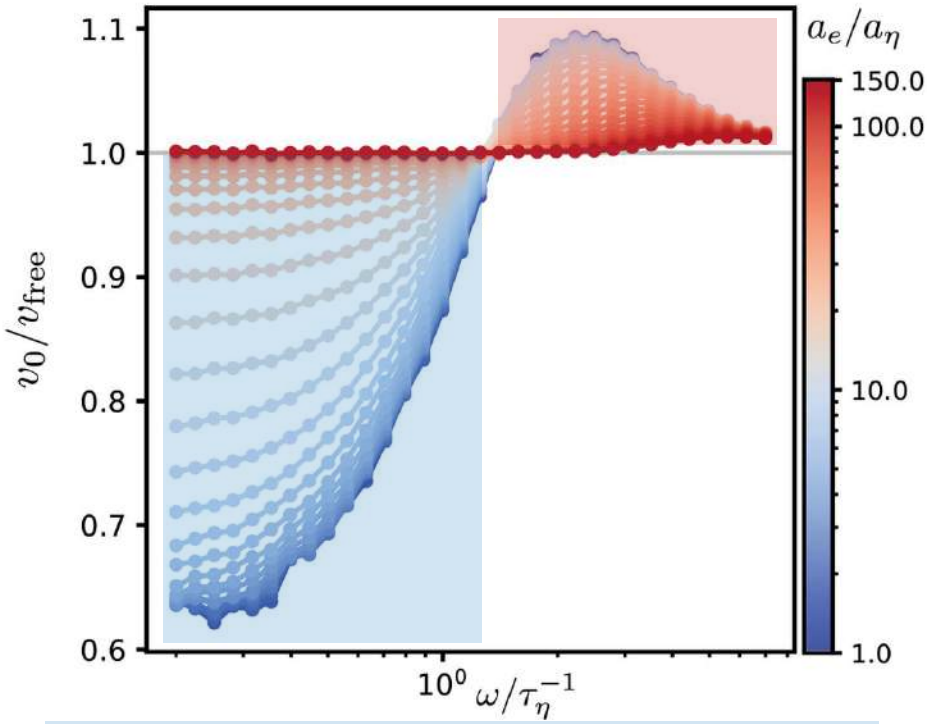
What about resonance?

Is there a resonance between the particles and the vortex?



Resonance —— forced particles & vortex

v_0/v_{free} v.s. ω



Normalize v_0 by the free fall velocity v_{free}

What determines the natural frequency ω_0?

Small forcing frequency:

$$\frac{v_0}{v_{\text{free}}} < 1$$

- Decreased mobility
- Particle trapping

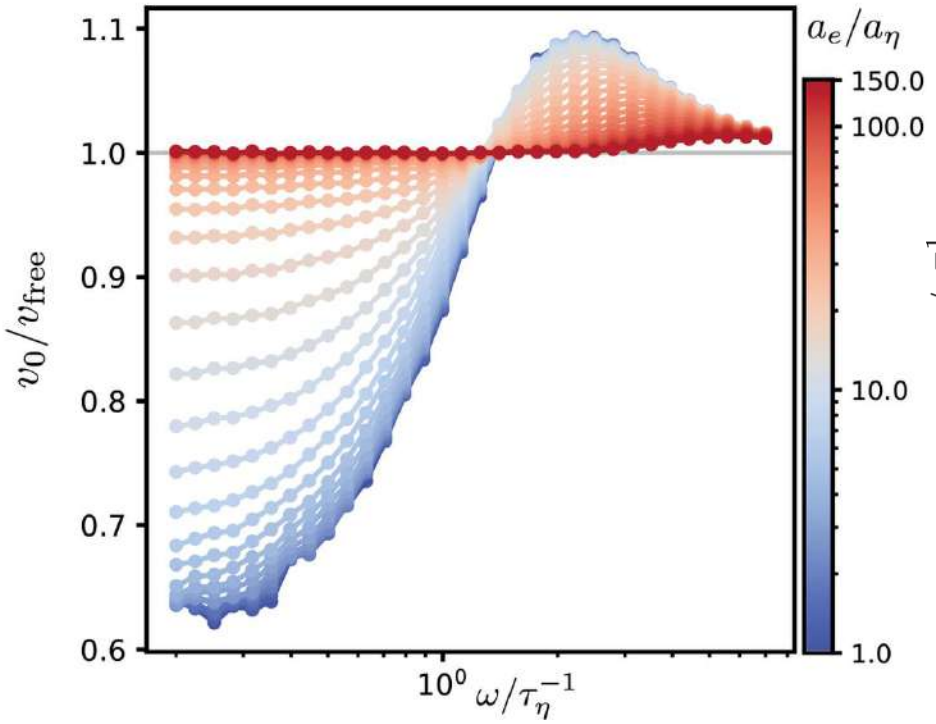
Around resonance frequency:

$$\frac{v_0}{v_{\text{free}}} > 1$$

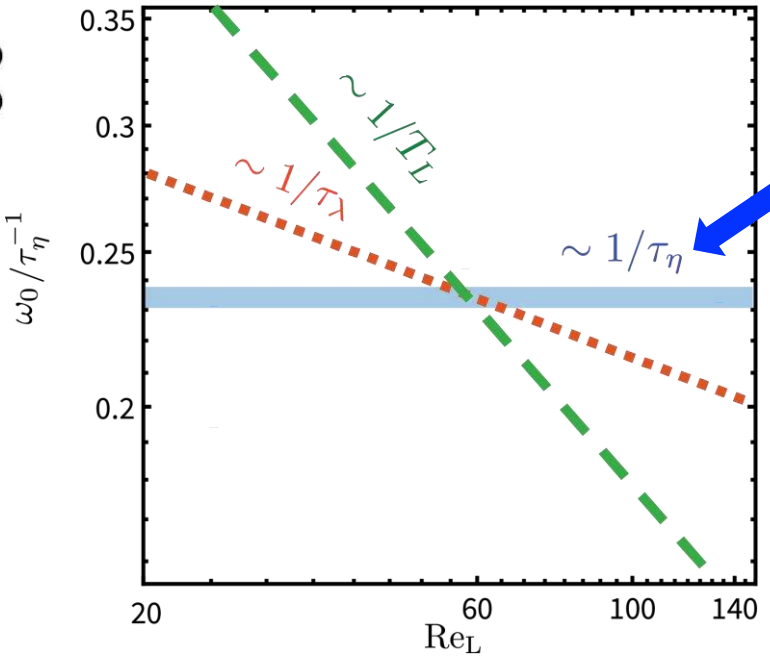
- Enhanced mobility
- Resonance

Resonance —— Reynolds number dependence of ω_0

v_0/v_{free} v.s. ω



Re v.s. ω_0



ω_0 scales with Kolmogorov frequency?

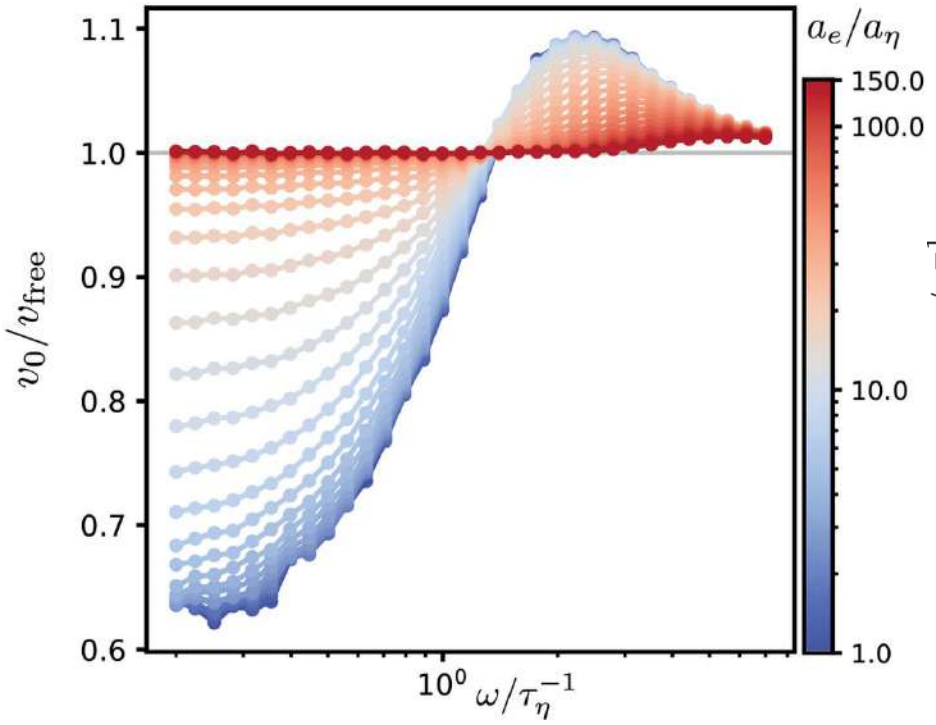
Three characteristic time scales related to the turbulence

- Integral time scale: T_L
- Taylor micro scale: τ_λ
- Kolmogorov scale: τ_η

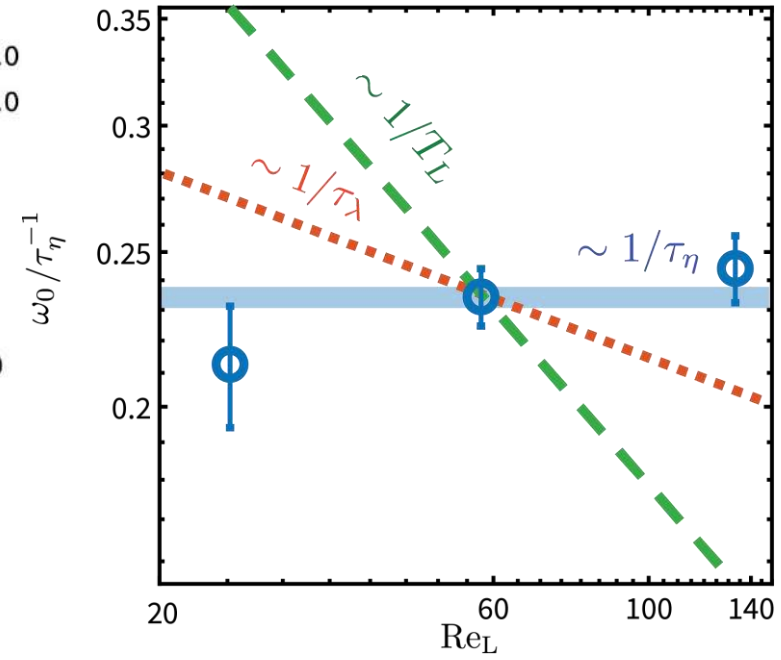
→ Three different frequency scales

Resonance —— Reynolds number dependence of ω_0

v_0/v_{free} v.s. ω



Re v.s. ω_0



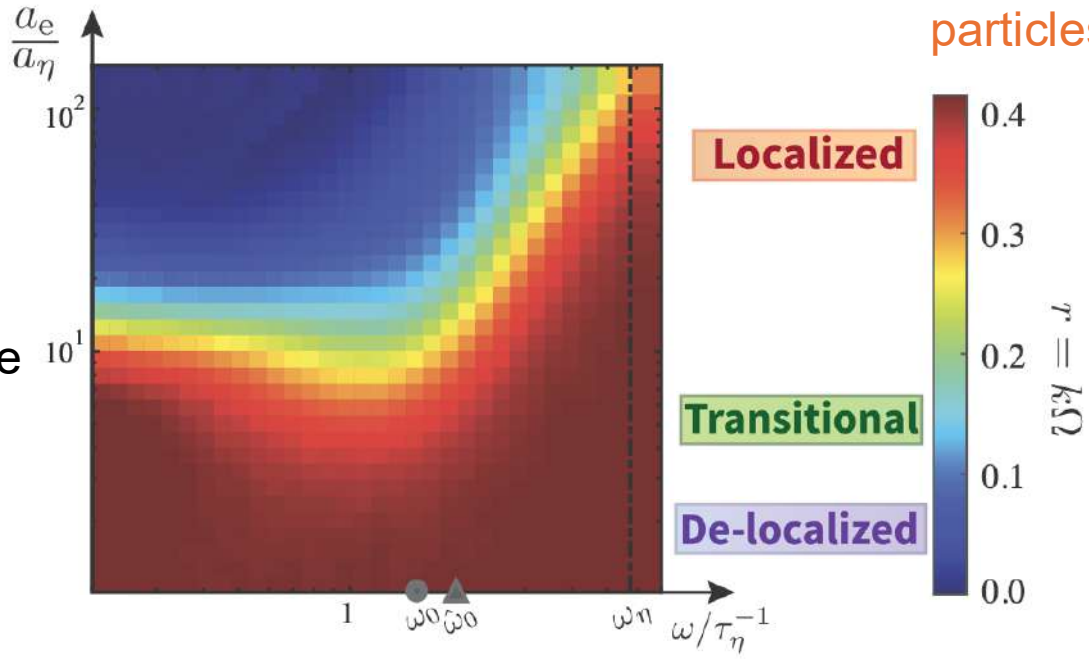
YES!

- ω_0 scales most closely as the Kolmogorov frequency $\omega_\eta \sim 1/\tau_\eta$.
- Physically: ω_0 is related to the **effective depth (curvature)** of the potential well.

Time to sum up all the rich phenomena in one **phase diagram**!

Phase diagram —— summary of simulation results

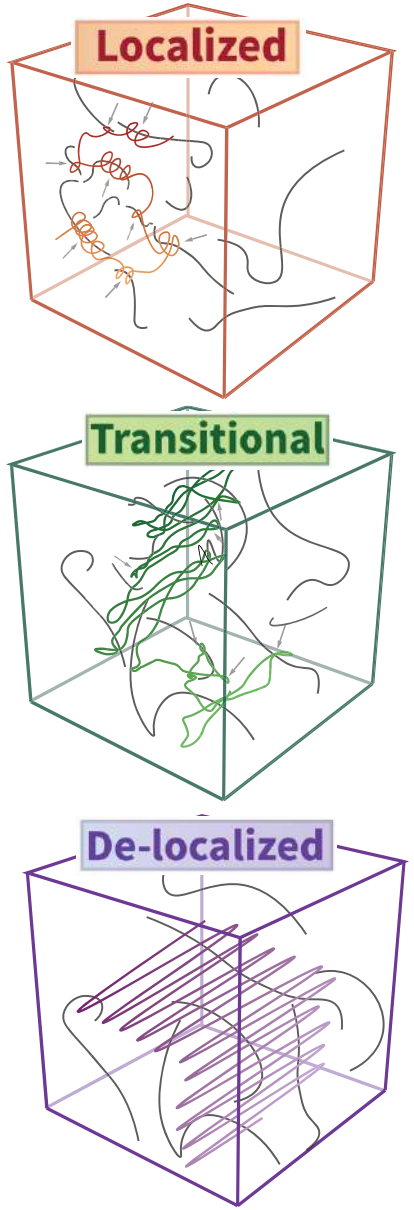
r: fraction of trapped particles



Forcing amplitude

Forcing frequency

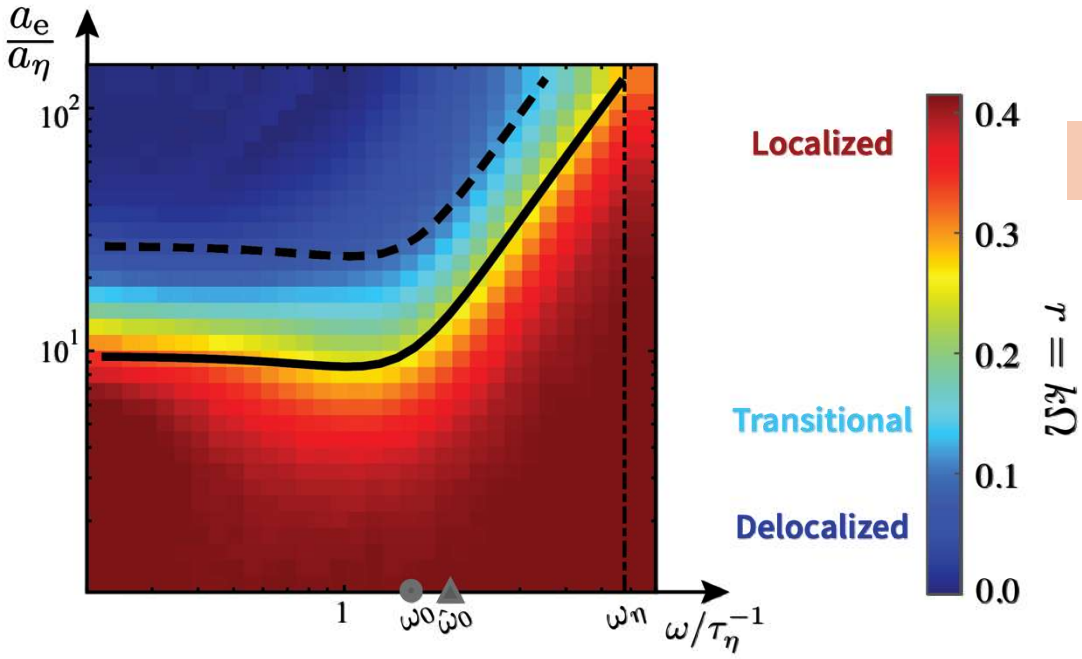
Quantitative classification?

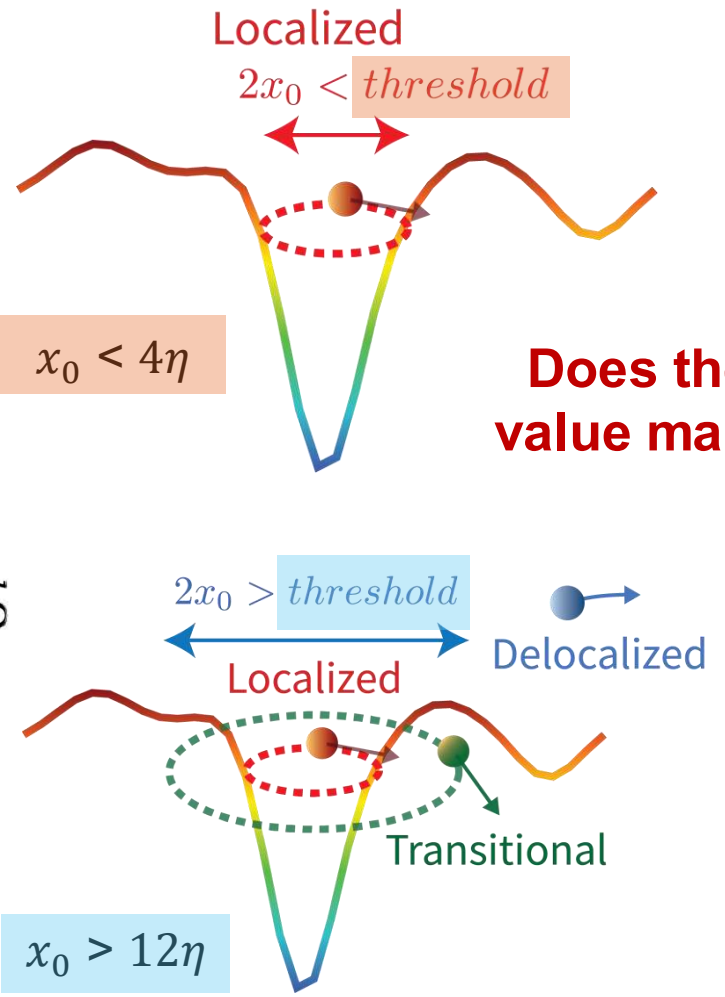


YES!!

We can do more by the **analogue model**!

Phase diagram —— prediction from the analogue model



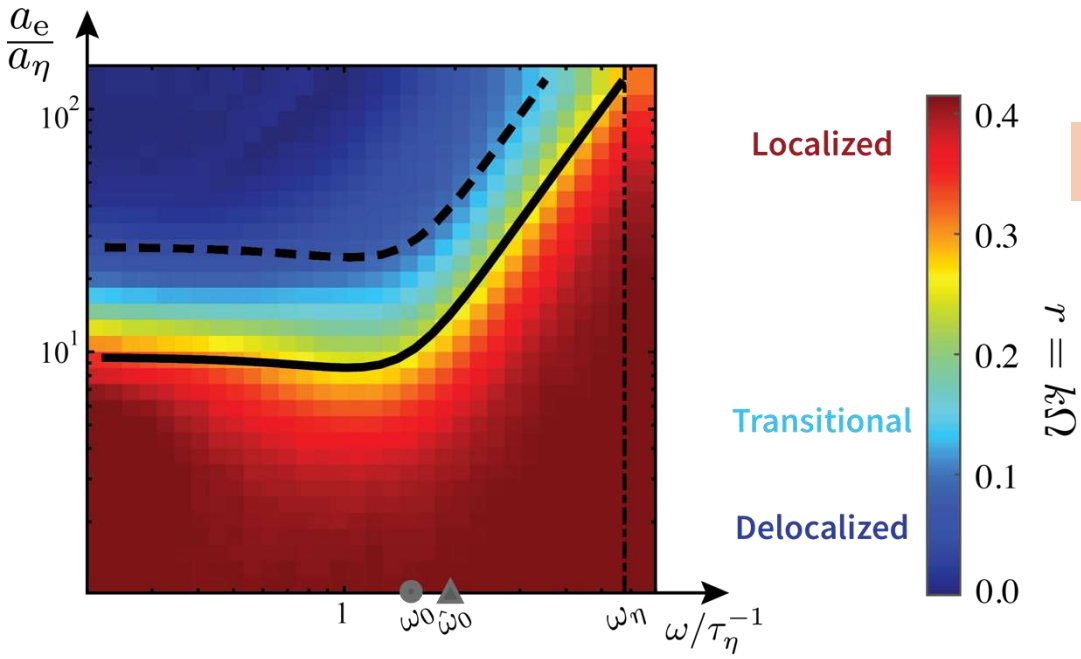


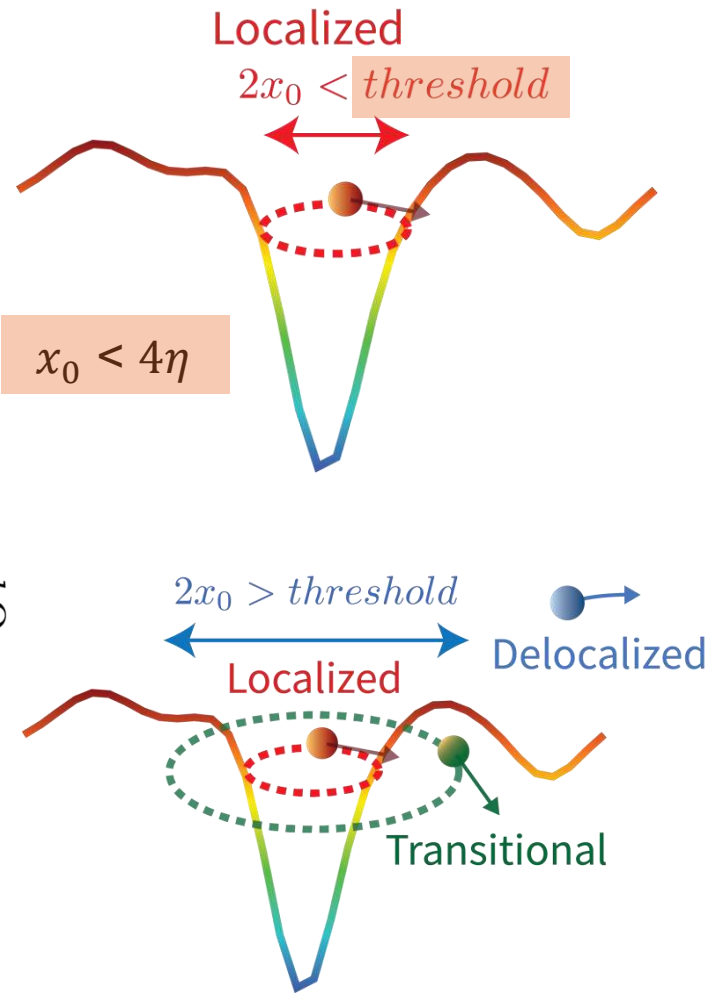
Does the probed value make sense?

- Physically: the **effective radius** of the vortex.

Probe the characteristics of turbulent small scale structures

Phase diagram —— prediction from the analogue model





- Physically: the **effective radius** of the vortex.
- Comparable to other research results $(4\eta \sim 5\eta)$ by identification and statistical analysis of vortex structures[1-3]

1. A Ghira, G Elsinga, C Da Silva, Phys. Rev. Fluids 7, 104605 (2022). 2. M Tanahashi, S Iwase, T Miyauchi, J. Turbul. 2, 006 (2001).
3. C Da Silva, RJ Dos Reis, JC Pereira, J. Fluid Mech. 685, 165–190 (2011)

Summary of topic 1

- Three distinct regimes: localized, transitional, and delocalized.
- A resonance phenomenon is observed (ω_0 scales closely as $1/\tau_\eta$).
- Forced bubbles can be used as probes for small scale structures in turbulence.

More information:

Wang, Ziqi, Xander M. de Wit, and Federico Toschi. "Localization–delocalization transition for light particles in turbulence." PNAS, 121.38 (2024): e2405459121.

Focus of today

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Oscillatory forcing
+
Translational dynamics

Localization-delocalization transition
for light particles in turbulence

TOPIC 2

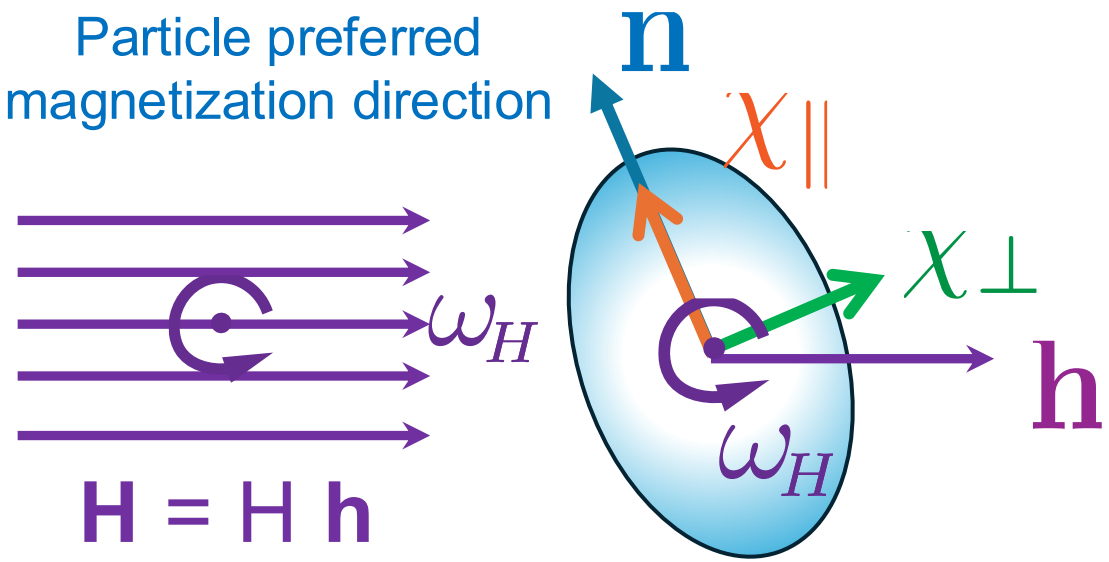
Rotating magnetic field
+
Rotational dynamics

Stochastic resonance of rotating particles in turbulence

Small light magnetic particles

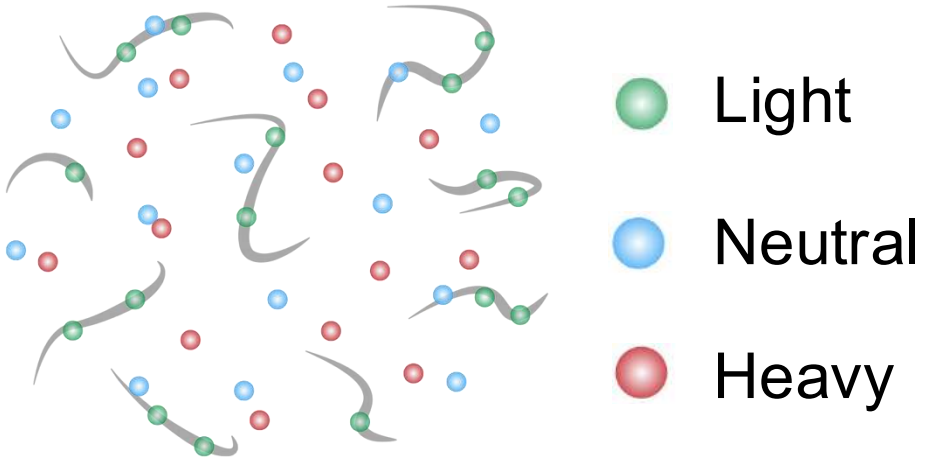
Easy to control

Magnetic field → Control → Rotational dynamics

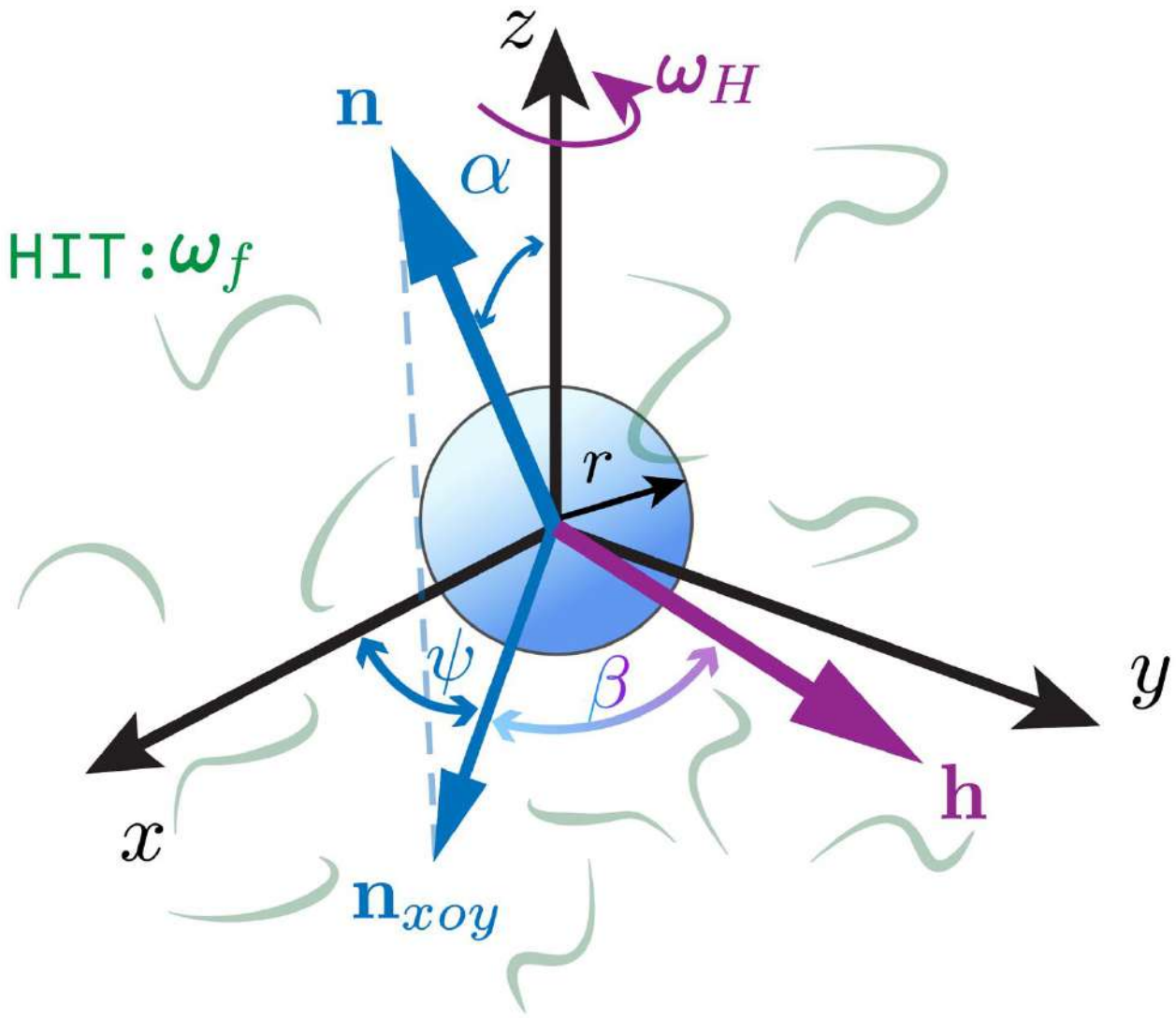


Preferential concentration

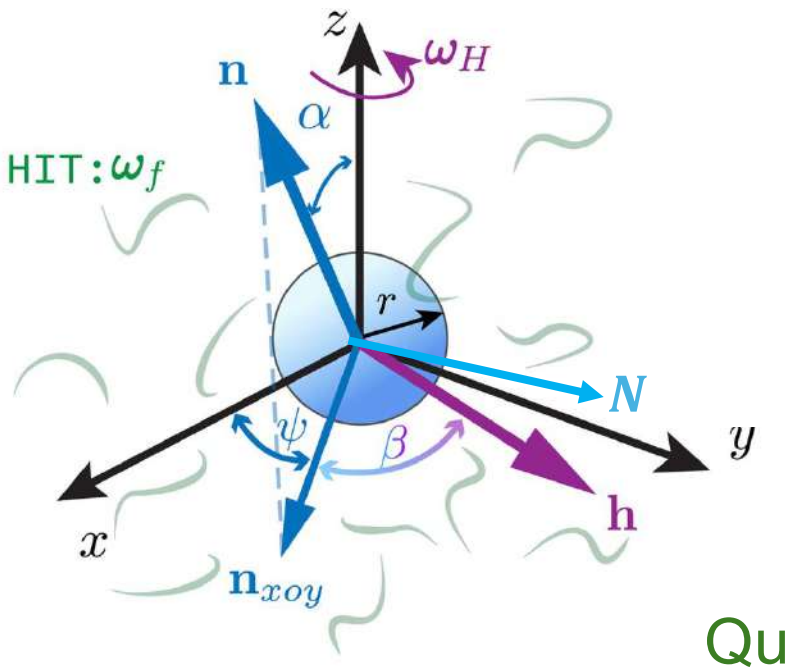
Light particle → Explore → High vorticity



Theoretical representation



Theoretical representation——Expressed explicitly in angle



$$\boldsymbol{n} = (\sin\alpha\cos\psi, \sin\alpha\sin\psi, \cos\alpha)^T$$

Torque balance
Overdamped limit

$$\boldsymbol{T}_M \cdot \hat{\boldsymbol{z}} + \boldsymbol{T}_D \cdot \hat{\boldsymbol{z}} = 0$$
$$\boldsymbol{T}_M \cdot \boldsymbol{N} + \boldsymbol{T}_D \cdot \boldsymbol{N} = 0$$

Simplify

Magnetic field rotation drag

Turbulence torque

Quiescent fluid case

$$\dot{\beta} = \omega_H - \frac{\omega_a}{2}\sin 2\beta - \omega_{f,z}$$

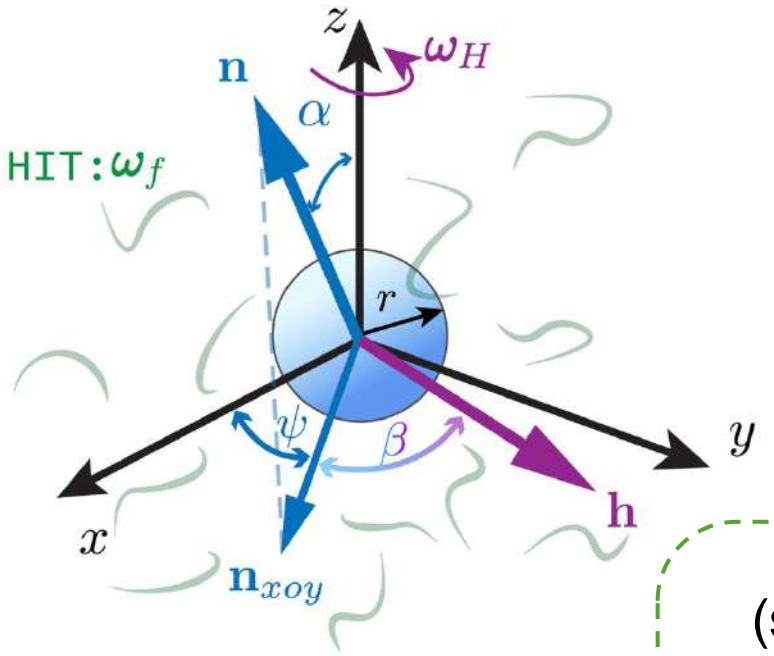
Magnetic torque

$$\omega_a = \mu \mathcal{V}_p H^2 (\chi_{\parallel} - \chi_{\perp})/\xi_r$$

Magnetic field strength

Anisotropy in magnetic susceptibility

Theoretical representation——Quiescent fluid case



Magnetic field rotation drag

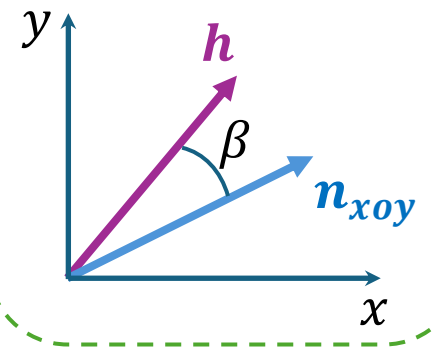
$$\dot{\beta} = \omega_H - \frac{\omega_a}{2}\sin 2\beta$$

Magnetic torque

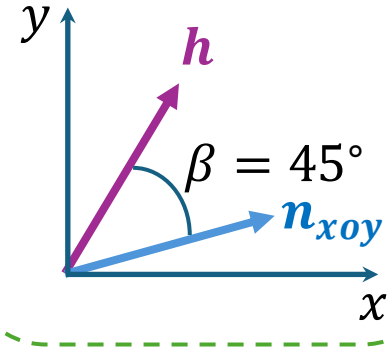
Critical frequency: $\omega_{cr} = \frac{\omega_a}{2}$

Critical ratio: $\left[\frac{\omega_H}{\omega_a}\right]_{cr} = \frac{1}{2}$

$\omega_H < \omega_{cr}$
(sub-critical)
↓
"phase locked"



$\omega_H = \omega_{cr}$
(critical)

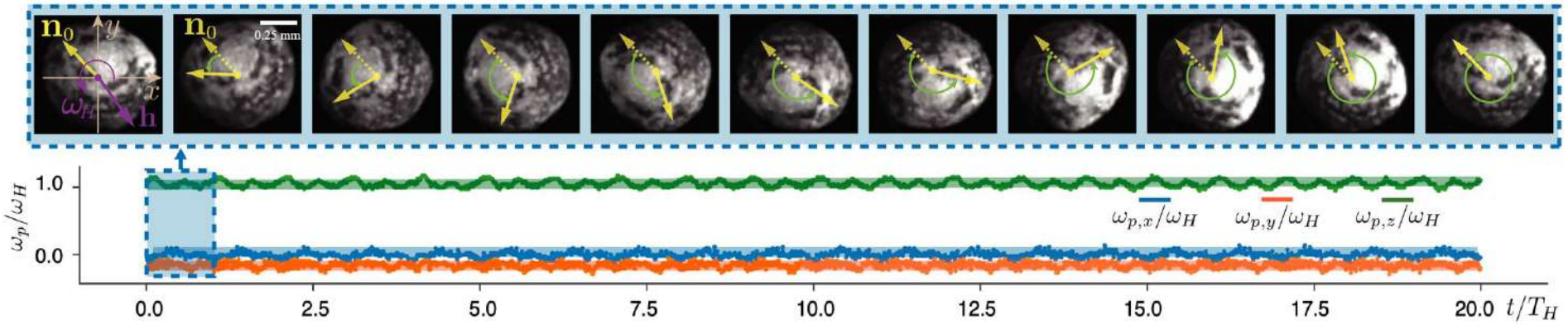


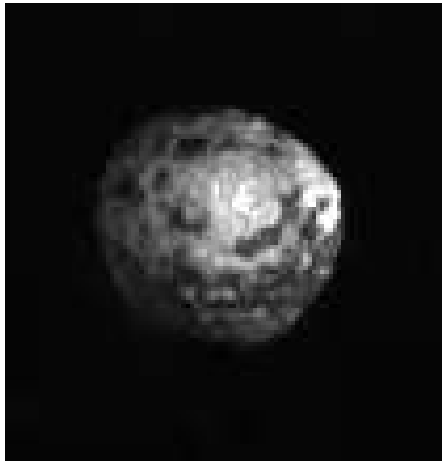
$\omega_H > \omega_{cr}$
(super-critical)
↓
"back and forth"

Magnetic torque cannot balance the response drag torque: β changes periodically over time

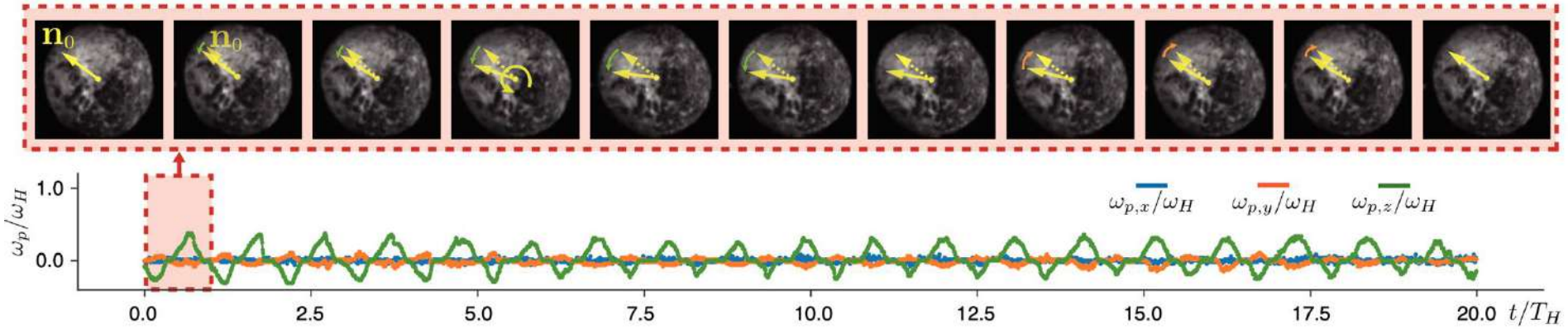
Quiescent fluid case——Experiments

➢ “Phase-locked” regime



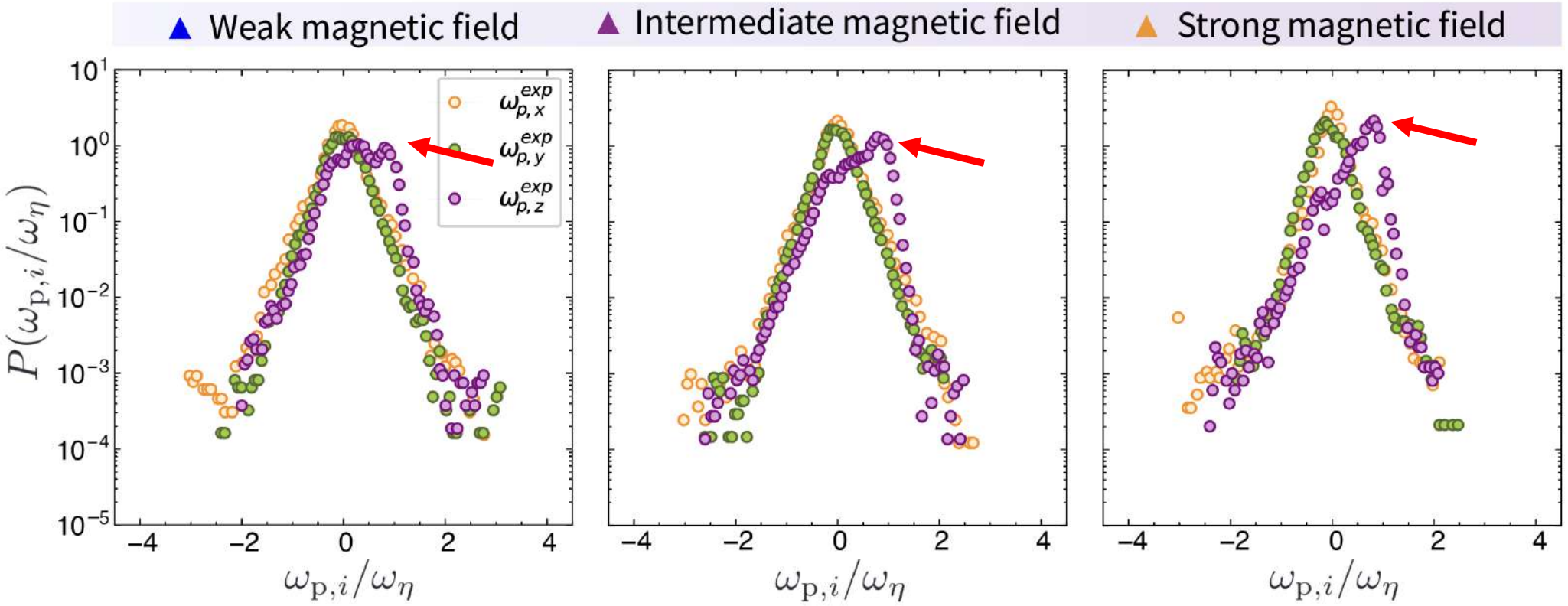


➢ “Back-and-forth” regime



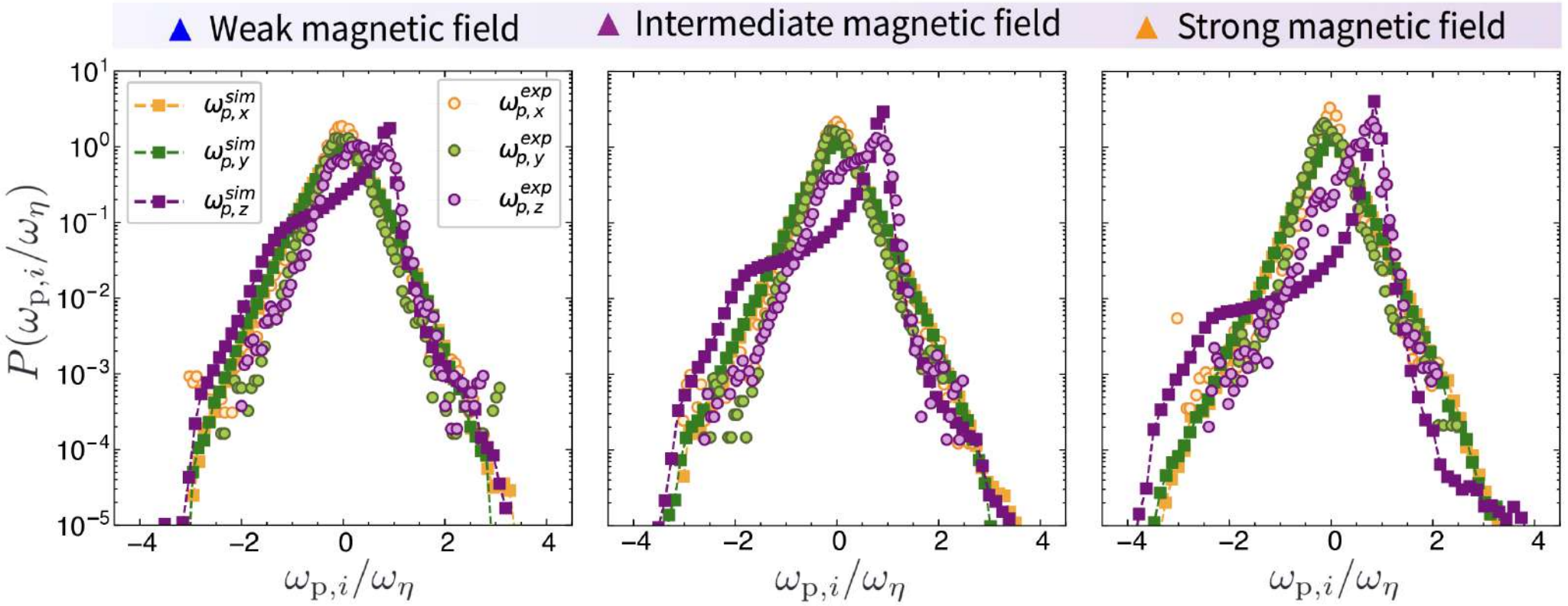
Switching on turbulence——Experiments

PDF of the particle angular velocity



- A dominant peak in the z-direction: $\omega_{p,z} \approx \omega_H$.

Switching on turbulence——Experiments + Simulations



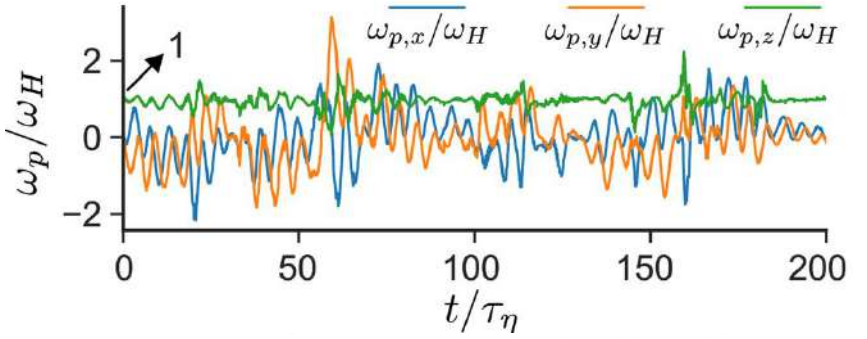
- **Good agreement** on the peak positions!

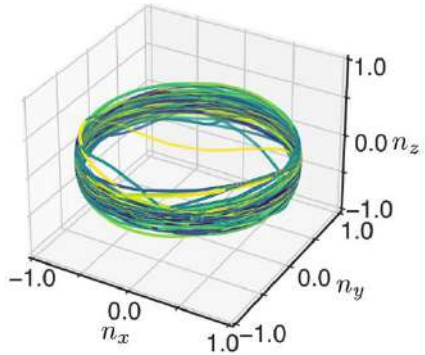
Theoretical model → Capture → Main physics

Three regimes!

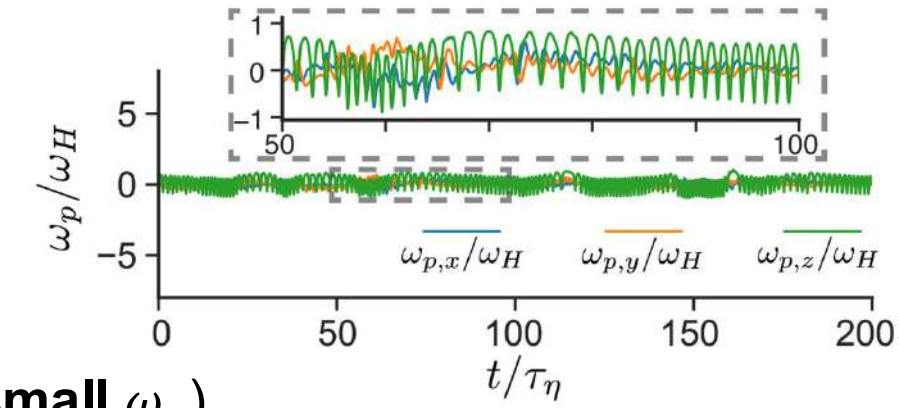
- **Strong** magnetic field intensity (**large** ω_a)

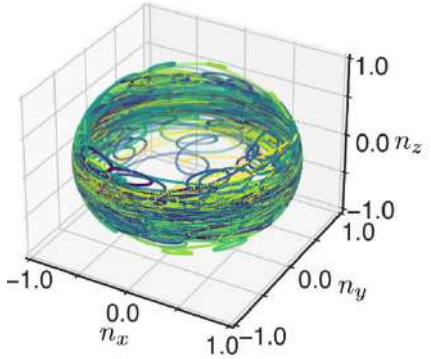
Phase-locked
(small ω_H)

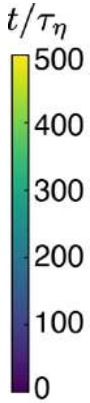




Back-and-forth
(large ω_H)

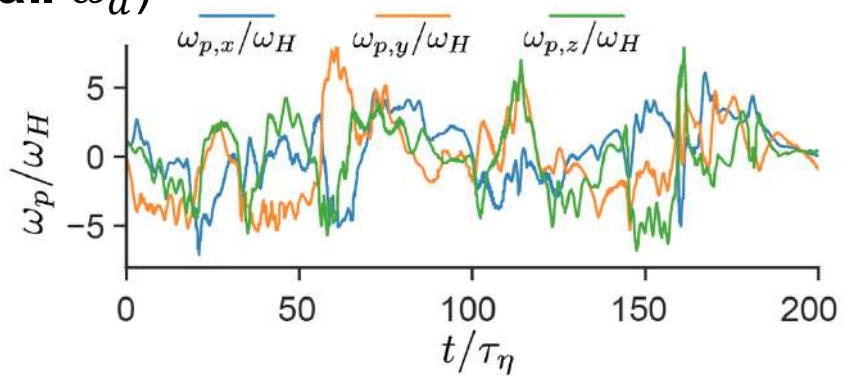


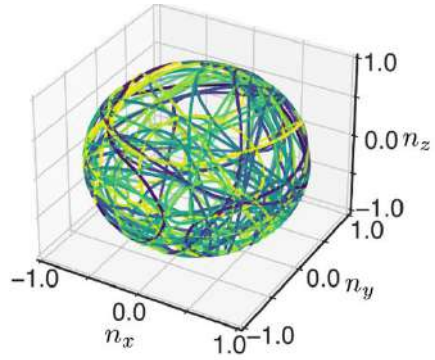




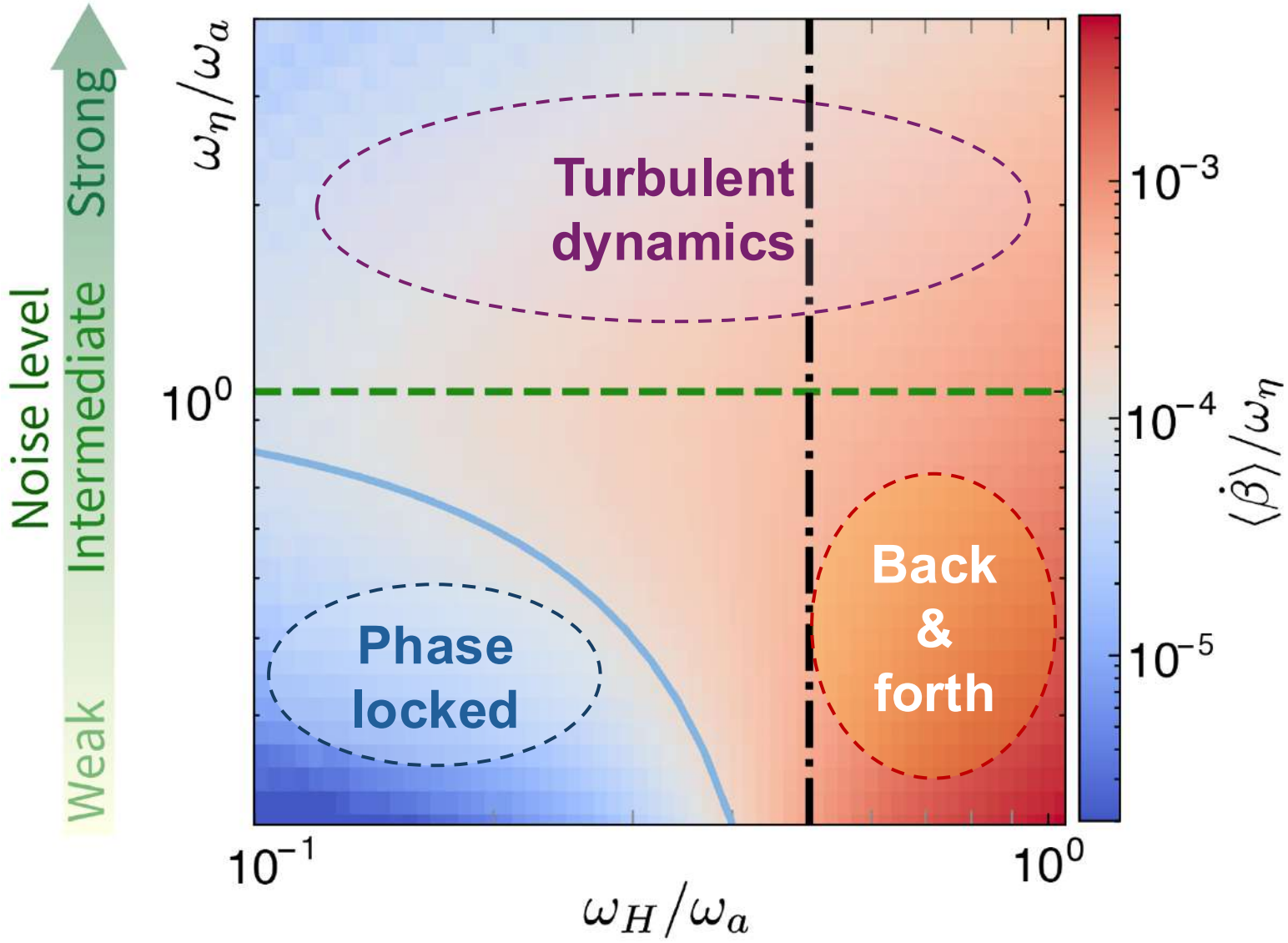
- **Weak** magnetic field intensity (**small** ω_a)

Turbulent dynamics

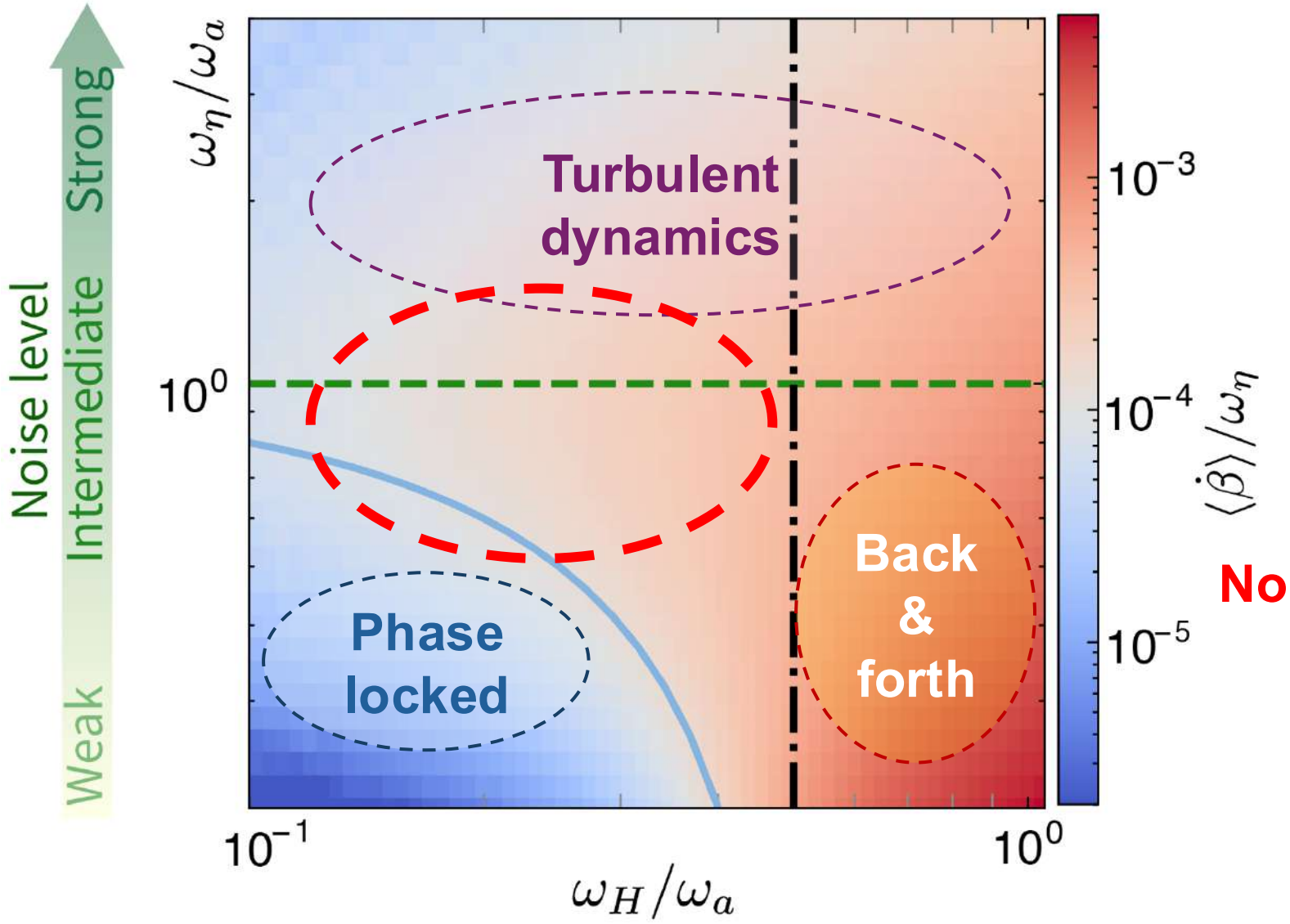




Phase diagram

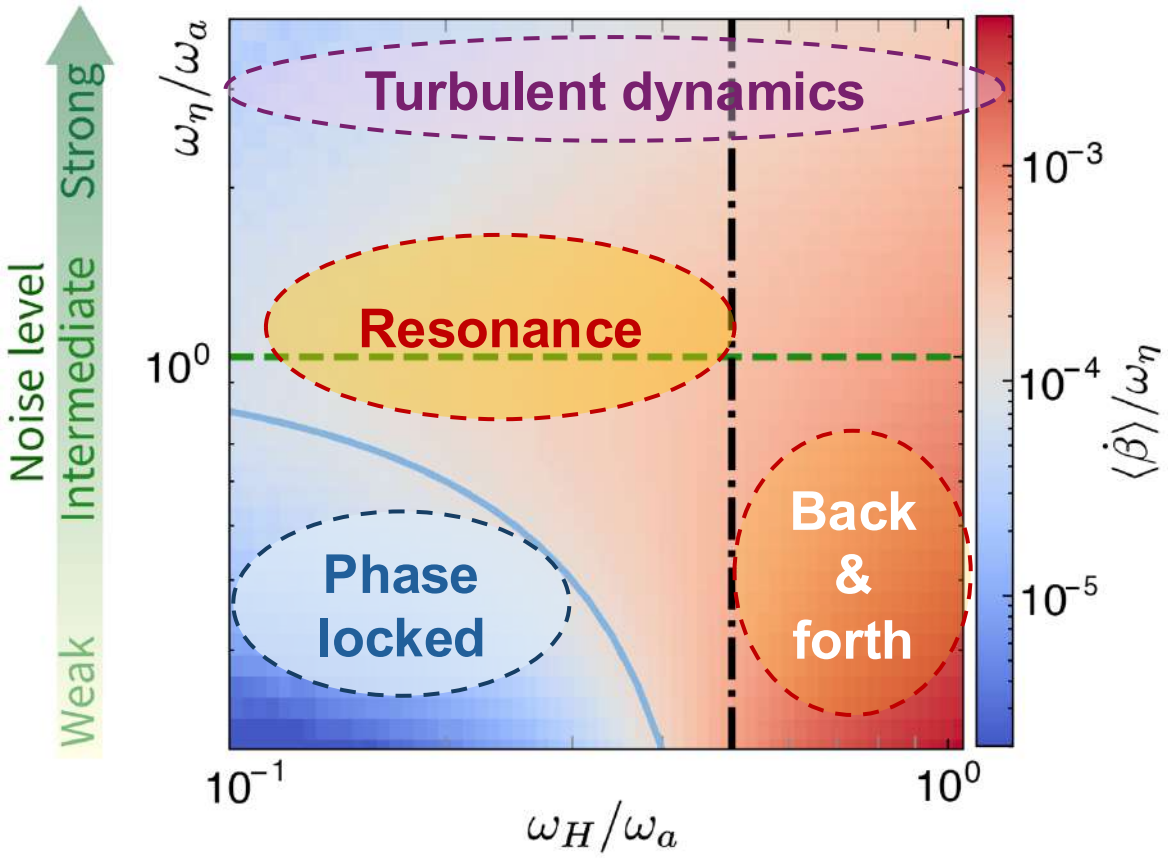


Phase diagram

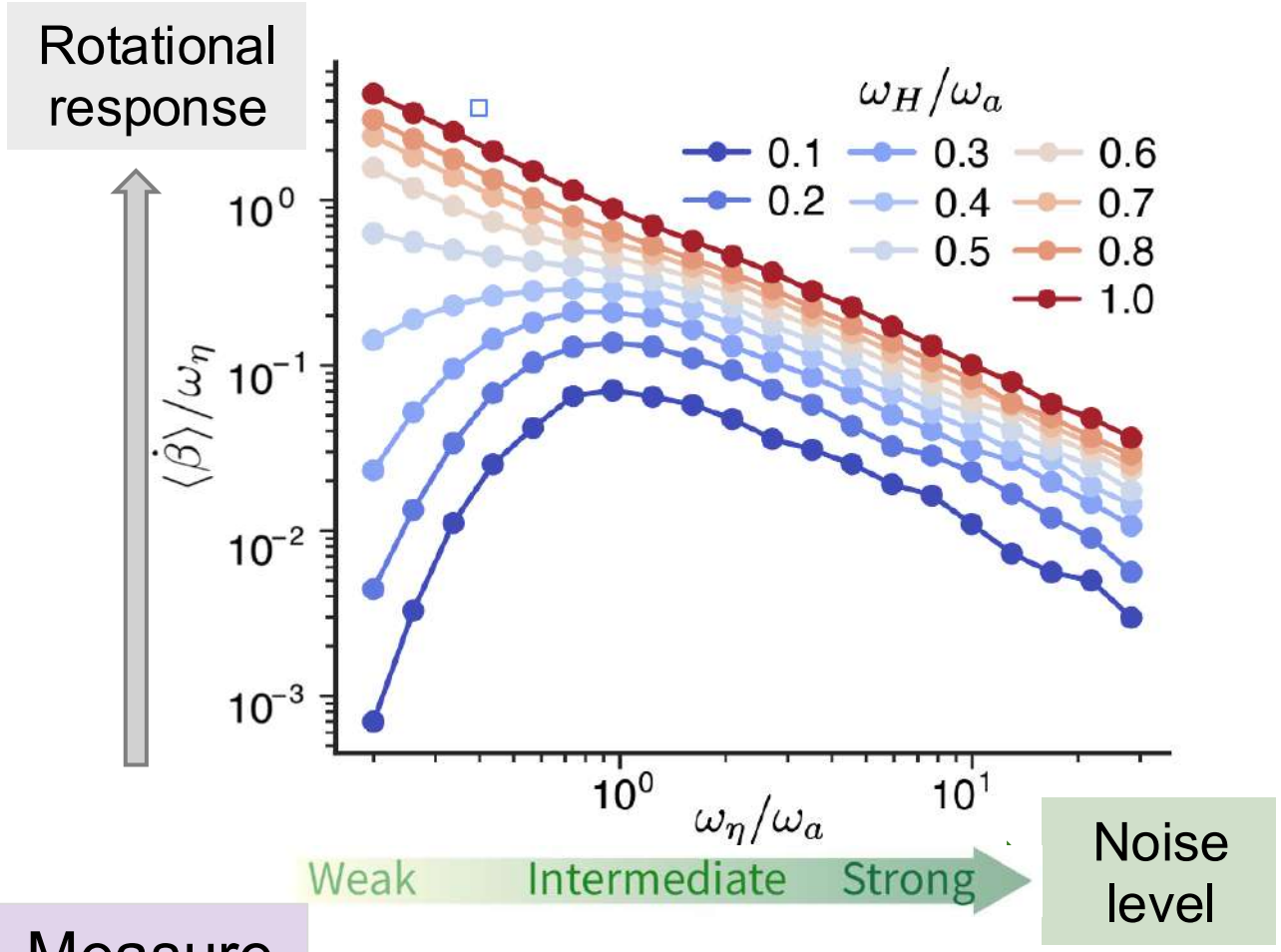


Non-monotonic?
Why?

Stochastic resonance



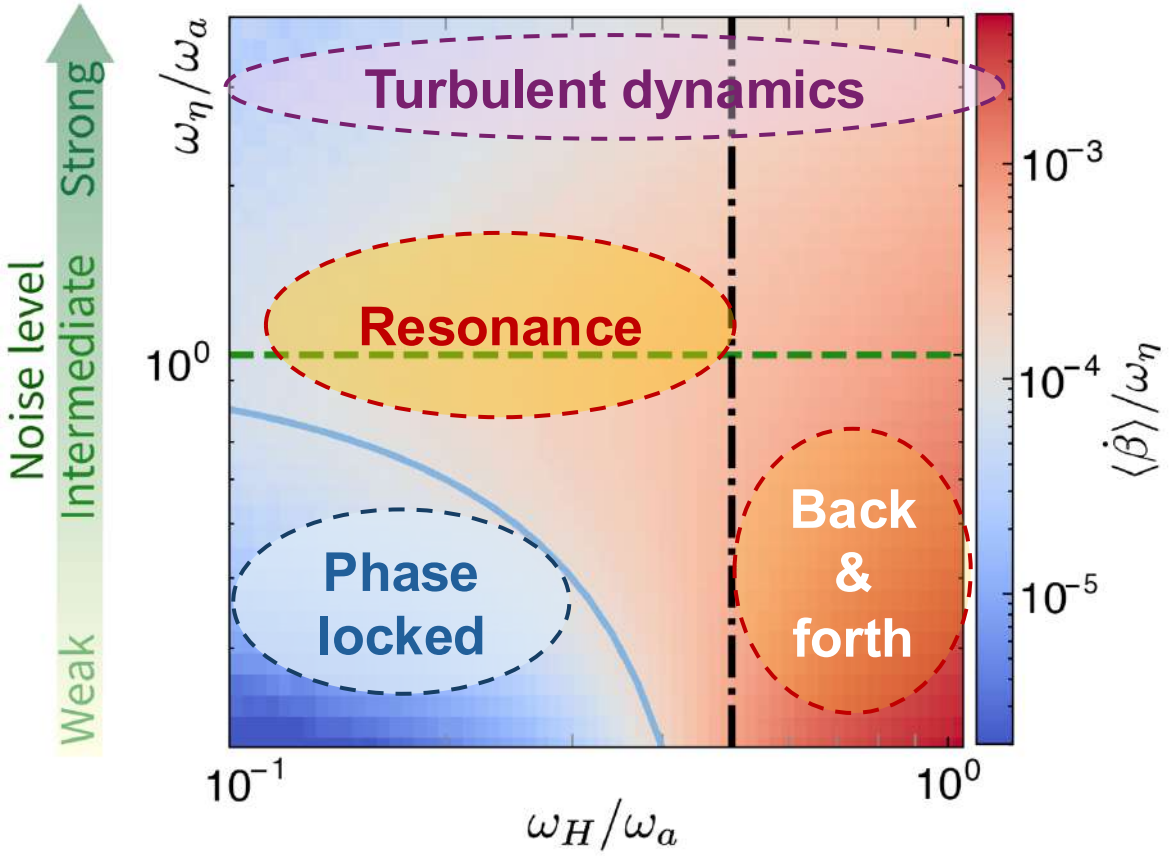
Resonance arises at the transition from the "phase-locked" regime to the "turbulent-dynamics" regime.



Measure turbulent vorticity

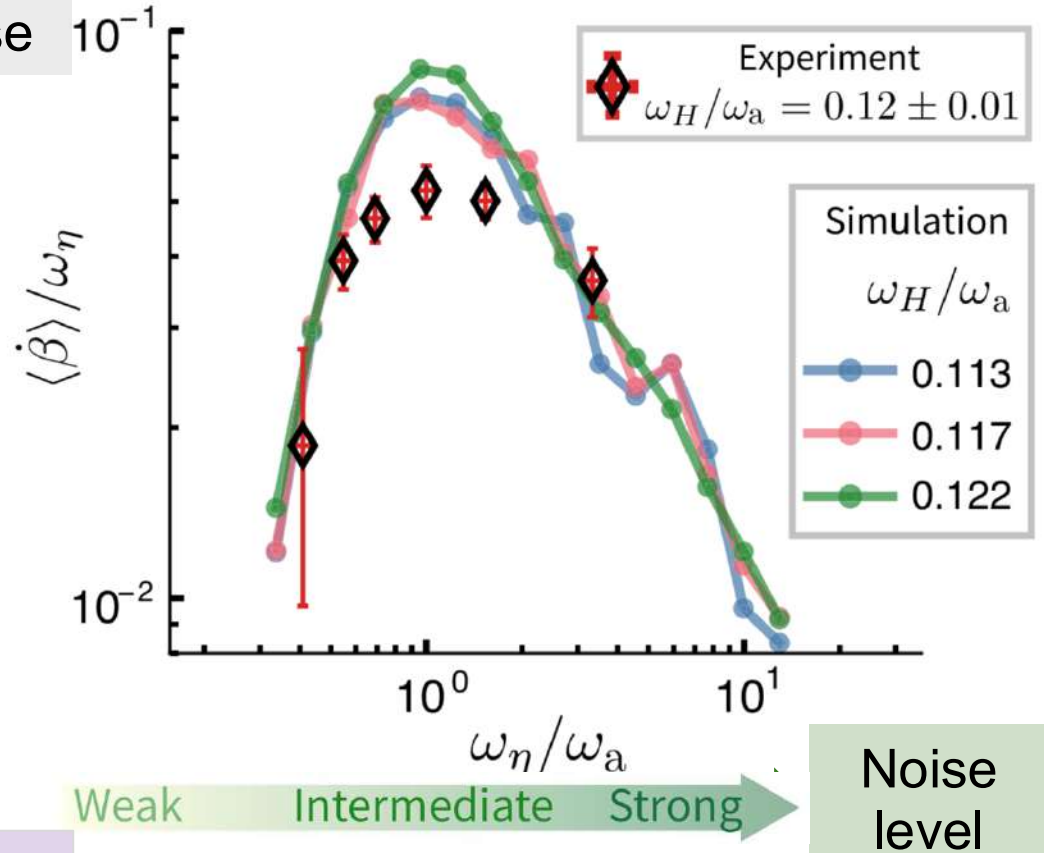
- Resonant peak!
- Peak position: $\omega_a \sim \omega_\eta$.

Stochastic resonance



Resonance arises at the transition from the "phase-locked" regime to the "turbulent-dynamics" regime.

Rotational response



Noise level

Measure turbulent vorticity

- Resonant peak!
- Peak position: $\omega_a \sim \omega_\eta$.

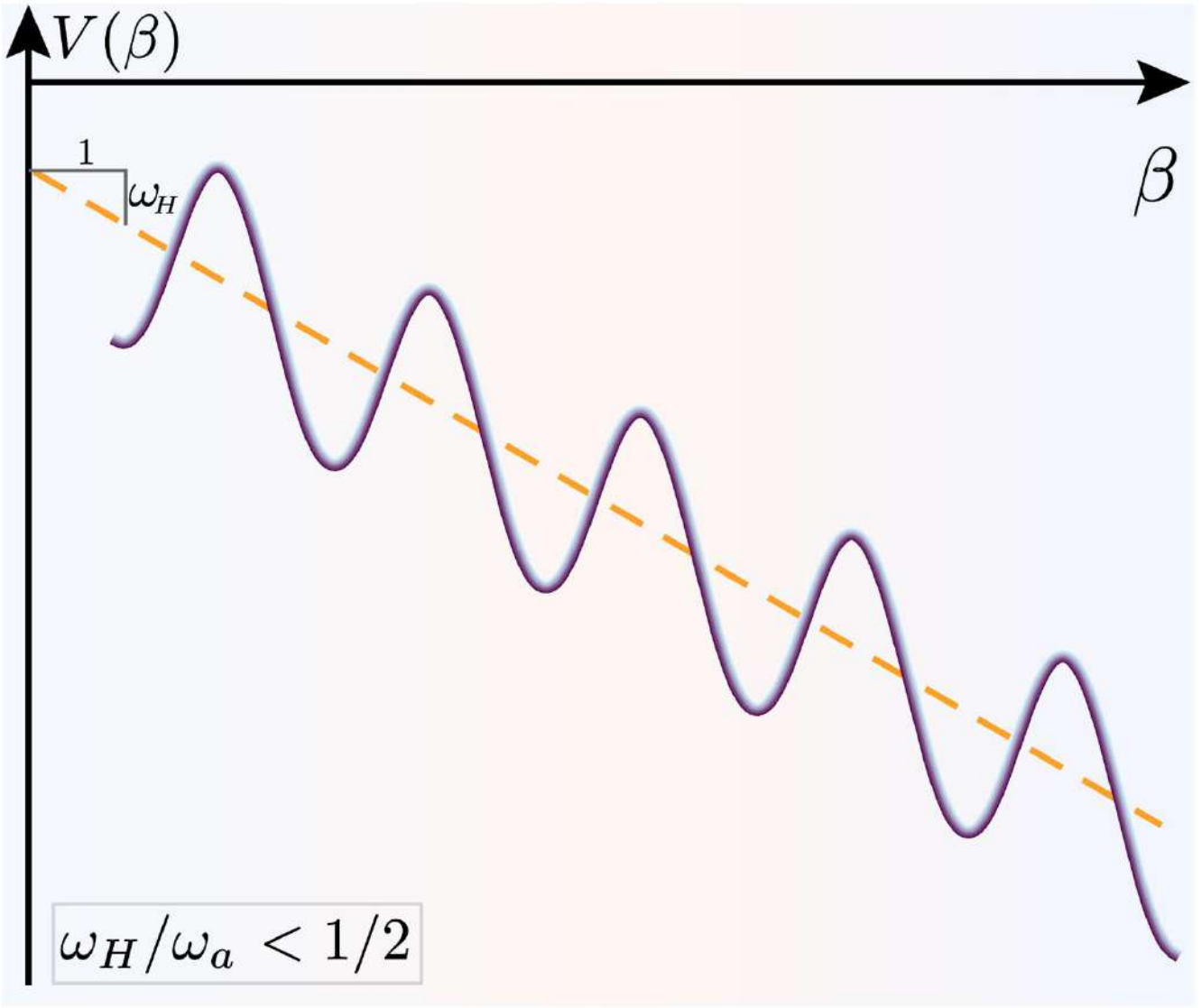
Stochastic resonance: mechanism

$$\dot{\beta} = \omega_H - \frac{\omega_a}{2}\sin 2\beta - \omega_{f,z}$$

$$\dot{\beta} = -\frac{dV(\beta)}{d\beta} - \omega_{f,z}$$

Potential well

$$V(\beta) = -\left(\omega_H \beta + \frac{\omega_a}{4}\cos 2\beta\right)$$



See also: Benzi, R. (2010). Stochastic resonance: from climate to biology. *Nonlinear Processes in Geophysics*, *17*(5), 431-441.

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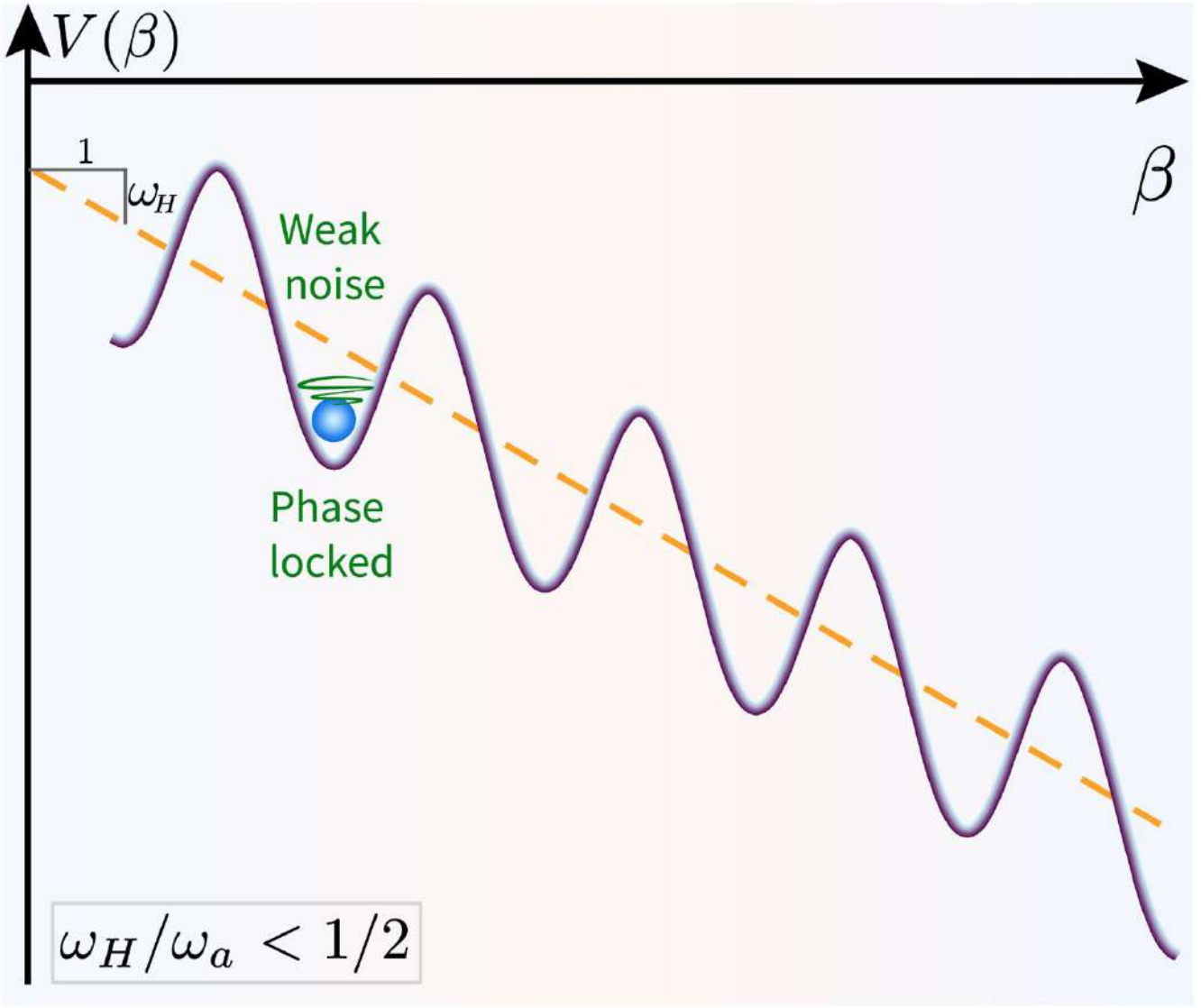

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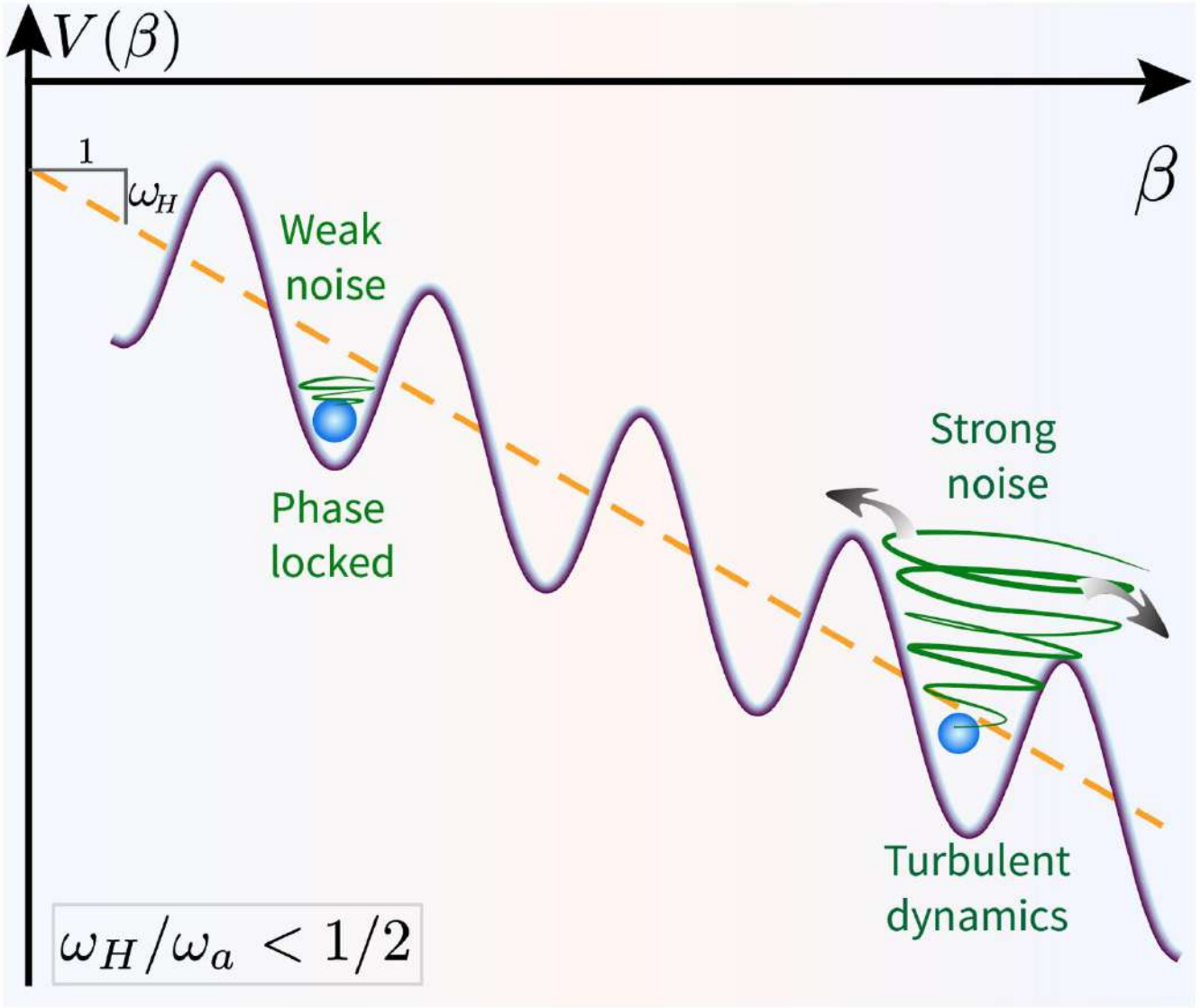

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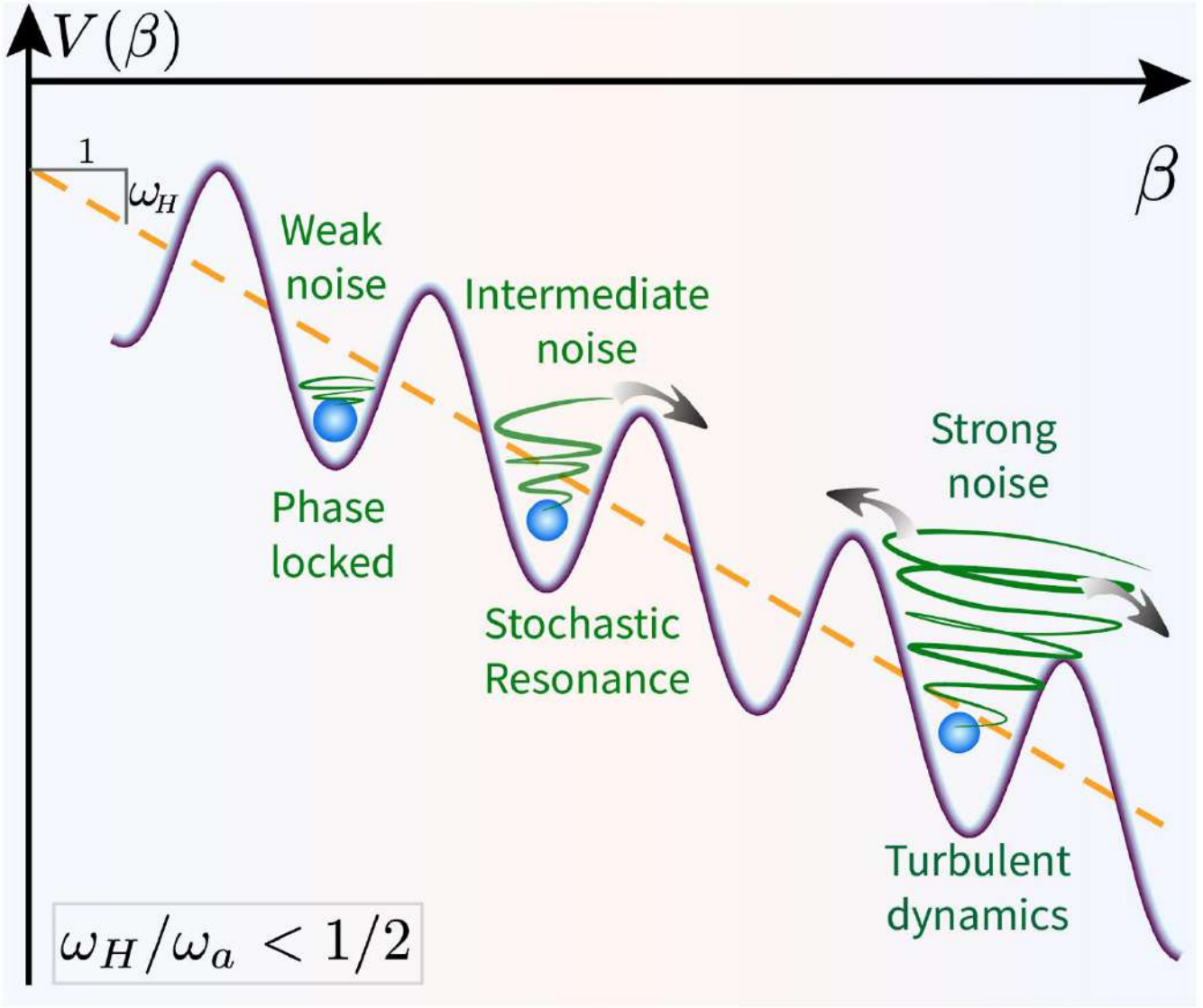
Potential well

$$V(\beta) = -\left(\omega_H \beta + \frac{\omega_a}{4}\cos 2\beta\right)$$

- As turbulent vorticity aids the particle in “escaping” the magnetic field.



See also: Benzi, R. (2010). Stochastic resonance: from climate to biology. *Nonlinear Processes in Geophysics*, *17*(5), 431-441.

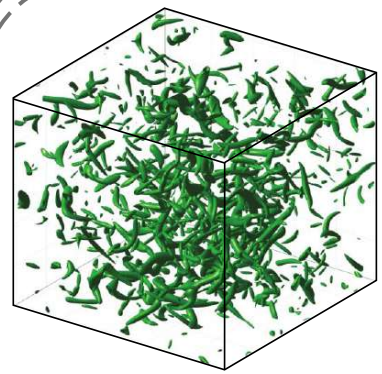
Summary of topic 2

- Three distinct regimes: phase-locked, back-and-forth, turbulent dynamics.
- Stochastic resonance of particle rotational dynamics in turbulence ($\omega_a \sim \omega_\eta$).
- Forced magnetic bubbles can be used as probes for vorticity in turbulence.

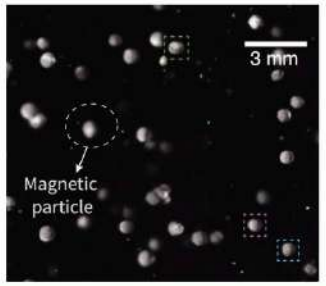
More information:

Ziqi Wang, Xander M. de Wit, Roberto Benzi, Chunlai Wu, Rudie PJ Kunnen, Herman JH Clercx, and Federico Toschi. "**Stochastic resonance of rotating particles in turbulence**." Nature Communications 16, no. 1 (2025): 10376.

Highlights

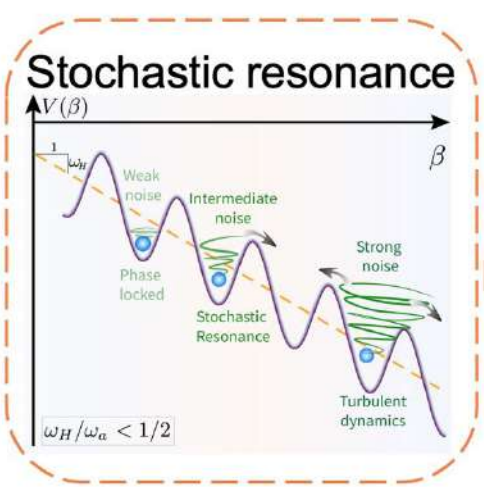


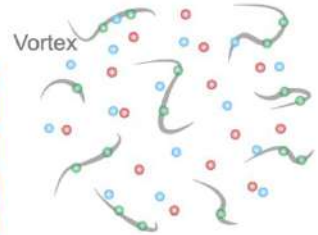
"turbulence microscope"

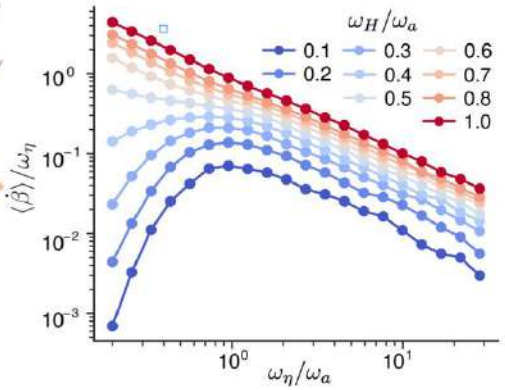


ω_H

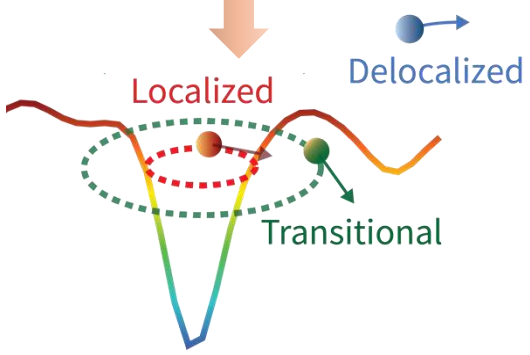
H = H h

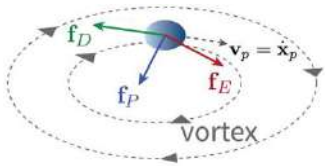




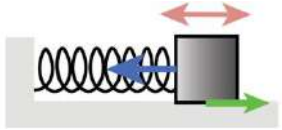


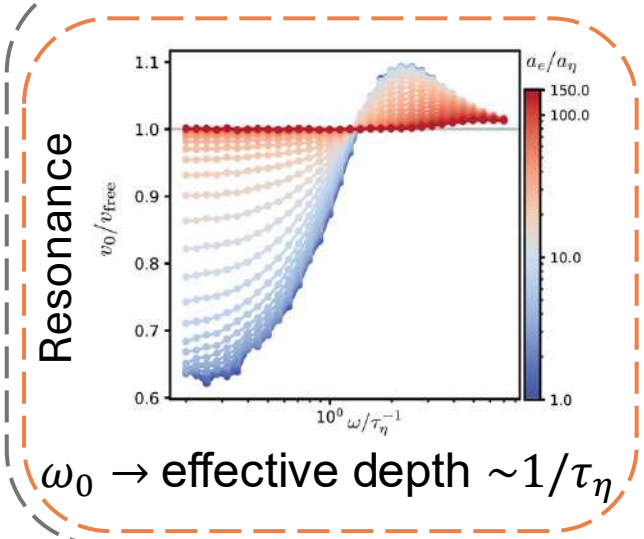
External force





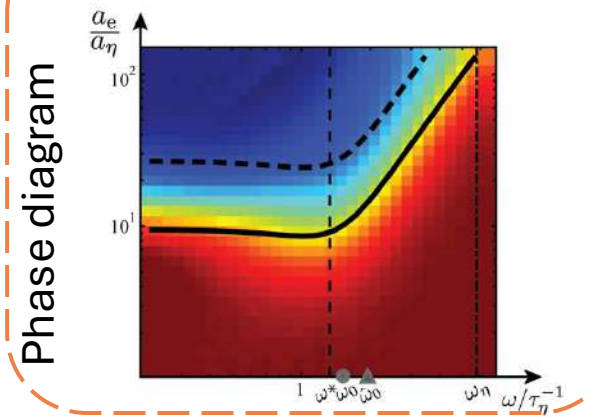
Analogy





Kolmogorov scale

effective radius: 4η



Thanks!


Xander M. de Wit


Roberto Benzi


Chunlai Wu


Rudie PJ Kunnen


Herman JH Clercx


Federico Toschi

- Ziqi Wang, Xander M. de Wit, and Federico Toschi. "**Localization–delocalization transition for light particles in turbulence**." PNAS, 121.38 (2024): e2405459121.
- Ziqi Wang, Xander M. de Wit, Roberto Benzi, Chunlai Wu, Rudie PJ Kunnen, Herman JH Clercx, and Federico Toschi. "**Stochastic resonance of rotating particles in turbulence**." Nature Communications 16, no. 1 (2025): 10376.